Equations

2

Unit Overview

In this unit you will extend your knowledge of equations as you study several ways to solve multistep equations, and you will apply your understanding to application problems. You will model and solve problems involving systems of equations.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- legend
- persuasive
- coincide

Math Terms

- evaluate
- consecutive terms
- constant difference
- linear
- slope
- discrete data
- continuous data
- coefficient
- constant term
- slope-intercept form
- direct variation
- system of linear equations
- solution to a system of equations

ESSENTIAL QUESTIONS

? How can you write and solve linear equations?

? How can graphs be used to interpret solutions of real-world problems?

EMBEDDED ASSESSMENTS

These assessments, following Activities 10, 13, and 15, will give you an opportunity to demonstrate how you can use your understanding of equations to solve real-world problems.

Embedded Assessment 1:

Embedded Assessment 2:

Embedded Assessment 3:

UNIT 2

Getting Ready

1. What is the difference between an expression and an equation?

2. Write an expression for the following:
   a. one more than twice a number
   b. a number decreased by six
   c. two-thirds of a number

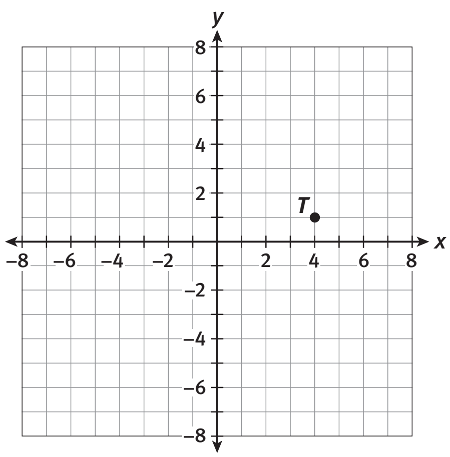
3. Evaluate the following expressions if $x = 4.1$ and $y = 2.3$.
   a. $2x + 3$
   b. $16 - 5y$
   c. $x + y$

4. Complete the table below so that the data is linear.

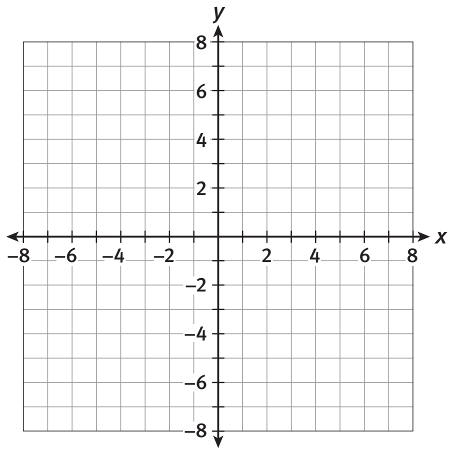
Input	Output
1	
2	6
3	10
4	

5. A line contains the points (2, 5) and (4, 6):
   a. Where does it cross the x-axis?
   b. Where does it cross the y-axis?

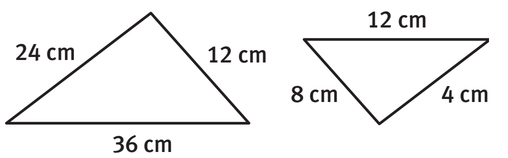
6. Use the graph below to:
   a. Plot and label the points $R(3, 5)$ and $S(6, 0)$.
   b. Give the coordinates of point T.



7. Draw a horizontal line that contains the point (2, 3) and a vertical line that contains (1, 4).



8. Determine if the triangles pictured below are similar, and give a reason for your conclusion.



Writing Expressions

Pebbles in the Sand

Lesson 9-1 Representing Patterns

Learning Targets:

- Identify and represent patterns using models, tables, and expressions.
- Write and evaluate algebraic expressions that represent patterns with constant differences.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Use Manipulatives, Think-Pair-Share

People have been investigating number patterns for thousands of years. ***Legend*** has it that Pythagoras and his students arranged pebbles in the sand to represent number patterns. One pattern they studied is shown below.

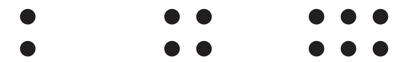


Figure 1 Figure 2 Figure 3

1. Draw the fourth, fifth and sixth figures.

2. Organize the number of pebbles in each figure into a table.

Figure Number	Number of Pebbles

3. Extend the pattern to determine how many pebbles are in the 10th figure.

4. Describe the patterns you observe in the pebble drawings and the table in words.

5. How many pebbles are in the 53rd figure? Explain your reasoning.

6. Write a numeric expression using the number 3 for the number of pebbles in the third figure.

My Notes

ACADEMIC VOCABULARY

A ***legend*** is a story handed down by tradition that is popularly regarded as historical but unverified.

My Notes

MATH TIP

An *expression* is a mathematical phrase using numbers or variables or both. $1 + 1$ and $3x - 5$ are examples of expressions.

MATH TERMS

You do not solve an expression; you **evaluate** it for a specific value. To do this, substitute a value for the variable and simplify.

7. Write a similar numeric expression using the number 7 for the number of pebbles in the seventh figure.

8. Let n represent the figure number.

a. Use n to write an expression that could be used to determine the number of pebbles in figure n.

b. What value would you substitute for n to determine the number of pebbles in the third figure?

c. Check to see that your expression from part a is correct by ***evaluating*** it for the value you chose in part b.

d. Use your expression to determine the number of pebbles in the 100th figure.

Check Your Understanding

A pattern of small squares is shown. Use the pattern to answer Items 9–13 that follow.

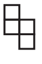


Figure 1

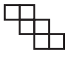


Figure 2

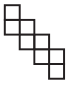


Figure 3

9. How many small squares are in each figure?

10. Draw the fourth, fifth, and sixth figures and determine the number of small squares in each figure.

11. Create a table to organize the number of squares in each figure into a table.

12. Describe in words the patterns you see in the square pattern and in the table.

13. How many squares would be in the 10th figure? Explain your reasoning.

My Notes

14. Another pebble arrangement is shown below.

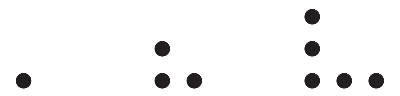


Figure 1 Figure 2 Figure 3

a. Draw the fourth, fifth, and sixth figures.

b. Organize the number of pebbles in each figure in the table below.

Figure Number	Pebbles	Difference in Number of Pebbles
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

c. Describe any patterns you observe in the pebble drawings and in the table above.

15. Subtract ***consecutive terms*** in the pebbles column and record this information in the last column in the table.

16. Reason quantitatively. How does the ***constant difference*** in the new column relate to the patterns you observed?

CONNECT TO AP

The ability to identify patterns allows you to understand and describe different types of functions and provides a foundation for understanding rate of change in calculus.

MATH TERMS

Consecutive terms follow each other directly in a sequence. In the table below, 5 and 8 are *consecutive terms*. There is a **constant difference** between the terms in this table that is equal to 3.

n	term	
1	5	3
2	8	3
3	11	3
4	14	

My Notes

17. The number of pebbles in a specific figure can be written using repeated additions of the constant difference. For example, the third figure is $1 + 2 + 2$ or $1 + 2(2)$.

Figure Number	Pebbles	Expression Using Repeated Addition
1	1	$1 + 2(0)$
2	$1 + 2$	$1 + 2\ (1)$
3	$1 + 2 + 2$	
4		
5		

a. Write the number of pebbles in the fourth and fifth figure using repeated addition of the constant difference.

b. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in figure n.

c. What value would you substitute for n to determine the number of pebbles in the third figure?

d. Check to see that your expression from part a is correct by evaluating it when $n = 5$.

e. Use your expression to determine the number of pebbles in the 100th figure.

My Notes

Check Your Understanding

18. A pattern of pebbles is shown. Use the pattern to answer parts a–c.

a. Draw the fourth, fifth, and sixth figures.
b. Create a table to organize the number of pebbles in each figure for the first six figures.
c. What is the constant difference?
d. Write the number of pebbles for each figure using repeated addition of the constant difference.
e. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in figure n.

LESSON 9-1 PRACTICE

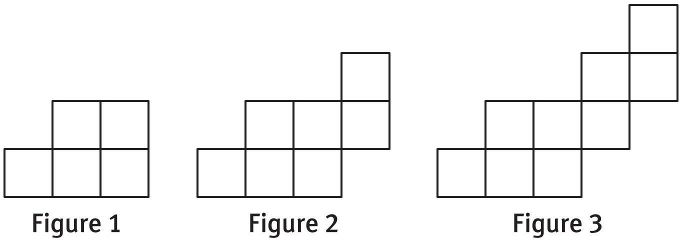
19. Check to see that your expression from Item 18 part e is correct by evaluating when $n = 6$.

20. Use your expression from Item 18 part e to determine the number of pebbles in the 51st figure.

21. **Critique the reasoning of others.** The expression that represents the number of squares in the nth figure of a pattern is given by $2 + 3(n - 1)$. Mia claims the constant difference is 2. Do you agree? Explain.

My Notes

22. A pattern of small squares is shown.



a. Draw the fourth, fifth, and sixth figures.
b. Create a table showing the number of the figure and the number of squares in each figure.
c. Describe the patterns you observe in the square drawings and in your table.
d. What is the constant difference?
e. How many small squares are in the first figure?
f. Let n represent the figure number. Use n to write an expression that could be used to determine the number of squares in the nth figure.

23. Compare the expression you found in Item 22 part f with the expression you found in Item 17 part b. How are they the same? How are they different?

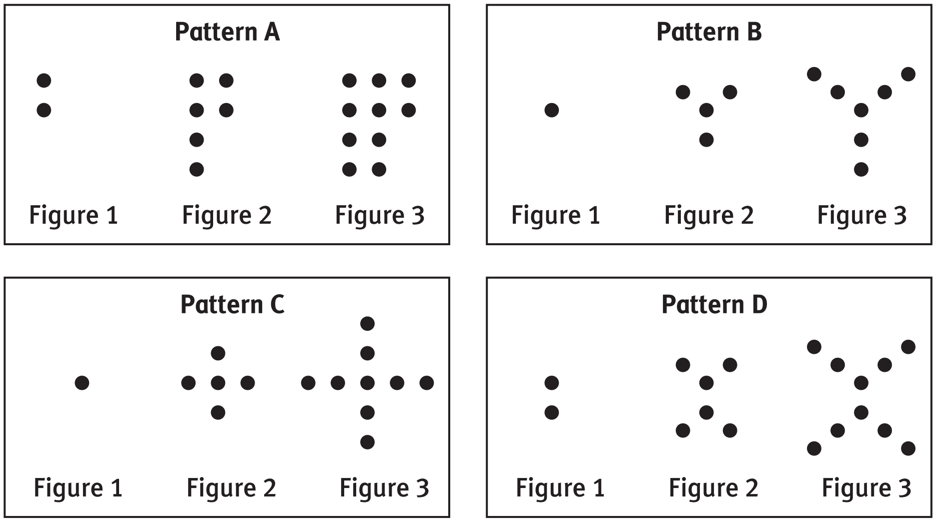
My Notes

Learning Targets:

- Identify patterns that do not have a constant difference.
- Write and evaluate algebraic expressions that represent patterns that do not have a constant difference.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Discussion Group, Group Presentation

1. Four different pebble patterns are shown. Your teacher will assign one to your group. Use your selected pattern to answer parts a–e that follow and then prepare a group presentation of your results.



a. Draw a few additional figures and then organize the information in a table. Identify the constant difference.

My Notes

b. Describe the pattern in words.

c. **Reason abstractly**. Write an expression using the variable n that could be used to determine the number of pebbles in figure n.

d. Use your expression to determine the number of pebbles in the 10th, 53rd, and 200th figures.

Figure	10th	53rd	200th
A and D			
B			
C			

e. For the pattern you selected, is it possible to have a figure with 100 pebbles? Explain your reasoning.

2. Based on the class's work for Item 1, how does the constant difference in a pebble pattern relate to the algebraic expression that can be written to describe the pattern?

My Notes

Check Your Understanding

Tables representing two pebble patterns are shown below.

Pebble Pattern A					
Figure	1	2	3	4	5
Pebbles	3	7	11	15	19

Pebble Pattern B					
Figure	1	2	3	4	5
Pebbles	2	7	12	17	22

3. What is the constant difference for each pebble pattern shown in the tables?

4. For each pebble pattern, use the variable n to write an expression for the nth figure.

5. How many pebbles are in the 50th figure for each pebble pattern?

6. Both pebble patterns have 7 pebbles in Figure 2. If the patterns continue, will they ever have the same number of pebbles as another figure? Explain your reasoning.

7. The Pythagoreans also studied the following pebble pattern.

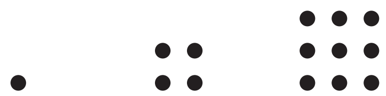


Figure 1 Figure 2 Figure 3

a. How many pebbles are there in the fourth, fifth, and sixth figures?

b. Does this pattern have a constant difference? Explain your response.

c. Describe the pattern in words.

d. How many pebbles are there in the 10th figure? How did you determine your answer?

My Notes

e. How many pebbles are there in the 40th figure? In the *n*th figure?

f. The Pythagoreans called the numbers in this pattern the square numbers. Why do you think this is so?

8. The numbers in the pebble pattern shown below are called the rectangular numbers.

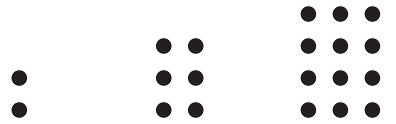


Figure 1 Figure 2 Figure 3

a. How many pebbles are in the fourth, fifth, and sixth figures?

b. Describe the pattern in words. Is there a constant difference? Explain your response.

c. Describe how to find the number of pebbles in the 10th figure.

d. How many pebbles are there in the 30th figure? In the nth figure?

9. The Pythagoreans called the numbers represented by the pebbles in this pebble pattern the triangular numbers.

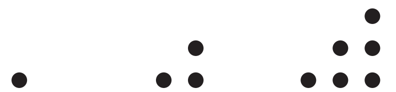


Figure 1 Figure 2 Figure 3

a. Why do you think the Pythagoreans called these numbers triangular?

My Notes

b. How is the triangular number pebble pattern related to the pebble pattern of the rectangular numbers?

c. Use your response to part b to write an algebraic expression for the number of pebbles in the nth triangular number.

d. Verify your expression by substituting $n = 4$. Is the result the number of pebbles in the fourth triangular number?

e. Use your expression to predict the number of pebbles in the 30th triangular number.

Check Your Understanding

10. Is the number 72 a square number, rectangular number, or triangular number? Explain your reasoning.

Use the figures below to answer Items 11–15.

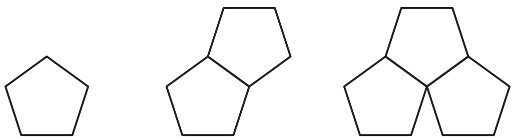


Figure 1 Figure 2 Figure 3

11. Assume each side of each pentagon is 1 cm. What is the perimeter of each figure shown?

12. Draw the next three figures and determine the perimeter of each.

13. Organize the results of Items 11 and 12 in a table. What would be the perimeter of the 10th figure? Explain your reasoning.

14. Use n to represent the figure number. Write an expression that could be used to determine the perimeter of the nth figure.

15. Use your expression to determine the perimeter of the 50th figure.

My Notes

LESSON 9-2 PRACTICE

16. **Critique the reasoning of others.** Nate claims 56 is a rectangular number because a rectangle with base 4 and height 14 can be formed. What is his error?

17. What is the sixth square number? How did you get your answer?

18. Use the expression from Item 9 part c to show that 45 is a triangular number.

19. Use the figures below to answer parts a–d. Assume each figure is a regular octagon with sides of 1 foot.

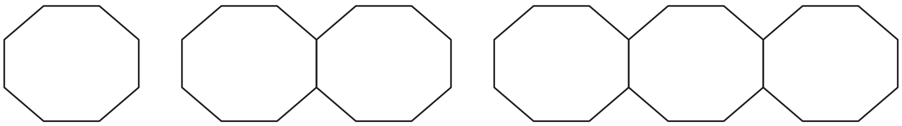


Figure 1 Figure 2 Figure 3

   a. What is the perimeter of each figure?
   b. Draw the next three figures and determine the perimeter of each.
   c. Use n to represent the figure number. Write an expression that could be used to determine the perimeter of the nth figure.
   d. Verify the expression in part c by substituting $n = 6$.

20. **Model with mathematics.** Octagonal blocks are being used to make a walkway along a garden. Use your expression from Item 19 part c to find the perimeter of the walkway if 30 octagonal blocks, each side 1 foot long, are used for the walkway.

ACTIVITY 9 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 9-1

1. Use the figures below to answer parts a–c.

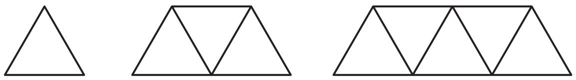


Figure 1 Figure 2 Figure 3

   a. What is the perimeter of each figure shown? Assume each side is 1 unit.
   b. Draw the next three figures.
   c. What would be the perimeter of the 10th figure? Justify your response.

2. Write an expression, using the variable n, that could be used to determine the perimeter of the nth figure in Item 1. Use the expression to determine the perimeter of the 50th figure.

3. A pattern of pebbles is shown.

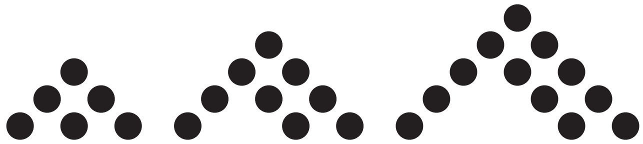
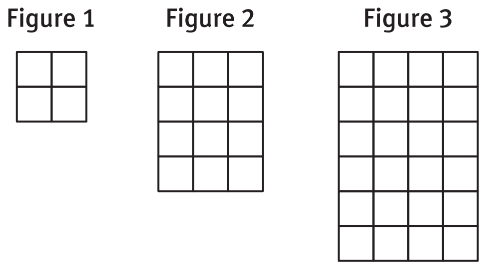


Figure 1 Figure 2 Figure 3

   a. Draw the fourth, fifth, and sixth figures.
   b. Create a table showing the number of the figure and the number of pebbles in each figure.
   c. Describe the patterns you observe in the pebble drawings and the table in words.
   d. What is the constant difference?
   e. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in the nth figure.
   f. Use the expression in part e to determine the number of pebbles in the 51st figure.

Lesson 9-2

4. Use the pattern of unit squares shown to answer parts a–c.



a. What is the area of each figure if each small square has an area of 1 unit?

b. Draw the next three figures in the pattern and determine the area of each.

c. What would be the area of the 10th figure? Justify your response.

5. Write an expression that could be used to determine the area of the *n*th figure in Item 4. Use the expression to determine the area of the 35th figure.

6. Use the figures from Item 4 to answer parts a–c.

a. What is the perimeter of each of the six figures?

b. Is there a constant difference? Explain.

c. Write an expression that could be used to find the perimeter of the *n*th figure.

d. Use the expression in part c to find the perimeter of the 35th figure.

MATHEMATICAL PRACTICES
Model with Mathematics

7. Use the expression $\frac{n(n+1)}{2}$.

a. Create a pattern using circles or dots and show the first three figures.

b. Determine the number of circles or dots in the 10th figure. Explain how you determined the number of dots in this figure.

Solving Equations

ACTIVITY 10

Bags and Cubes
Lesson 10-1 Solving Linear Equations with Models

My Notes

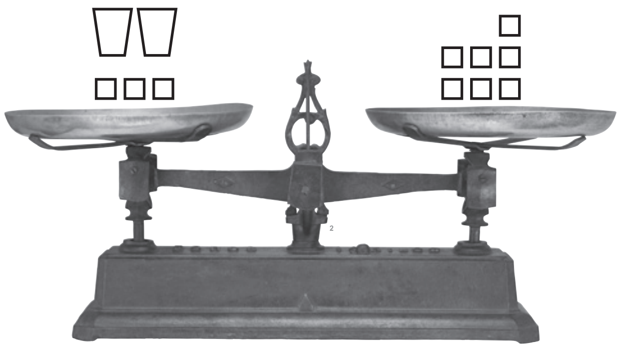
Learning Targets:

- Solve linear equations with rational number coefficients.
- Solve linear equations by using the Distributive Property and collecting like terms.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Visualization, Create Representations, Use Manipulatives, Sharing and Responding

Solving linear equations is a way to solve problems in the real world. Creating a model of a problem can help you break the problem down into parts that you can visualize.

Some small lightweight plastic bags contain an equal number of centimeter cubes. They are placed on a balance scale with some additional cubes.



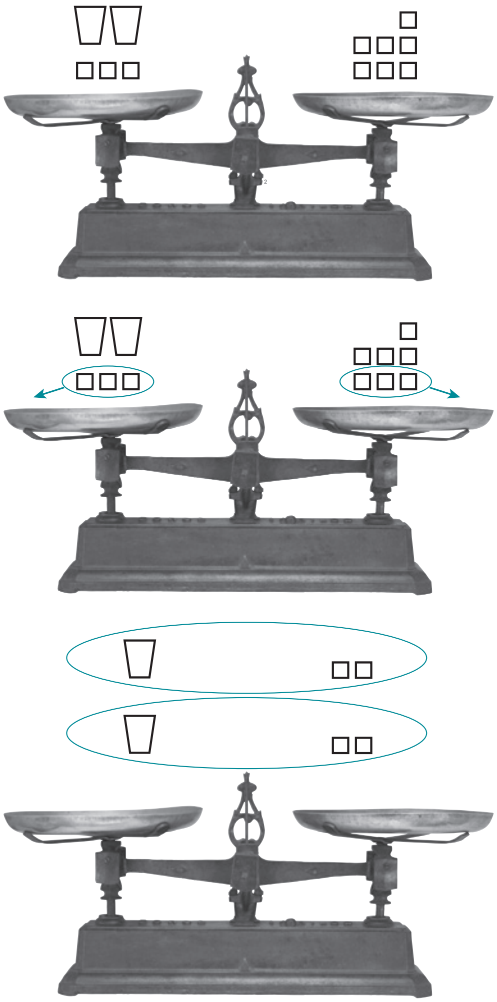
1. If the scale is balanced, how many cubes must be in each bag? Explain how you determined your answer.

2. Write an equation to represent the diagram shown above. Let x represent the number of cubes in a bag.

My Notes

3. One way to solve the problem involves removing equal amounts from both sides and then regrouping the remaining cubes. The diagrams below illustrate this process. The first diagram is the original diagram. Write an explanation for both the second and third diagrams to the right of each.

Diagram **Explanation**

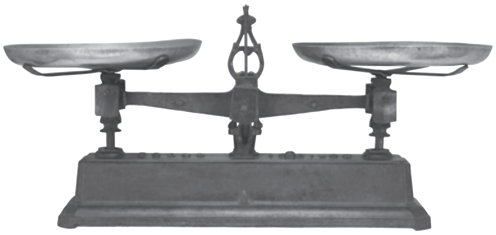


4. From the diagrams shown above, how many cubes are in each bag?

5. Model each of the following problems using bags and cubes, then determine how many cubes are in a bag. Record your work using a diagram like the balance scale. Write an equation for the original problem, an explanation for each step, and the solution. Let x be the number of cubes in a bag.

My Notes

a. If 3 bags are on one side of the scale and 12 cubes are on the other side, how many cubes must be in each bag to maintain the balance?



b. If 3 bags and 10 cubes are on one side of the scale and 16 cubes are on the other side, how many cubes must be in each bag to maintain the balance?

The solution to a bags-and-cubes equation can be determined algebraically by using inverse operations to solve the equation.

Example A

Solve $3x + 4 = 7$.

Step 1: Use inverse operations. Subtract 4 from both sides.

$$3x + 4 - 4 = 7 - 4$$
$$3x = 3$$

Step 2: Use inverse operations. Divide both sides by 3.

$$\frac{3x}{3} = \frac{3}{3}$$
$$1x = 1$$

Step 3: Use the Multiplicative Identity Property to isolate the variable.

$$x = 1$$

Solution: In the equation, $3x + 4 = 7$, $x = 1$.
Substitute 1 for x in the equation to verify: $3(1) + 4 = 3 + 4 = 7$.

MATH TIP

Equations can also be solved by working vertically.

$$\begin{array}{rcl} 3x + 4 & = & 7 \\ -4 & & -4 \\ \hline \frac{3x}{3} & = & \frac{3}{3} \\ x & = & 1 \end{array}$$

My Notes

Try These A

Solve each of the following algebraically, using inverse operations.

a. $2x + 0.4 = 18$

b. $-\frac{1}{3}x - 4 = 12$

c. $1.5 = 5x - 2$

d. $3x + 10 = 16$

e. $14 = 3x - 2$

6. **Construct viable arguments.** How do the steps to solve an equation algebraically represent the bags-and-cubes process from previous items?

7. Use properties of operations or other explanations to describe each step in solving the equation $3(x - 2) = 8$.

$3(x - 2) = 8$	Original equation
$3x - 6 = 8$	a.
$3x - 6 + 6 = 8 + 6$	b.
$3x = 14$	c.
$\frac{3x}{3} = \frac{14}{3}$	d.
$1x = \frac{14}{3}$	e.
$x = \frac{14}{3}$	f.

MATH TIP

The *Distributive Property* is used to multiply a factor by both addends in parentheses. For example:

$$a(b + c) = a(b) + a(c)$$

Conversely, the Distributive Property is used to factor and combine addends. For example:

$$ab + ac = a(b + c)$$

My Notes

Check Your Understanding

8. Kathleen is spending summer vacation hiking in Canada and wants to dress appropriately for the day's hike. The local news reports that the high temperature is expected to be 20° Celsius. Use the formula $F = \frac{9}{5}C + 32$ where C is the degrees Celsius and F is the degrees Fahrenheit to determine the local temperature in degrees Fahrenheit.

9. Solve for the variable in the equation: $-3x + 11 = 26$.

10. Solve $\frac{5}{2}y = 10$ for y.

11. Solve for p in the equation $0.3p + 1.75 = 6.25$.

LESSON 10-1 PRACTICE

12. What is the value of x in the equation $7x - 30 = 180$?

13. Solve $-\frac{3}{4}x = 36$ for x.

14. **Make sense of problems.** Describe each step in solving the equation $3y + 7 - 5y = 13$.

$3y + 7 - 5y = 13$	Original equation
$-2y + 7 = 13$	a.
$\underline{\quad -7 \quad -7}$	
$-2y = 6$	b.
$\frac{-2y}{-2} = \frac{6}{-2}$	
$y = -3$	c.

15. Solve for x in the equation $\frac{(x+2)}{7} = 14$.

16. Solve the equation algebraically: $-3x + 6 = -15$.

My Notes

Learning Targets:

- Use linear equations with one variable to model and solve real-world and mathematical problems.
- Solve linear equations with variables on both sides of the equation by using the Distributive Property and collecting like terms.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Think-Pair-Share, Create Representations, Guess and Check

Often, equations have variables on both sides of the equal sign. This type of equation requires multiple steps to solve. Many times these equations are most efficiently solved algebraically. The following is an example of an algebraic solution to a multistep equation.

Example A

Solve $12x + 5 = 6x + 17$.

Step 1: Use inverse operations to combine like variables on one side of the equation. Subtract $6x$ from both sides.

$$\begin{array}{r} 12x + 5 = 6x + 17 \\ \underline{-6x \qquad\; -6x \qquad} \\ 6x + 5 = 17 \end{array}$$

Step 2: Use inverse operations. Subtract 5 from both sides.

$$\begin{array}{r} 6x + 5 = 17 \\ \underline{-5 \quad -5} \\ 6x = 12 \end{array}$$

Step 3: Use inverse operations. Divide both sides by 6.

$$\frac{6x}{6} = \frac{12}{6}$$
$$1x = 2$$

Step 4: Use the Multiplicative Identity Property to isolate the variable.

$$x = 2$$

Solution: In the equation $12x + 5 = 6x + 17$, $x = 2$.
Substitute 2 for x in the equation to verify: $12(2) + 5 = 6(2) + 17$

$$24 + 5 = 12 + 17$$
$$29 = 29$$

Try These A

Solve each equation and provide an explanation for each step.

a. $2 - \frac{3}{2}(x + 1) = 6x - 9$

b. $2(x + 0.7) = 6(x - 0.8)$

c. $2x - 10 = \frac{1}{3}(x - 9)$

d. $8 - x = 3x + 2(x - 4)$

e. $\frac{1}{2}(18x - 8) = 6x - \frac{7}{2}$

My Notes

Equations can often be written to model mathematical and real-world situations. Solving these equations provides solutions to the problems.

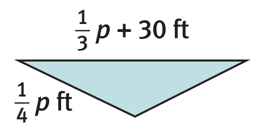
Example B

Kurt is surveying a triangular plot of land. He was told that the longest side of the region is 30 feet longer than $\frac{1}{3}$ the perimeter, and the shortest side is $\frac{1}{4}$ the perimeter. He also knows that the longest side is 60 feet longer than the shortest side of the region. What is the perimeter of the parcel of land?

Step 1: Create a model.

longest side = shortest side + 60 ft

$\frac{1}{3}p + 30\text{ ft} = \frac{1}{4}p + 60\text{ ft}$



Step 2: Solve the equation.

$\frac{1}{3}p + 30 = \frac{1}{4}p + 60$

$\frac{1}{3}p = \frac{1}{4}p + 30$

$\frac{1}{12}p = 30$

$p = 360$ feet

Solution: The perimeter of the triangular plot of land is 360 feet.

Example C

Find a number such that one-half the number decreased by two is equal to one-third the number increased by one.

Step 1: Create a model. Let x = the number.

half the number $- 2 = \frac{1}{3}$ the number $+ 1$

$\frac{1}{2}x - 2 = \frac{1}{3}x + 1$

Step 2: Solve the equation.

$\frac{1}{2}x - \frac{1}{3}x - 2 = \frac{1}{3}x + 1 - \frac{1}{3}x$

$\frac{1}{6}x - 2 + 2 = 1 + 2$

$\frac{1}{6}x = 3$

$x = 18$

Step 3: Check the solution.

$\frac{1}{2}x - 2 = \frac{1}{3}x + 1$

$\frac{1}{2}(18) - 2 = \frac{1}{3}(18) + 1$

$9 - 2 = 6 + 1$

$7 = 7$

Solution: The number is 18.

Try These B–C

Create a model and solve.

a. Two-fifths a number increased by six is equal to three-fourths the number decreased by 8.

My Notes

b. Marcus has created a budget for his upcoming trip to the theme park. Admission is 40% of the budget. He plans to spend 32% of his money on food, 23% on souvenirs, and save 5% for emergencies. He knows the admission will be \$6 more than he will spend on food and souvenirs. How much money will Marcus need to take to the park?

1. John and Danell know that regular exercise strengthens your heart. They know that a person's heart rate should not exceed a certain limit during exercise. The maximum rate R is represented by the equation $R = 0.8(220 - y)$ where y is the person's age in years. Determine the age of a person whose maximum heart rate during exercise is 164.

MATH TIP

Use the decimal form of a percent in a formula where r is the annual interest rate. For example, $3\% = 0.03$.

2. The formula $A = p + prt$ gives the amount of money A in an account where the initial deposit is p and the money grows at a simple annual interest rate of r for t years. Assume no other deposits or withdrawals are made.

a. How much money is in the account after 5 years if \$20 is deposited and the annual interest rate is 4%?

b. How much was deposited in the account if the amount in the account after 6 years is \$17.70 and the interest rate is 3%?

c. What would the annual interest rate have to be for \$50 to grow to \$70 in 10 years?

3. The set of numbers $\{\frac{1}{2}, 3, 6, 0.17, 0, 11\}$ contains possible solutions to the following equations. Determine which of these numbers is a solution to each of the following equations. Show all work to justify your conclusions.

a. $9x + 5 = 4(x + 2) + 5x$

b. $7x - 10 = 3x + 14$

c. $3x - 12 = 3(x + 1) - 15$

An equation has ***no solution*** if there is no value for the variable that will create a true mathematical statement. An equation has ***infinitely many solutions*** if there are an unlimited number of values for the variable that will create a true mathematical statement.

My Notes

4. **Critique the reasoning of others.** Mikayla, Ryan, and Gabriella solved the following three equations as shown below. Each student has reasoned their equation as having *no solution, one solution,* or *infinitely many solutions,* respectively. Why are the students correct here?

Mikayla

$$\begin{aligned} 2x + 5 &= 5(x - 7) - 3x \\ 2x + 5 &= 5x - 35 - 3x \\ 2x + 5 &= 2x - 35 \\ -2x \quad &\quad -2x \\ 5 &= -35 \end{aligned}$$

Ryan

$$\begin{aligned} 2(x - 3) + 5 &= 4(x - 1) \\ 2x - 6 + 5 &= 4x - 4 \\ 2x - 1 &= 4x - 4 \\ -2x \quad &\quad -2x \\ -1 &= 2x - 4 \\ +4 \quad &\quad +4 \\ \frac{3}{2} &= \frac{2x}{2} \\ \frac{3}{2} &= x \end{aligned}$$

Gabriella

$$\begin{aligned} 6(x + 1) - 10 &= 6x - 4 \\ 6x + 6 - 10 &= 6x - 4 \\ 6x - 4 &= 6x - 4 \\ -6x \quad &\quad -6x \\ -4 &= -4 \end{aligned}$$

5. Write a real-world problem for the equation $7x - 10 = 3x + 14$.

Check Your Understanding

Solve each of the following. Show your work.

6. $\frac{1}{3}(2x - 4) + 5 = -\frac{2}{3}(x + 1)$
7. $2(x+1) = 3x - x + 2$
8. $6x - 5 = 5(x - 3) + x$
9. $3(2x + 4) = 6(5x + 2)$
10. Write a real-world problem for the equations in Items 6-8.
11. Create a model and solve.
    a. In a recent basketball game between the Storm and the Chargers the number of points scored by the Storm in the first half was equal to the number of points scored by the Chargers in the second half of the game. In the first half the Storm scored one-fourth the total points scored in the game. During the second half the Chargers scored 16 points less than one-third the points scored in the game. How many total points were scored during the game?
    b. Seven-tenths of a number decreased by thirteen is equal to three-tenths the number increased by 13. Find the number.

My Notes

LESSON 10-2 PRACTICE

12. Solve for x: $180 - x = 10 + 2(90 - x)$.

13. The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the base of the triangle and h is the height of the triangle. The area of the triangular sail of a sailboat is 126 ft^2. The base is 12 ft. Find the height of the sail.

14. Solve for x: $8 + 2x - 4 = 6 + 2(x - 1)$.

15. Write a real-world problem for the equation in Item 14.

16. How is an equation different from an expression?

17. Create a model and solve.

a. Five-sixths a number decreased by three is equal to five-eighths the number increased by 7.

b. Elleanna is planning to join a DVD club and is investigating the options for membership. Option 1 is a \$20 membership fee and \$1.25 rental change for each DVD rented. Option 2 has no rental fee and changes \$2.50 for each DVD rented. How many DVDs would Elleanna have to rent in order for the total cost of Option 1 to be equal to Option 2?

18. Reason abstractly. What does it mean for an equation to have no solution? How is it possible for one equation to have many solutions?

ACTIVITY 10 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 10-1

Solve the equations below.

1. $2x + 8 = 15$
2. $35 = 3x + 14$
3. $2x + 7 + 2x = 19$
4. $5 - 2(x + 5) = 30$
5. $3.5x - 20 = 2.4x + 13$
6. Determine whether the equation $y - y = y$ is *sometimes*, *always*, or *never* true. Justify your response with examples.
7. Which equation is *not* equivalent to $2x + 15 = 35$?
   A. $\frac{4}{5}x = 8$
   B. $2x - 5 = x + 25$
   C. $2x + 7 = 27$
   D. $\frac{4}{3}x = x + \frac{10}{3}$
8. Which of the following could be the first step when solving the equation $4 - 3(x + 6) = 28$?
   A. Add 3 to both sides.
   B. Subtract 4 from both sides.
   C. Divide by -3 on both sides.
   D. Subtract $4 - 3$.
9. Create an equation whose solution is all real numbers.
10. Create an equation that has no solution.
11. The formula for the area of a rectangle is $A = lw$, where l is the length of the rectangle and w is the width of the rectangle. The area of the cell phone screen is $7.5\ \text{in}^2$. The width is 2.5 in. Find the length of the screen.

Lesson 10-2

Solve for x.

12. $6 + 0.1x = 0.15x + 8$

13. $0.8x - 11 = 0.3x + 41$

14. $8x + 3 - 10x = -2(x - 2) + 3$

15. $2(x - 11) = 6(x + 3)$

16. Which of the following could be the first step when solving the equation $5 + 6(x - 1) = 11 + 5x$?
A. Multiply $(x - 1)$ by 6.
B. Subtract 1 from both sides.
C. Divide by 5 on both sides.
D. Subtract $5x$ from $6(x - 1)$.

17. Solve $3(x + 1) + 1 + 2x = 2(2x + 2) + x$.

18. What is the next step in solving the equation $\frac{4}{5}x = 36$?
A. $\frac{4}{5} - \frac{4}{5}x = 36 - \frac{4}{5}$
B. $\left(\frac{5}{4}\right)\frac{4}{5}x = 36\left(\frac{5}{4}\right)$
C. $\frac{4}{5}x = 36 - (5 - 4)$
D. $\frac{4}{5}x = 36 - (4 * 5)$

19. Which equation is *not* equivalent to $6x - 1 = 11 + 5x$?
A. $x = 12$
B. $6x = 12 + 5x$
C. $x - 1 = 11$
D. $\frac{6}{10}x = 3$

20. Which choice is a solution for the equation $7x + 11 = 10x - 25$?
A. -10
B. 5
C. 12
D. -6

21. Solve $2x + 1 = x - 3$ for x.

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

22. Why is it important to be able to use properties of numbers and equality when solving an equation?

Muhammad Yunus is the founder of the Grameen Bank and a recipient of the Nobel Peace Prize for his work to find solutions to extreme poverty. The Grameen Bank issues microcredit loans and is the world's original microlending organization. Microcredit loans are one way to help people move out of extreme poverty. The loans involve lending very poor people small amounts of money to start a new business or to expand an existing one.

1. Malik wants to open a bicycle repair shop in India. He will need 800 rupees to buy the tools and equipment necessary to start his business. After that, he will need 1,400 rupees per month to keep his business running.

a. Copy and complete the table below for Malik's total costs.

Number of Months in Business	Total Costs to Operate (in rupees)
1	
2	
3	
6	
n	

b. Use the completed table to write an expression to represent Malik's total costs to operate his business in terms of the number of months he has been in business.

c. Copy and complete the table below for Malik's total profit in terms of the months he is in business. Use the fact that Malik estimates he will make 3,600 rupees in sales each month.

Number of Months in Business	Total Profit (in rupees)
1	
2	
3	
6	
n	

MATH TIP

Profit P is defined as Revenue R, or money brought in, minus Costs C, or money paid out.

$$P = R - C$$

d. Use the completed table to write an expression to represent Malik's total profits from his business in terms of the number of months he has been in business.

e. Assume $1.00 is equivalent to 43 rupees. How much profit, in dollars, will Malik make in 1 year?

2. Maria lives in Honduras. She is applying for a microcredit loan to start a street food business. The start-up costs for her business include $65 for the cart and $900 for a freezer, plus the cost of bulk foods. She estimates her total start-up cost will be $1,053.

a. If b represents the cost of the bulk foods, write and solve an equation to find the cost of the bulk foods Maria plans to buy.

b. Maria has monthly costs of $420 and expects monthly sales of $725. Use the formula $P = n(R - C)$, where P represents profit, R represents monthly revenue or sales, and C represents monthly costs, to find the number of months it will take her to make $2,000 in profit.

3. An organization has raised $5,000 that it would like to use to provide $200 microloans to residents of a particular community. Of the $5,000 they raised, $400 must be used for administrative costs like the salaries of employees who set up the loans and the paperwork required. Write and solve an equation to determine how many microloans the organization can provide.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-e, 2a-b, 3)	• Clear and accurate understanding of solving expressions and equations.	• Solving expressions and equations with few if any errors.	• Difficulty solving expressions and equations.	• Incorrect or incomplete solving of expressions and equations.
Problem Solving (Items 1e, 2a-b, 3)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-d, 2a-b, 3)	• Writing accurate expressions and equations from a table or a problem situation. • Accurately creating tables to represent a problem situation.	• Writing expressions and equations that usually result in the correct answer. • Creating tables to represent a problem situation.	• Errors in writing expressions and equations to represent a problem situation. • Errors in creating tables to represent a problem situation.	• Writing inaccurate or incomplete expressions or equations. • Creating inaccurate or incomplete tables to represent a problem situation.
Reasoning and Communication (Items 1e, 2b)	• Reasoning to round a numerical answer and clearly communicating the answer in the requested units.	• Giving a correct answer.	• Failing to round an answer to a reasonable number or neglecting the units.	• An incomplete or inaccurate answer.

Exploring Slope

ACTIVITY 11

High Ratio Mountain

Lesson 11-1 Linear Equations and Slope

My Notes

Learning Targets:

- Understand the concept of slope as the ratio $\frac{\textit{change in } y}{\textit{change in } x}$ between any two points on a line.
- Graph proportional relationships; interpret the slope and the y-intercept (0, 0) of the graph.
- Use similar right triangles to develop an understanding of slope.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking The Text, Discussion Groups, Sharing and Responding, Interactive Word Wall

Misty Flipp worked odd jobs all summer long and saved her money to buy passes to the ski lift at the High Ratio Mountain Ski Resort. In August, Misty researched lift ticket prices and found several options. Since she worked so hard to earn this money, Misty carefully investigated each of her options.

High Ratio Mountain Ski Resort

Student Lift Ticket prices

Daily Lift Ticket	\$30
10-Day Package	\$80 upon purchase and \$20 per day (up to 10 days)
Unlimited Season Pass	\$390

1. Suppose Misty purchases a daily lift ticket each time she goes skiing. Complete the table below to determine the total cost for lift tickets.

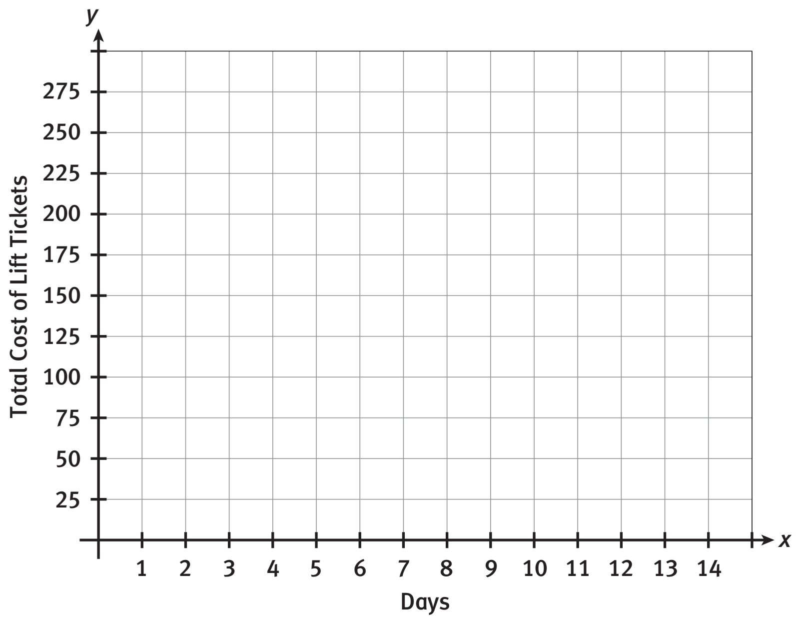
Number of Days	0	1	2	3	4	5	6
Total Cost of Lift Tickets							

2. According to the table, what is the relationship between the cost of the lift tickets and the number of days?

My Notes

3. Let d represent the number of days for which Misty bought lift tickets and C represent Misty's total cost. Write an equation that can be used to determine the total cost of lift tickets if Misty skis for d days.

4. **Model with mathematics.** Plot the data from the table on the graph below. The data points appear to be ***linear***. What do you think this means?



5. Label the leftmost point on the graph point A. Label the next 6 points, from left to right, points B, C, D, E, F, and G.

6. **Reason quantitatively.** According to the graph, what happens to the total cost of lift tickets as the number of days increases? Justify your answer.

7. Describe the movement, on the graph, from one point to another.

A to B: Vertical Change ________ Horizontal Change ________

B to C: Vertical Change ________ Horizontal Change ________

C to D: Vertical Change ________ Horizontal Change ________

D to E: Vertical Change ________ Horizontal Change ________

E to F: Vertical Change ________ Horizontal Change ________

F to G: Vertical Change ________ Horizontal Change ________

MATH TIP

Vertical change is the number of spaces moved up or down on a graph. "Up" movement is represented by a positive number. "Down" is a negative number.

Horizontal change is the number of spaces moved right or left on a graph. Movement to the right is indicated by a positive number. Movement to the left is indicated by a negative number.

My Notes

8. a. The movements you traced in Item 7 can be written as ratios. Write ratios in the form $\frac{\text{vertical change}}{\text{horizontal change}}$ to describe the movement from:

A to B: B to C:

C to D: D to E:

b. Vertical change can also be described as the *change in y*. Similarly, the horizontal change is often referred to as the *change in x*. Therefore, the ratio $\frac{\text{vertical change}}{\text{horizontal change}}$ can also be written as $\frac{\text{change in } y}{\text{change in } x}$. Determine the *change in y* and *change in x* from A to C in Item 4. Write the ratio as $\frac{\text{change in } y}{\text{change in } x}$.

Continue to use the data from Item 4. Determine the *change in y* and *change in x* for each movement described below. Then write the ratio $\frac{\text{change in } y}{\text{change in } x}$.

c. From B to E: **d.** From A to E:

e. From B to A: **f.** From E to B:

READING AND WRITING MATH

When writing a ratio, you can also represent the relationship by separating each quantity with a colon. For example, the ratio 1:4 is read "one to four."

My Notes

9. Describe the similarities and differences in the ratios written in Item 8. How are the ratios related?

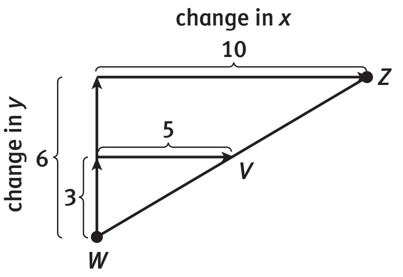
10. Make sense of problems. What are the units of the ratios created in Item 8? Explain how the ratios and units relate to Misty's situation.

11. How do the ratios relate to the equation you wrote in Item 3?

MATH TIP

In similar triangles, corresponding angles are congruent and corresponding sides are in proportion.

12. The ratio $\frac{\text{change in } y}{\text{change in } x}$ between any two points on a line is constant. Use the diagram below and what you know about similar triangles to explain why the $\frac{\text{change in } y}{\text{change in } x}$ ratios are equivalent for the movements described.



$$\frac{\text{change in } y}{\text{change in } x} = \frac{3}{5} = \frac{6}{10}$$

From W to V:

From W to Z:

My Notes

The ***slope*** of a line is determined by the ratio $\frac{\text{change in } y}{\text{change in } x}$ between any two points that lie on the line.

- The slope is the *constant rate of change* of a line. It is also sometimes called the *average rate of change*.
- All linear relationships have a *constant rate of change*.
- The slope of a line is what determines how steep or flat the line is.
- The y-intercept of a line is the point at which the line crosses the y-axis, $(0, y)$.

MATH TERMS

Slope is the ratio of vertical change to horizontal change, or $\frac{\textit{change in } y}{\textit{change in } x}$.

13. Draw a line through the points you graphed in Item 4. Use the graph to determine the slope and y-intercept of the line. How do the slope and y-intercept of this line relate to the equation you wrote in Item 3?

READING MATH

The slope of a line, $\frac{\textit{change in } y}{\textit{change in } x}$, is also expressed symbolically as $\frac{\Delta y}{\Delta x}$.

Δ is the Greek letter delta, and in mathematics it means "change in."

14. Complete the table to show the data points you graphed in Item 4. Use the table to indicate the ratio $\frac{\text{change in } y}{\text{change in } x}$ and to determine the slope of the line.

Number of Days	Total Cost of Lift Tickets
0	
1	
2	
3	
4	
5	
6	

change in y:

change in x:

$\frac{\text{change in } y}{\text{change in } x}$:

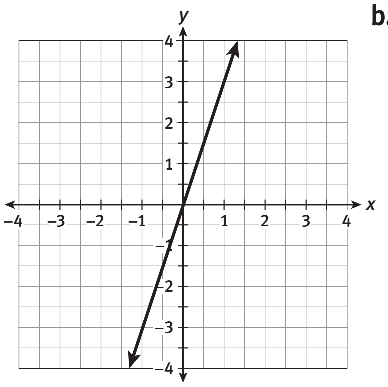
slope:

My Notes

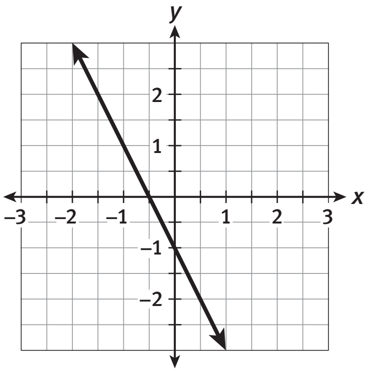
Check Your Understanding

15. Find the slope and the y-intercept for each of the following. Remember to use the ratio $\frac{\text{change in } y}{\text{change in } x}$.

a.



b.



c.

x	y
0	0
1	2.5
2	5
4	10

d.

x	y
-1	4
0	2
1	0
3	-4

e. Look back at the figure for Item 12. Would a point P that is 9 units up from point W and 15 units to the right be on the line that contains points W, V, and Z? Use similar triangles to explain your answer.

16. John is longboarding at a constant rate down the road. If 2 minutes after he leaves his house he is 1,000 feet away and at 5 minutes he is 2,500 feet from his house, what would his average rate of change be?

CONNECT TO SPORTS

Longboards are larger than the more trick-oriented skateboards. Longboards are heavier and sturdier than skateboards. Some people even use them instead of bicycles.

My Notes

LESSON 11-1 PRACTICE

The Tran family is driving across the country. They drive 400 miles each day. Use the table below to answer Items 17–20.

Day	Total Miles Driven
1	400
2	800
3	
4	
5	

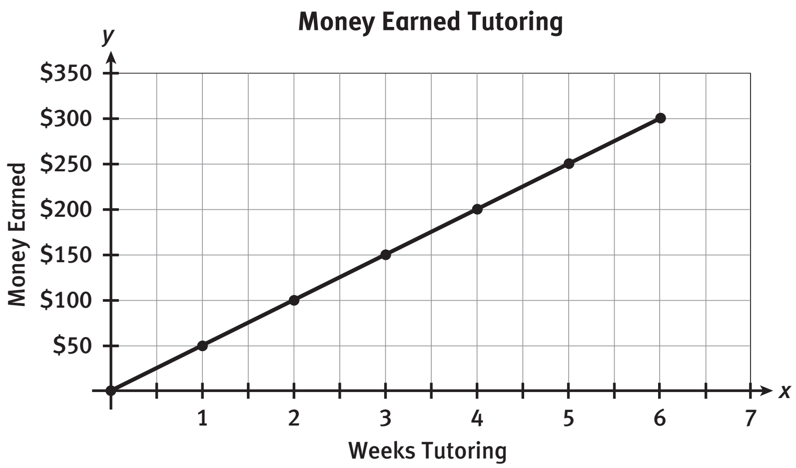
17. Complete the table.

18. Draw a graph for the data in the table. Be sure to title the graph and label the axes. Draw a line through the points.

19. Write an equation that can be used to determine the total miles, M, driven over d days.

20. Find the slope and the y-intercept of the line you created, using the graph you drew or the equation you wrote. Explain what each represents for the Tran family's situation.

The graph below shows the money a student earns as she tutors. Use the graph to answer Items 21–24.



21. What is the slope of the line?

22. What is the y-intercept of the line?

23. Write an equation that can be used to determine how much money, D, the student has earned after w weeks.

24. **Attend to precision.** Calculate how much money the student will have earned after 52 weeks.

Lesson 11-2
More on Linear Equations and Slope

My Notes

Learning Targets:

- Understand the connections among proportional relationships, lines, and linear equations.
- Graph proportional relationships; interpret the slope and the y-intercept $(0, y)$ of graphs.
- Examine linear relationships as graphs and as equations to solve real-world problems.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Sharing and Responding, Construct an Argument, RAFT

Remember that Misty had saved her money to buy passes to the ski lift at the High Ratio Mountain Ski Resort.

High Ratio Mountain Ski Resort

Student Lift Ticket prices

Daily Lift Ticket	\$30
10-Day Package	\$80 upon purchase and \$20 per day (up to 10 days)
Unlimited Season Pass	\$390

1. Suppose Misty purchased the 10-day ticket package that costs \$80 plus \$20 per day.

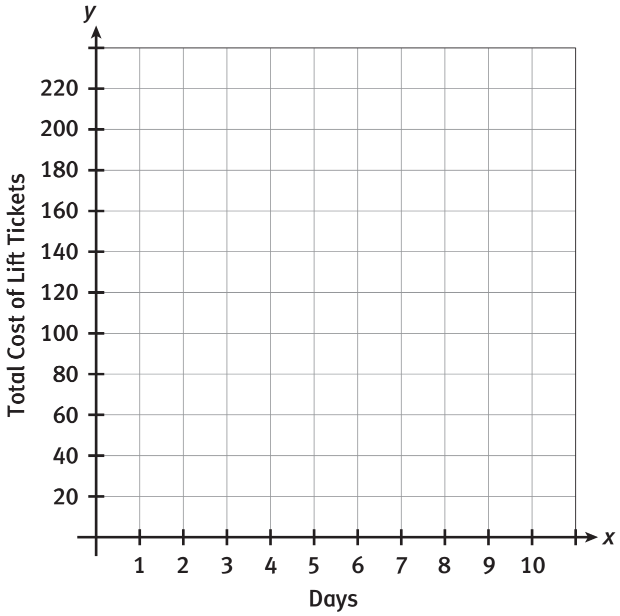
a. Complete the table below to determine the total cost of the lift tickets in the 10-day package for 0 through 6 days. Be sure to include the initial cost of \$80.

Number of Days	0	1	2	3	4	5	6
Total Cost of Lift Tickets							

b. Reason quantitatively. Explain how you know the data in the table are linear.

My Notes

2. Plot the data from the table on the given axes. Then draw a line through the points you plotted.



3. Determine the slope and the y-intercept of the line graphed in Item 2. Explain how these values relate to Misty's situation.

4. Let d represent the number of days Misty plans to ski and let K represent Misty's total cost. Write an equation that could be used to determine Misty's total cost if she bought a 10-day package.

5. Compare and contrast the lines associated with the data for the daily lift tickets in Item 4 from Lesson 11-1 and the data for the 10-day package in Item 2. Include the similarities and differences in their equations.

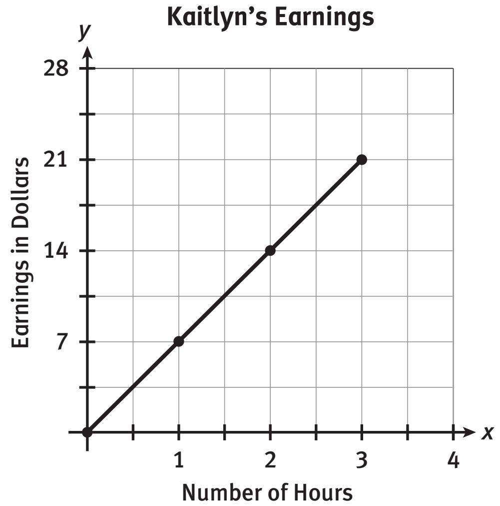
My Notes

Check Your Understanding

6. Veronika and Kaitlyn work in the ski shop at High Ratio Mountain. Because Veronika works on-call and doesn't have a set schedule she earns \$10 plus \$6 for each hour she is called in to work. Kaitlyn's earnings are modeled by the graph shown.

a. Write equations to represent each girl's earnings.

b. State and interpret the slope and y-intercept of each girl's equation you wrote in part a.



7. A line with a slope of $\frac{-1}{2}$ contains the point (2, 3). Use the My Notes section to graph the point and use the slope to give the coordinates of three other points on the line.

8. Although \$390 seemed a little expensive, Misty considered the unlimited season pass.

a. First, she compared the season pass to the daily lift tickets at \$30 each. How many times would Misty have to go skiing before she would save money with the \$390 season pass? Explain your reasoning.

b. Complete the table below for the total cost of the unlimited season pass for 0 through 6 days.

Number of Days	0	1	2	3	4	5	6
Total Cost of Lift Tickets							

c. Explain how you know the data in the table are linear.

d. What is the rate of change of the cost of the tickets with the unlimited season pass?

My Notes

9. Next Misty compared the price of an unlimited season pass to two 10-day packages that she would use for 20 days of skiing. Which package would be the best buy? Why? Justify your answer.

10. Express regularity in repeated reasoning. If Misty skis the following number of days, which of the three packages should she purchase? Explain your reasoning for each.

a. 6 days

b. 8 days

c. 13 days

d. 16 days

11. Construct viable arguments. Write a ***persuasive*** letter to Misty based on your analysis that makes a recommendation for which package she should purchase. Include multiple representations (graphs, tables, and/or equations) to support your reasoning. Provide a concluding statement that summarizes your reasoning.

ACADEMIC VOCABULARY

A synonym for ***persuasive*** is convincing.

Check Your Understanding

Emily rides her bike 24 miles in 2 hours.

12. Create a ratio of Emily's miles per hour.

13. Use the ratio you created to determine how far Emily can ride in 5 hours.

14. If Emily rode her bike for 42 miles at the rate you determined, how long did she ride?

15. Raine rides her bike 37 miles in 3 hours. If Raine started at the same time as Emily and also rode her bike at a constant rate for 42 miles, who finished first? Explain your reasoning.

My Notes

LESSON 11-2 PRACTICE

Lucas conducted an experiment on the rate of water evaporation. He placed 500 mL of water in a measuring cup at room temperature and collected data on the amount of water in the cup every day for a week. His results are shown in the data table below:

Day	Water Measurement (mL)
0	500
1	475
2	450
3	425
4	400
5	375
6	350
7	325

16. What is the rate of change in the amount of water in the cup?

17. On a graph of these data, what would be the y-intercept, and what does it represent in Lucas's experiment?

18. Write an equation that can be used to determine the water level, l, after d days.

19. At this same rate of evaporation, how many days will it take for all of the water to evaporate?

Lucas wanted to see how lowering the temperature of the water would change the results of his experiment. He placed 500 mL of water in a measuring cup, but placed the cup in the refrigerator, and collected data as before for a week. His results are shown in the data table below:

Day	Water Measurement (mL)
0	500
1	490
2	480
3	470
4	460
5	450
6	440
7	430

20. What is the rate of change in the amount of water in the cup?

21. Write an equation that can be used to determine the water level, l, after d days.

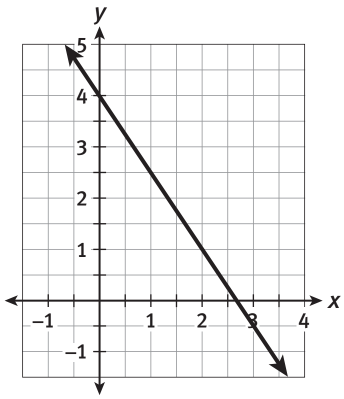
22. **Make sense of problems.** At this rate of evaporation, how many more days will it take for all of the water to evaporate in the refrigerator experiment than in the room-temperature experiment?

ACTIVITY 11 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 11-1

1. Misty determined that she gets 64 miles per 2 gallons of gas from her car as she drives from her house to the ski slope.
   a. Create a ratio of Misty's miles per gallon.
   b. Using the ratio you found in part a, determine how far Misty can go on 1 gallon of gas.
   c. How many miles could Misty travel on a full tank of 12 gallons of gas?
2. Brynn rides her bike 84 miles in 4 hours.
   a. Create a ratio of Brynn's miles per hour.
   b. Using the ratio you found in part a, determine how far Brynn can ride in 7 hours.
   c. If Brynn rides 57 miles at the rate indicated, how long will she ride? Justify your response.
3. What is the slope of the line shown?



A. -4
B. $-\frac{3}{2}$
C. $\frac{3}{2}$
D. 4

4. Find the slope and y-intercept of the line represented by each of the following.

a.

x	y
0	1
3	7
6	13

b.

x	y
−3	10
0	4
4	−4

c.

x	y
0	−3
2	−1
4	1

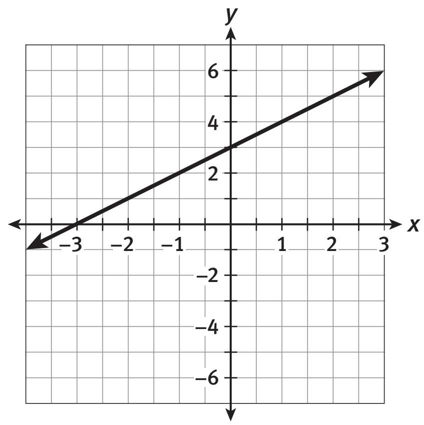
d.

x	y
−1	5
0	9
4	25

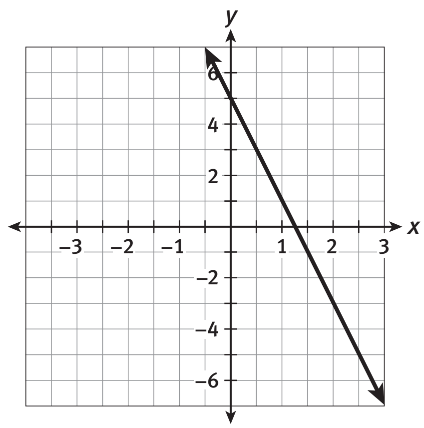
Lesson 11-2

5. Find the slope and y-intercept for each of the following graphs.

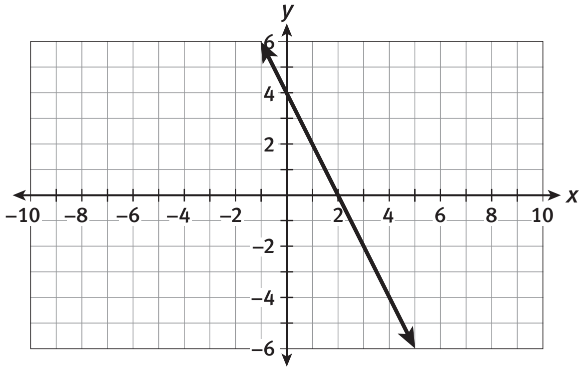
a.



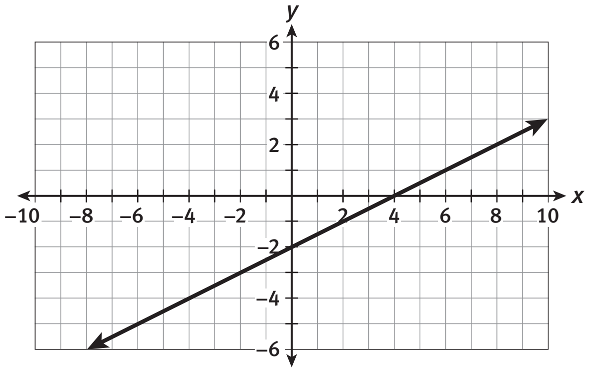
b.



c.



d.



6. If a line has a slope of $-\frac{1}{2}$ and contains the point (2, 3), then it must also contain which of the following points?

A. (−2, 6)
B. (0, 5)
C. (1, 2)
D. (4, 2)

7. If a line with a slope of $\frac{3}{4}$ contains the point with coordinates (−1, 4), give the coordinates of three other points that must also be on the line.

8. Complete the following tables so that the data are linear.

a.

x	y
0	1
1	3
2	5
3	
4	9

b.

x	y
0	10
1	16
2	22
3	
4	34
5	40

9. Which of the following situations, when graphed, would create a linear graph? Explain your reasoning.

A. the height of a wedding cake as 5-inch layers are added
B. the speed of each car passing through an intersection
C. the weight of a sandbag as shovelfuls of dirt are added

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

10. How does the value of the slope of a line affect the steepness of the line?

Slope-Intercept Form

Leaky Bottle

Lesson 12-1 Identifying Slope Using Tables and Graphs

My Notes

Learning Targets:

- Graph linear relationships represented in different forms.
- Write an equation in the form $y = mx + b$ to model a linear relationship between two quantities.
- Interpret the meaning of slope and y-intercept in a problem context.

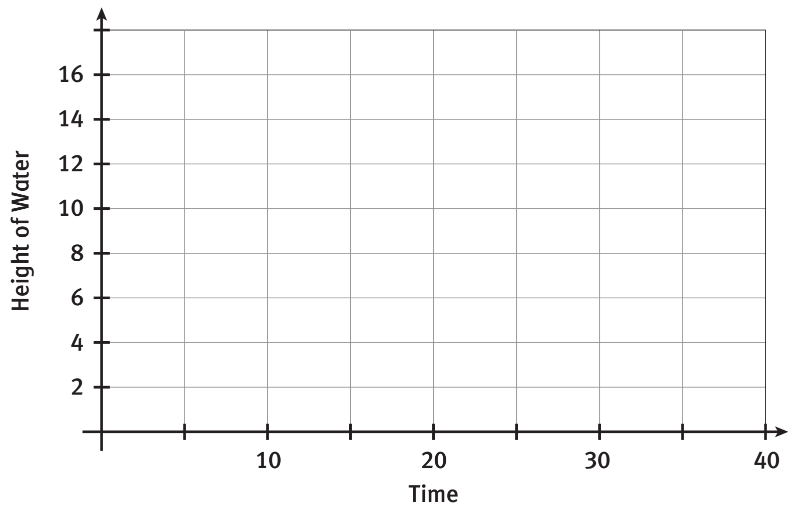
SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Create Representations, Look for a Pattern, Discussion Group, Sharing and Responding

Owen's water bottle leaked in his book bag. He did the following experiment to find how quickly water drains from a small hole placed in a water bottle.

1. Follow the steps below and fill in the table.
- Get a water bottle and a container to catch the water.
- Poke a very small hole in the bottom of the water bottle.
- Ensure the hole is facing down and open the bottle cap.
- Draw a line on the bottle every 5 seconds to mark the water level.
- After the water is drained from the bottle, measure the heights at each of the times you marked.

Time in Seconds	0	5	10	15	20	25	30	35	40
Height of Water (cm)									

2. Graph the data on the grid below.



3. Does the relationship between time and the height of the water appear to be linear? Explain your reasoning.

My Notes

Discrete data are data that can only have certain values, such as the number of people in your class. On a graph of discrete data, there will be space between every two possible values. ***Continuous data*** can take on any value within a certain range, for example, height. On a graph, continuous data have no breaks, holes, or gaps.

4. Are the data you collected continuous or discrete? Justify your response.

5. Draw a line through the points on your graph.

a. Find the slope of the line.

b. Determine the y-intercept of the line.

c. Describe the meaning of the slope of the line in this problem situation.

d. Describe the meaning of the y-intercept of the line you drew in this problem situation.

6. Write an equation that gives the height of the water H given the time t.

7. How does the ***coefficient*** of t in your equation relate to the experiment? Be certain to include appropriate units.

8. How does the ***constant term*** in the equation relate to the experiment? Be certain to include appropriate units.

WRITING MATH

The y-intercept of a line is written as an ordered pair, $(0, y)$, where y is the value at which the line intersects the y-axis.

MATH TERMS

The **coefficient** of a variable is the number being multiplied by the variable. The **constant term** is the term without a variable.

My Notes

9. Attend to precision. Find the slope of the line that passes through the points in the table.

x	*y*
0	11
2	7.5
4	4

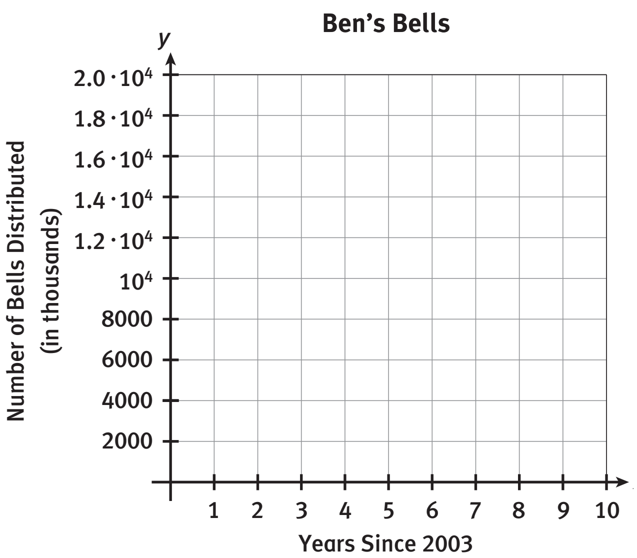
Check Your Understanding

Ben's Bells, a community service organization, began hanging ceramic wind chimes randomly in trees, on bike paths, and in parks around the country in 2003. A written message on each bell asked the finder to simply take one home and pass on the kindness. One thousand bells were hung in 2003. The number of bells increased by 2,000 each year after 2003.

10. Complete the table below to indicate the number of bells that were hung each year after 2003.

Years Since 2003	0	1	2	3
Number of Bells Distributed				

11. Graph the data on the grid below.



12. Is the data continuous or discrete? Justify your response.

13. Use the data in the table or the graph to write an equation that gives the number of bells distributed, *N*, given the years since 2003, *t*.

14. Using the graph in Item 11, determine the slope and *y*-intercept of the line. What do these values represent in this real-life situation?

My Notes

LESSON 12-1 PRACTICE

15. Graph the data in the table. Find the slope of the line created by connecting the data points in the table.

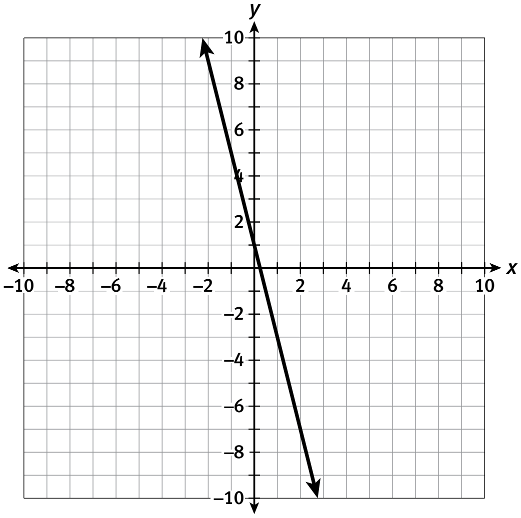
x	y
0	10
3	8.5
6	7

16. Examine the table of values. Write an equation that gives the value of y for any given value of x. State the slope and y-intercept of the line that your equation represents.

x	-3	-2	-1	0	1	2	3
y	15	10	5	0	-5	-10	-15

17. a. Find the slope of the line in the following graph.

b. Determine the y-intercept of the line.



18. MyCar Rental charges \$15 per day to rent one of its cars plus an \$8 service charge.

a. Write an equation that gives the total cost, y, for the number of days, x, that a car is rented.

b. State the slope and the y-intercept of the line that your equation represents.

19. Mariella is driving a distance of 100 miles. She drives at a constant rate of 60 miles per hour. The equation that represents the distance she has left to go is $y = 100 - 60x$, where x is the number of hours she has already driven.

a. Graph the equation.

b. State the slope and the y-intercept of the graph from part a.

c. Reason abstractly. Explain what the slope and y-intercept of the line represent in this item.

My Notes

Learning Targets:

- Compare different proportional relationships represented in different ways.
- Graph linear relationships and identify and interpret the meaning of slope in graphs.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Construct an Argument, Discussion Groups, Look for a Pattern

Slope can be seen visually as the slant or the steepness of a line. Using a graph, one can determine the slope of a line (as well as the y-intercept). One can also compare the slopes of different lines by considering their graphs or their equations.

1. For each linear equation in the tables below:
- Complete the table of values.
- Graph, using a different color for each line.
- Determine the slope.

a.

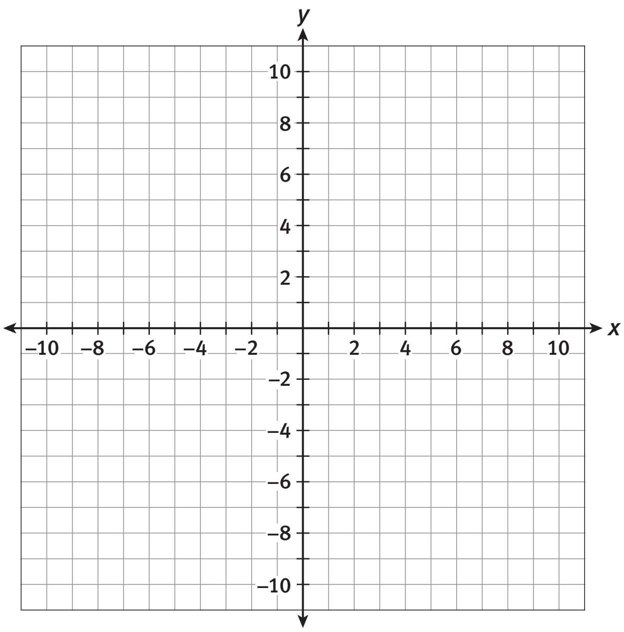
x	$y = x$	y
-3	$y = -3$	-3
-2		
-1		
0		
1		
2		
3		

b.

x	$y = 2x$	y
-3	$y = 2(-3)$	-6
-2		
-1		
0		
1		
2		
3		

c.

x	$y = 4x$	y
-2	$y = 4(-2)$	-8
-1.5		
-1		
0		
1		
2		
3		



My Notes

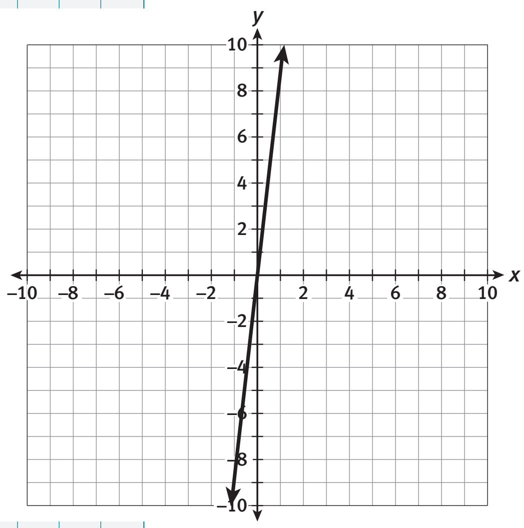
2. How does the slope you found for each linear equation relate to the coefficient of x in the equation?

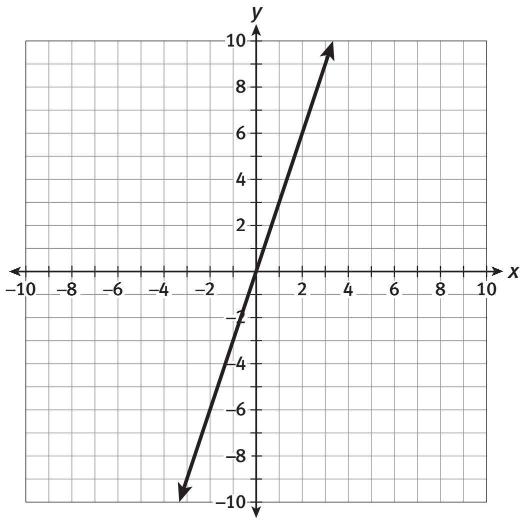
3. How does the slope you found for each linear equation relate to the steepness of each line?

4. Write an equation of a line that is:
   a. steeper (increasing) than $y = 3x$

   b. steeper (decreasing) than $y = -6x + 2$

5. **Critique the reasoning of others.** Jordan and Alex disagree about the following two graphs. Jordan feels that the line in the left graph is steeper, while Alex feels the line in the right graph is steeper. Which line is steeper? Justify your response to Jordan and Alex.





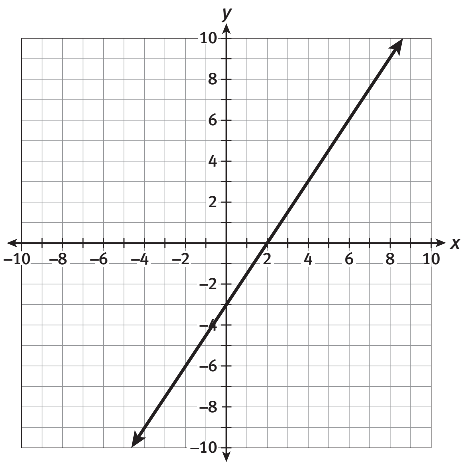
My Notes

Check Your Understanding

6. Write the equation of a line that has a slope that is greater than 1 but less than 2.

7. Graph the equation $y = 3x$. State the slope of the line.

8. The graph, table, and equation below represent 3 different linear relationships. Which one has the greatest rate of change? Justify your response.



x	y
0	3
1	6
2	9
3	12
4	15

$y = 0.5x - 4$

9. For each linear equation in the tables below:
- Complete the table of values.
- Graph, on the grid provided, using a different color for each line.
- Determine the slope.

a. $y = -x$

x	y
−3	
−2	
−1	
0	
1	
2	
3	

slope =

b. $y = -2x$

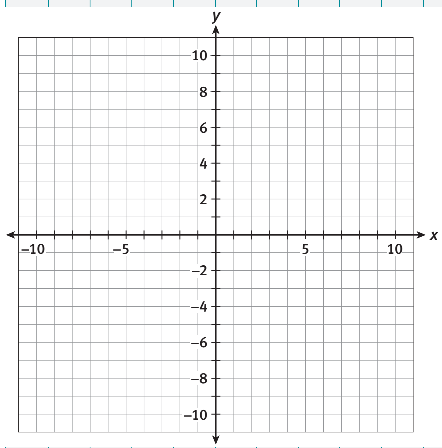
x	y
−3	
−2	
−1	
0	
1	
2	
3	

slope =

c. $y = -4x$

x	y
−2	
−1.5	
−1	
0	
1	
2	
3	

slope =



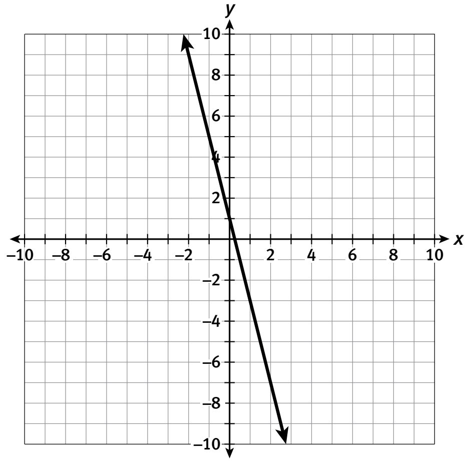
10. How do the slopes you found relate to the coefficients of x in the three equations?

My Notes

LESSON 12-2 PRACTICE

11. Complete the table of values and graph the equation $y = -5x$. State the slope of the line.

x	$y = -5x$	y
−3	$y = -5(-3)$	15
−2		
−1		
0		
1		
2		
3		



12. The table and the equation below represent different linear relationships. Which one has the greater rate of change? Explain your reasoning.

x	−2	−1	0	1	2
y	1	5	9	13	17

$y = 3x - 4$

13. How does the slope of a line relate to the steepness of the line?

14. Given the equations below, how can the steepness of the lines be determined without graphing the lines?
line A: $y = 8x$ line B: $y = -3x + 2$

15. Graph each equation.
- $y = \frac{1}{2}x$
- $y = x$
- $y = 2x$

a. State the slope of each equation.
b. **Reason quantitatively.** Which line is steepest? Justify your answer using the slope.

My Notes

Learning Targets:

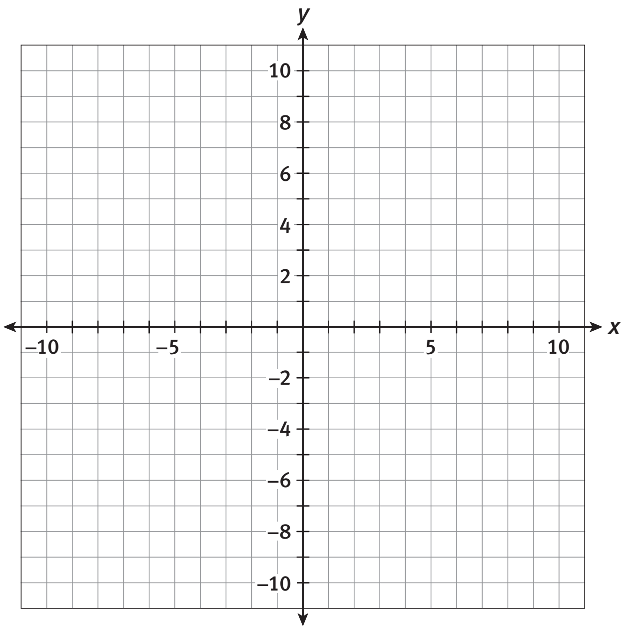
- Derive equations of the form $y = mx$ and $y = mx + b$ from their graphs.
- Graph linear relationships and identify and interpret the meaning of slope and y-intercept in graphs.

SUGGESTED LEARNING STRATEGIES: Close Reading, Discussion Groups, Sharing and Responding, Create Representations, Marking the Text

The ***slope-intercept form*** of a linear equation is $y = mx + b$ where m is the slope of the line and b is the y-intercept of the line.

1. For each linear equation below:
- Make a table of values.
- Graph on a blank grid like the one below, using a different color for each line.
- Determine the slope and y-intercept.

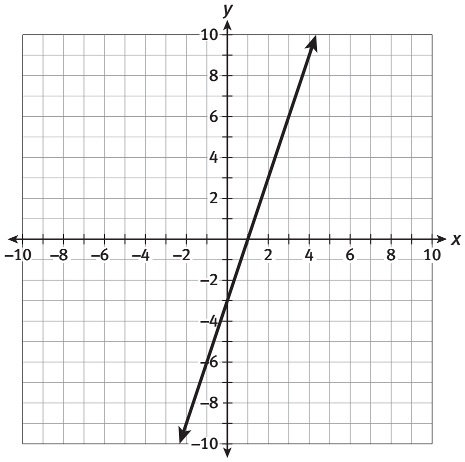
a. $y = \frac{1}{2}x$ **b.** $y = \frac{1}{4}x$ **c.** $y = \frac{1}{5}x$



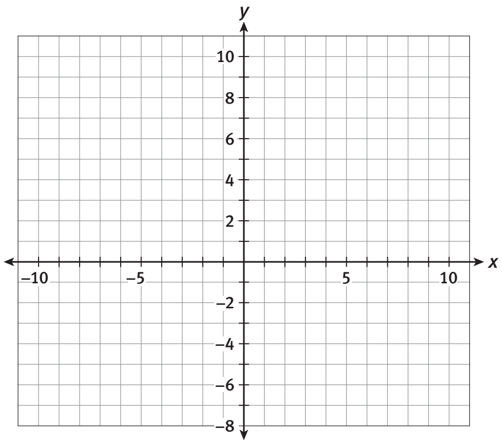
My Notes

2. The equation and graph below represent different linear relationships. Identify the slope and y-intercept of both linear relationships.

$$y = \frac{1}{2}x + 4$$



3. For each linear equation below:
- Make a table of values in the My Notes section.
- Graph using a different color for each line.
- Determine the y-intercept.
- Determine the slope.



a. $y = \frac{2}{3}x + 3$

b. $y = \frac{2}{3}x + 6$

c. $y = \frac{2}{3}x - 3$

Lesson 12-3
Linear Relationships Using Slope-Intercept Form

My Notes

4. How is the y-intercept related to the constant term in the equations you just graphed?

5. Explain how to graph a line using the slope and y-intercept.

6. **Make use of structure.** Identify the slope and y-intercept in each of the following equations.

a. $y = \frac{3}{2}x + 5$

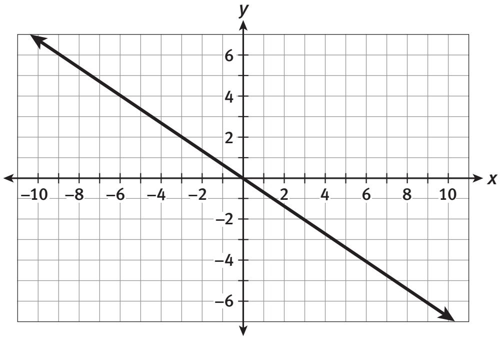
b. $y = -x + 1$

c. $y = 4x - 3$

Check Your Understanding

7. Make a table of values and graph the equation $y = -3x + 2$. State the slope of the line. Give the coordinates of the y-intercept.

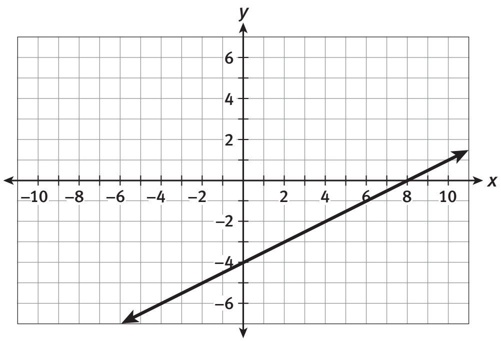
8. Write the equation of the line graphed.



9. Graph each of the following linear equations using the y-intercept and the slope.

a. $y = 2x + 4$ **b.** $y = -3x + 2$ **c.** $y = \frac{2}{3}x - 5$

10. Write an equation for the line graphed below.



11. Explain two different ways to graph a linear equation of the form $y = mx + b$ where m and b represent any real number.

My Notes

LESSON 12-3 PRACTICE

12. Graph the following linear equations. State the slope and y-intercept for each.

a. $y = 5x - 2$
b. $y = 2x + 10$
c. $y = -25x + 100$

13. What is the slope of the graph of $y = -2x + 6$?

A. 2 **B.** 6 **C.** –2 **D.** –6

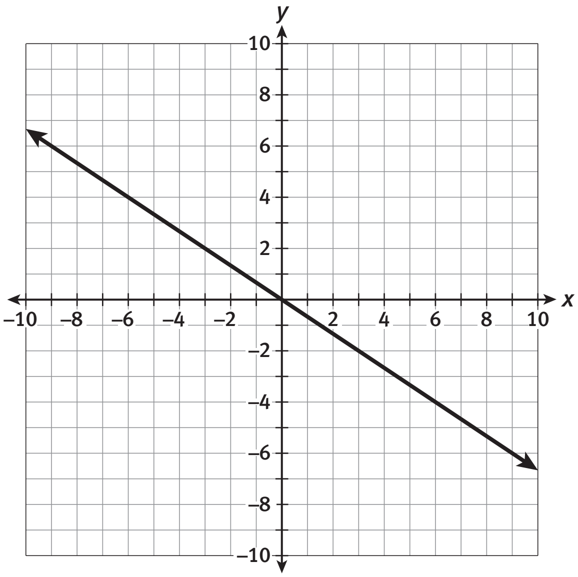
14. What is the y-intercept of the graph of $y = \frac{3}{5}x - 12$?

A. $(0, -12)$ **B.** $\left(0, \frac{3}{5}\right)$ **C.** $(0, 12)$ **D.** $\left(0, -\frac{3}{5}\right)$

15. Graph each of the following linear equations. State the slope and y-intercept for each.

a. $y = 5x$
b. $y = -4x$
c. $y = \frac{1}{5}x$

16. **Make sense of problems.** Write the equation of the line graphed below.



17. Celso is reading Lois Lowry's *The Giver*, which is 180 pages long. Celso reads 30 pages, on average, each hour. To model the number of pages he reads over time:

a. Create a table of values.
b. Construct a graph.
c. Write an equation.
d. State the meaning of the slope and y-intercept of the equation you wrote in part c.

ACTIVITY 12 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 12-1

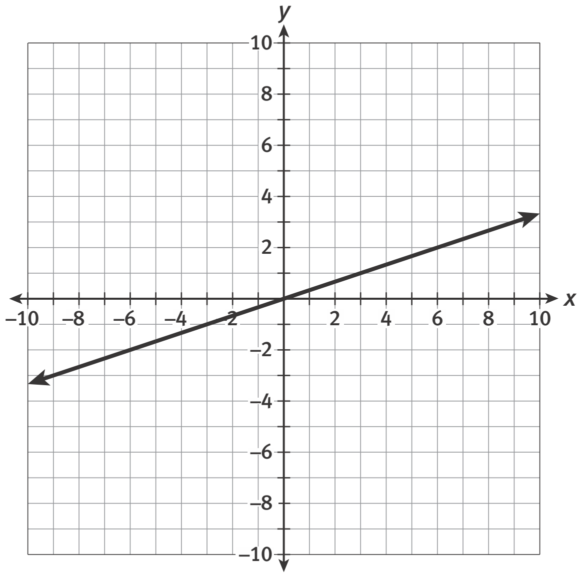
Owen found that he could use the equation $y = -3x + 24$ to model the amount of water, in mL, in his bottle after x seconds.

1. What is the slope and what does it represent in this problem situation? Be sure to include units.
2. What is the y-intercept and what does it represent in this problem situation? Be sure to include units.
3. Explain to Owen what would have to happen to the bottle for the slope to change to -4.
4. A line with a slope of -2 goes through the point (3, 5). It also goes through the point $(-2, p)$. What is the value of p?
   A. $p = 15$ **B.** $p = 3$ **C.** $p = -1$
5. A line with a slope of 3 goes through the point $(-2, -4)$. It also goes through the point $(0, p)$. What is the value of p?
6. Find the slope of the line that passes through the data points in the table.
   A. -3 **B.** 3 **C.** 0

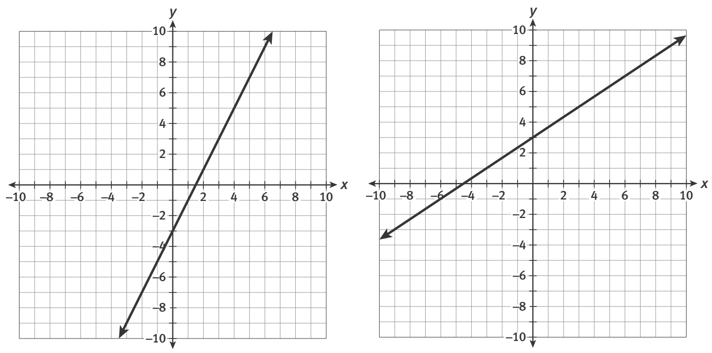
x	y
-4	-7
0	5
$\frac{1}{3}$	6

Lesson 12-2

7. What would happen to the slope of the graph of $y = \frac{2}{3}x - 3$ if the line were shifted 6 units up?
8. Explain how the slope of $y = 2x - 3$ can be used to graph the equation.
9. Compare and contrast the slopes for the equations $y = x$, $y = \frac{1}{3}x$, and $y = \frac{2}{3}x$. What conclusions can you draw about the slope of lines?
10. Write the equation of a line that is steeper than the line $y = \frac{1}{2}x$ but has a slope less than 1.
11. Write the equation of the line graphed below.



12. Two graphs are given below representing different linear relationships.



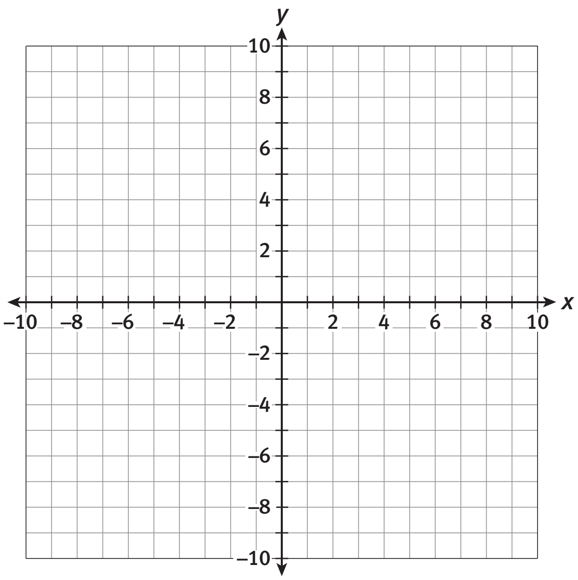
Which line has the larger slope? Explain your reasoning.

13. What is the slope of each line in Item 12? Explain the steps you used to find each slope.

Lesson 12-3

14. Identify and plot the y-intercept of the equation $y = \frac{1}{2}x + 3$. Then use the slope to determine and graph two more points on the line. Finally, sketch a line through the three points.
15. Explain how to graph the equation $y = 2x - 3$ without using a table of values.

16. Use the y-intercept and the slope to graph the following linear equations on the grid provided. Use a different color for each graph.

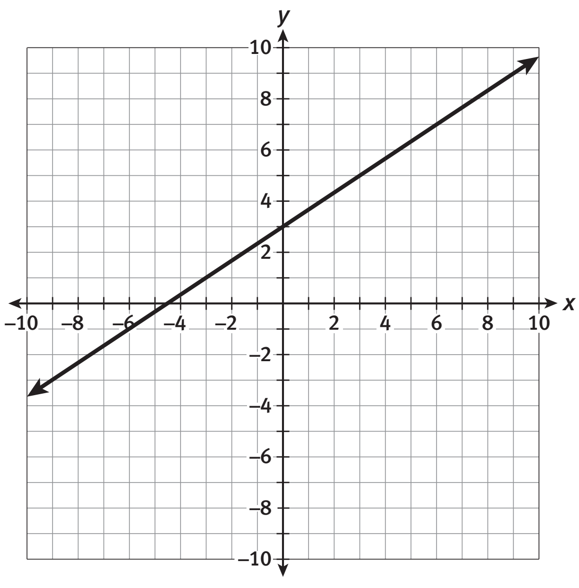


a. $y = \frac{1}{3}x - 2$

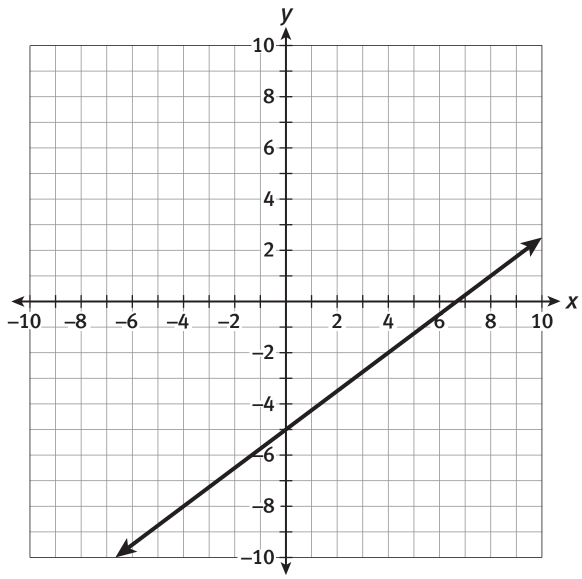
b. $y = -2x + 1$

c. $y = -3x + 4$

17. Write an equation for the line graphed below.



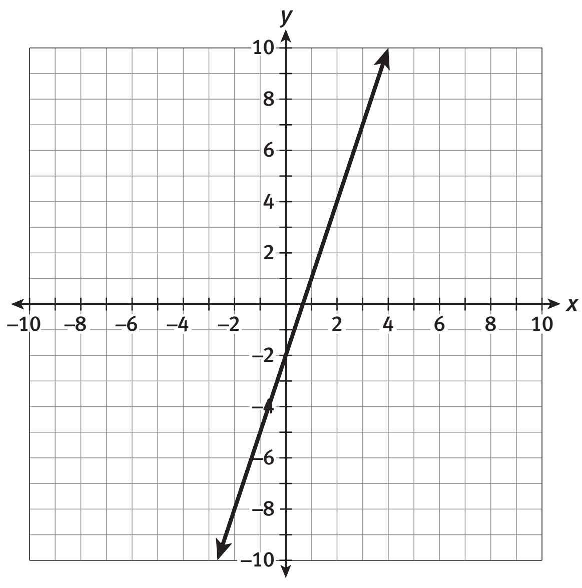
18. Identify the slope and y-intercept for the graph below.



MATHEMATICAL PRACTICES
Make Sense of Problems

19. The graph and equation below represent different linear relationships.

$$y = \frac{1}{4}x + 2$$



Identify the slope and y-intercept of both linear relationships.

Proportional Relationships

ACTIVITY 13

Vary Interesting

Lesson 13-1 Linear Proportional Relationships

My Notes

Learning Targets:

- Represent linear proportional situations with tables, graphs, and equations.
- Identify slope and y-intercept in these representations and interpret their meaning in real-life contexts.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Marking the Text, Look for a Pattern, Create Representations, Construct an Argument

Scarlett is spending her summer afternoons as a junior lifeguard at a local pool. Each afternoon she works, she is paid $10.00 plus $2.00 per hour. In the evenings she is umpiring for a recreational softball league at the local community center. They pay her $4.00 for each hour she works.

1. **Construct viable arguments.** Which job do you think is better? Explain your reasoning.

2. Scarlett wants to determine how much money she can possibly save by the end of the summer. To do this, she creates tables to track her earnings.

   Complete each of the two tables below to track Scarlett's earnings for the first 5 hours she works at each job.

Lifeguarding	
Time in Hours	Earnings
0	
1	
2	
3	
4	
5	

Umpiring	
Time in Hours	Earnings
0	
1	
2	
3	
4	
5	

My Notes

3. Do the earnings from either job have a constant rate of change? Explain your response. Share your ideas with a partner or in your group. Use precise math terms and academic vocabulary. As you listen to your peers, make notes of new words and how they can be used to describe mathematical concepts.

4. Consider Scarlett's job as a junior lifeguard.

a. Write an equation that could be used to determine the amount of money, y, that Scarlett would earn for x hours working at the pool.

b. Identify the slope in the equation and explain its meaning in the problem situation.

c. Identify the y-intercept in the equation and explain its meaning in the problem situation.

5. Consider Scarlett's job as an umpire.

a. Write an equation that could be used to determine the amount of money, y, that Scarlett would earn for x hours working as an umpire.

b. Identify the slope in the equation and explain its meaning in the problem situation.

c. Identify the y-intercept in the equation and explain its meaning in the problem situation.

As you listen to group and class discussions, be sure to ask for clarification of terms you do not understand. Record the terms and make notes about them in your math notebook.

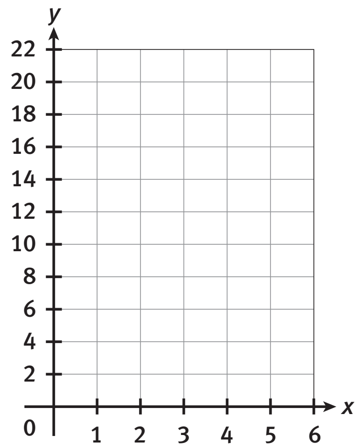
6. Describe the similarities and differences in the equations for the two jobs. How are these similarities and differences exemplified in the tables for the two jobs?

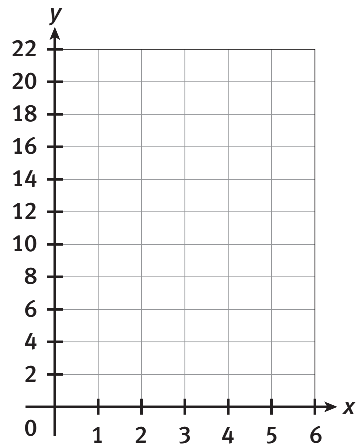
My Notes

DISCUSSION GROUP TIPS

When working with a partner or in groups, listen as group members describe their ideas and understanding, and ask for clarification of concepts that are unclear.

7. Graph the lines representing Scarlett's two jobs on the grids provided. Be sure to label the axes and include a title for each graph.





8. Compare and contrast the two graphs. How do the similarities and differences in the graphs relate to the equations and tables?

9. If Scarlett doubled the amount of time that she worked at the pool, would she double the amount of money that she made? Explain using examples.

My Notes

10. If she doubled the amount of time she worked as an umpire at the community center, would Scarlett double the amount of money that she made? Explain using examples.

Check Your Understanding

Use the two data tables below for Items 11–14.

A.

x	1	2	3	4
y	4	7	10	13

B.

x	1	2	3	4
y	3	6	9	12

11. Draw a graph for each data table. Be sure to label the axes.

12. Write an equation for each graph.

13. Consider the graph of A.
- **a.** What is the slope?
- **b.** What is the y-intercept?
- **c.** When the value of x doubles, does the value of y double?

14. Consider the graph of B.
- **a.** What is the slope?
- **b.** What is the y-intercept?
- **c.** When the value of x doubles, does the value of y double?

LESSON 13-1 PRACTICE

Nina wants to sign up for swim classes. The school charges \$6 per visit. The recreation center charges an initial fee of \$15 plus \$3 per visit.

15. Complete the tables to show the cost of lessons at the school and the center.

School Swim Classes

Visits	1	2	3	4
Cost (dollars)				

Recreation Center Swim Classes

Visits	1	2	3	4
Cost(dollars)				

16. a. Write an equation that can be used to find the total cost, y, for the number of visits, x, at the school swim classes from Item 15.

b. Write an equation that can be used to find the total cost, y, for the number of visits, x, at the recreation center swim classes from Item 15.

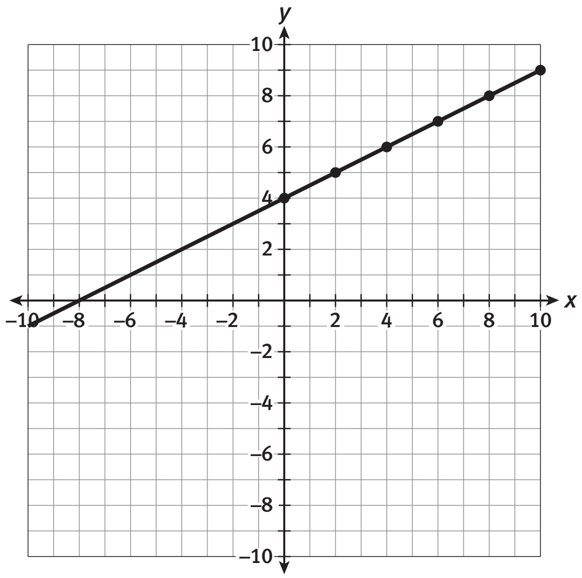
My Notes

17. a. Graph the equation you wrote for Item 16a. Be sure to label the axes.
b. Graph the equation you wrote for Item 16b. Be sure to label the axes.

18. a. Use your graph from Item 17a to find its slope and y-intercept.
b. Use your graph from Item 17b to find its slope and y-intercept.

19. Using the information from Items 15–18, where should Nina take her swim classes? Explain.

20. Critique the reasoning of others. Rayna wrote the equation for the graph shown as $y = \frac{1}{2}x$. Do you agree with Rayna? Explain your reasoning.



My Notes

Learning Targets:

- Solve problems involving direct variation.
- Distinguish between proportional and nonproportional situations using tables, graphs, and equations.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Sharing and Responding, Look for a Pattern, Discussion Groups

If changing one variable in an equation by a factor causes the other variable to change by the same factor, we say the two variables are **directly proportional** and call the equation relating the two variables a ***direct variation*** equation. Direct variation relationships describe many real-life situations.

MATH TERMS

A **direct variation** is a relationship between two variables x and y that can be written in the form $y = kx$, where k is any constant other than 0.

Direct variation equations are *linear* equations.

Example A

To relate time and distance for a car traveling at 55 mph, the equation $d = 55t$ can be used. Substitute values for t to see the direct variation relationship. What is the relationship between d and t?

Step 1: If $t = 2$ hours, the distance traveled, d, will be 110 miles.

Step 2: If the time is doubled so that $t = 4$ hours, the distance traveled, d, will also be doubled to 220 miles.

Solution: When the value for time, t, is doubled, the value for distance traveled, d, is also doubled. The variables d and t are directly proportional.

Try These A

Determine whether the relationship between the variables in the following equations is directly proportional or not. Explain your reasoning.

a. $y = 20x$

b. $y = 20 + x$

1. Consider the tables, graphs, and equations for Scarlett's two jobs in the previous lesson. Which of her jobs is an example of a directly proportional relationship? Justify your response with evidence from the table, graph, and equation.

My Notes

2. a. Create a table, an equation, and a graph for each of the following situations.

Situation A: The distance Lili walks is 2 times the number of minutes she has been walking.

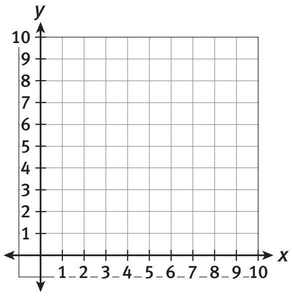
Situation B: The distance traveled by the rabbit in a critter race is consistently 1 inch more than twice the distance traveled by the iguana.

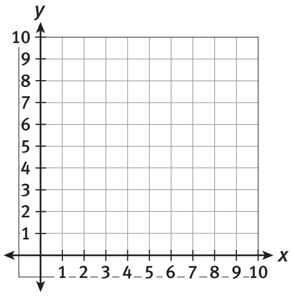
Situation C: The amount of water in the pitcher is half the time elapsed in seconds.

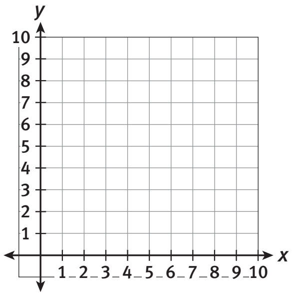
Situation A	
x	y
1	
2	
3	
4	
5	

Situation B	
x	y
1	
2	
3	
4	
5	

Situation C	
x	y
1	
2	
3	
4	
5	







Equation: ___________ Equation: ___________ Equation: ___________

b. Write a conjecture about the y-intercept of a directly proportional relationship.

My Notes

3. Which of the situations in Item 2 are examples of directly proportional relationships? Justify your response.

Check Your Understanding

Determine whether each of the following is an example of a directly proportional relationship or not. Explain your reasoning.

4. Michael eats three pieces of fruit each day.

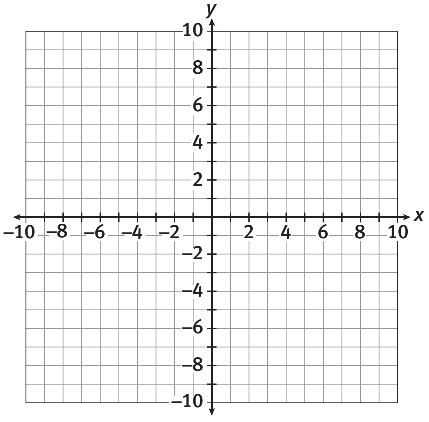
5. Admission to the school carnival costs $5.00 plus $0.75 per ride.

6. $y = x^2$

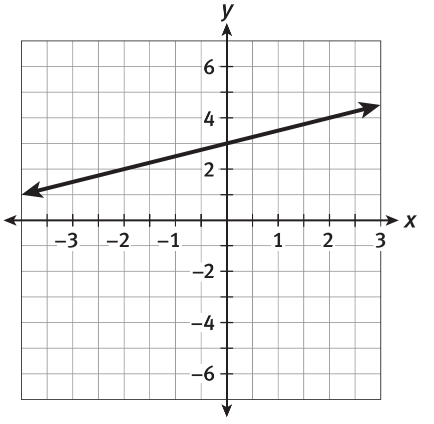
7.

x	y
2	32
4	128
6	288
8	512
10	800

8.



9. Kyra believes that the following graph shows a directly proportional relationship. Kwame disagrees. Who is correct? Justify your response.



My Notes

Direct variation equations can be written in the form $y = kx$. The value of k in the equation $y = kx$ is called the **constant of variation** or the **constant of proportionality.**

10. Use your tables, graphs, and equations from Item 2 to explain the relationship between the constant of variation, the rate of change, and the slope of a graph.

11. Model with mathematics. Direct variation equations are often used to solve real-world problems. In each of the following, determine the value of k, write an equation in the form $y = kx$ to represent the situation, and then use the equation to solve the problem.

a. The Chumas family is driving from North Carolina to California on vacation. They drive 330 miles in 5.5 hours. How many hours would it take them to drive 720 miles?

My Notes

b. Many animals age more rapidly than humans do. The chart below shows equivalent ages for dogs and humans. What is the equivalent dog age for a human who is 16 years old?

Dog Age	0	1	2	3	4	5
Human Age	0	7	14	21	28	35

Check Your Understanding

12. Ellen is training to compete in a half marathon. She currently runs at a rate of 4.5 miles per hour.

a. Create a table and a graph showing Ellen's distance as it relates to the amount of time she runs.

b. Write an equation to represent the relationship between the time she runs and the distance she runs.

c. Is the equation from part b a direct variation equation? Explain your reasoning.

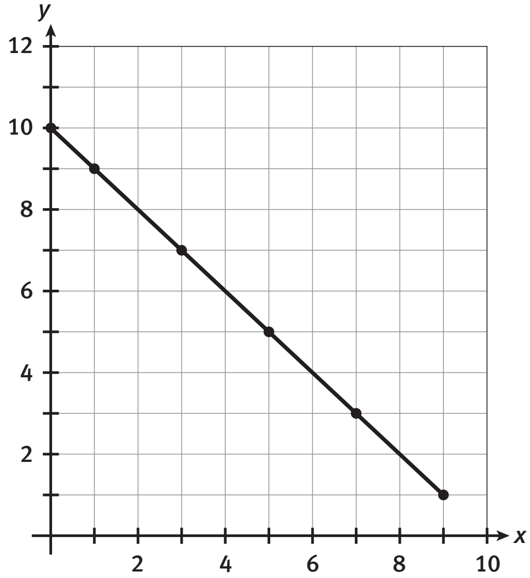
d. If Ellen runs 13.1 miles to complete the half marathon, how long will it take her to finish the race?

My Notes

LESSON 13-2 PRACTICE

For each representation below, tell whether the relationship is directly proportional or nonproportional.

13.



14.

a	*b*
0	0
2	200
4	400
6	600

15. $y = \frac{1}{3}x$

16. $d = 50 + 10h$

17.

x	0	5	10	15
y	1	26	51	76

18. Lupe reads 12 pages of her book every night.

19. **Make sense of problems.** Mrs. Tandy paid a $50 registration fee to join Fitness Club. She pays a monthly rate of $25. Write an equation to represent the total Mrs. Tandy pays for Fitness Club for *x* months. Determine if the equation you wrote represents a proportional relationship or a nonproportional relationship. Explain your reasoning.

ACTIVITY 13 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 13-1

1. Consider the following equations.
A: $y = 2x + 4$ B: $y = 2x$.
a. What are the slope and y-intercept for equation A?
b. What are the slope and y-intercept for equation B?

2. Kwame can type 25 words per minute.
a. Create a table that represents the speed at which Kwame can type.
b. Graph the data.
c. Write an equation that that represents the relationship between the number of words Kwame can type and his typing speed.
d. What is the slope in the equation you wrote in part c? What does the slope represent in this situation?
e. What is the y-intercept in the equation you wrote in part c? What does the y-intercept represent in this situation?

3. Ashleigh is hiking at a rate of 4 miles per hour.
a. Create a table to represent Ashleigh's distance after 1, 2, 3, 4, and 5 hours of hiking.
b. Graph the data.
c. Write an equation that represents the relationship between the time and her distance.
d. What is the slope of the equation you wrote in part c? What does the slope represent in this situation?
e. What is the y-intercept in the equation you wrote in part c? What does the y-intercept represent in this situation?

The tables show summer job earnings for Carolina and Monique. Use these tables for Item 4.

Carolina	
Hours	Earnings
1	$55
2	$60
3	$65
4	$70
5	$75

Monique	
Hours	Earnings
1	$10
2	$20
3	$30
4	$40
5	$50

4. Which equation shows the relationship between the hours Carolina works, x, and her earnings, y?
A. $y = \$5x$
B. $y = \$10x$
C. $y = \$5x + \$50$
D. $y = 10x + \$10$

Lesson 13-2

5. Linear relationships are (*sometimes*, *always*, or *never*) examples of directly proportional relationships.

6. Sketch a graph that represents a directly proportional relationship.

7. Sketch a graph that does not represent a directly proportional relationship.

8. Does the table shown represent a direct variation? Justify your response.

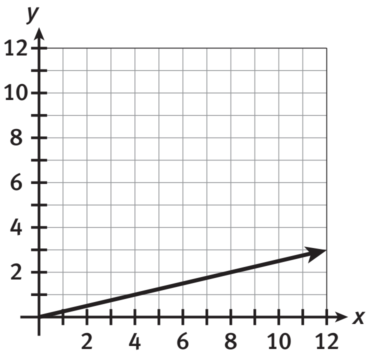
x	y
4	7
6	8
9	11
12	14

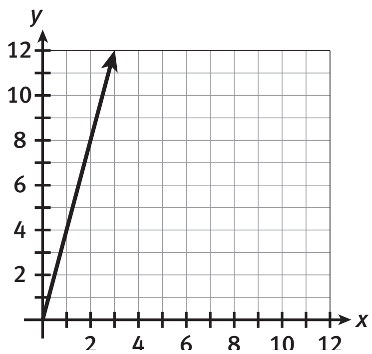
9. Find the constant of variation for the directly proportional relationship represented by the data in the table. Then write the equation that describes the relationship.

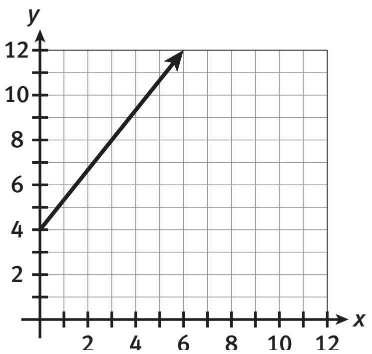
x	y
1	0.2
4	0.8
8	1.6
12	2.4

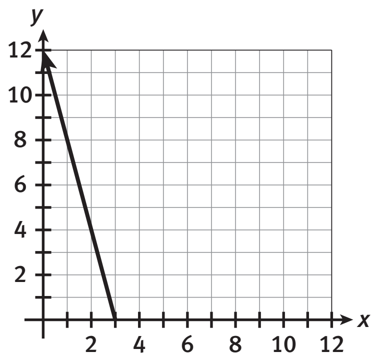
10. Which of the equations below are examples of direct variation? Explain your choices.
A. $y = 2x$
B. $y = 3x + 5$
C. $y = \frac{1}{2}x$
D. $6x = y$

11. The number of cups of flour used in a recipe varies directly with the number of cups of milk used in the recipe. For each cup of milk used, 4 cups of flour are used. If x represents the number of cups of milk and y represents the number of cups of flour, which of the following graphs represents the relationship between cups of milk and cups of flour used in the recipe?

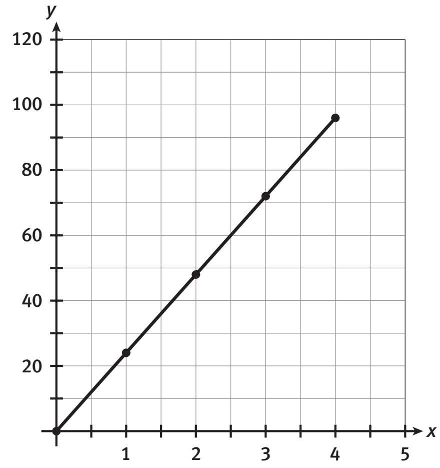
A.


B.


C.


D.


12. Tell whether the relationship shown is directly proportional or nonproportional. Explain your reasoning.



MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

13. Explain how to recognize a directly proportional relationship in an equation, in a table, and in a graph. Use examples to support your responses.

Linear Equations and Rates of Change

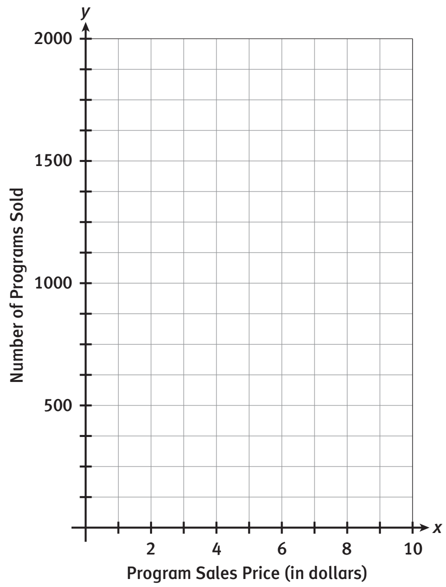
WHO IS THAT?

The Athletic Booster Club at C. Brown High School publishes a fall sports program for the Athletic Department. The purpose of the program is to raise funds for a new scoreboard in the football stadium and to highlight each of the athletes participating in fall sports. The program contains interviews with coaches and athletes, schedules of games and other events, and brief biographies of each athlete and coach.

The booster club members must consider how much to charge for each program, and how much money can be raised.

1. The Finance Chairman has researched previous years' sales records and organized the results in the following table. Graph the data on a grid like the one shown.

Booster Club Program Sales	
Program Price	**Number of Programs Sold**
\$2.50	1,625
\$4.00	1,250
\$5.00	1,000
\$5.50	875
\$7.50	375



2. The relationship between the price and the number of programs sold is linear. Justify this statement using both the table and the graph.

3. Draw a line through the points on your graph.

a. Determine the slope of the line through the points. Describe the meaning of the slope in this problem situation.

b. Use the data in the table, the slope determined in part a, and the graph to determine the y-intercept of the line through the data points. Describe the meaning of the y-intercept in this problem situation.

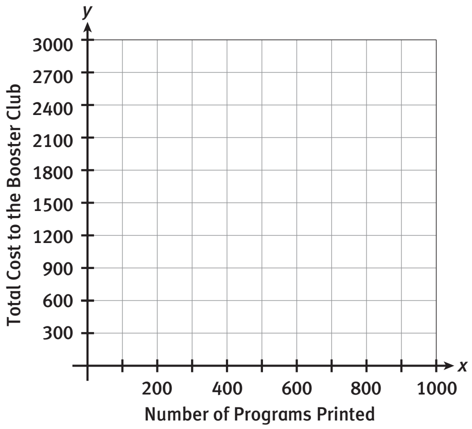
c. Write the equation of the line through the data points in slope-intercept form.

d. Is the relationship between the price of the programs and the number of programs sold directly proportional? Justify your response.

4. Assume the slope of the line was changed to -425.
   a. Write the equation that could be used to represent this new relationship between the price of the programs and the number of programs sold.
   b. Describe the meaning of the slope in this new equation.
   c. Describe any changes in the graph that would result from this change in the slope.

5. Assume the y-intercept of the original line was changed to 3,000.
   a. Write the equation that could be used to represent this new relationship between the price of the programs and the number of programs sold.
   b. Describe the meaning of the y-intercept in this new situation.
   c. Describe any changes in the graph that would result from this change in the y-intercept.

6. L & L Printing will print the programs for \$350 plus an additional \$1.50 per program printed.
   a. Write the equation, in slope-intercept form, that could be used to determine the total cost, C, to the Booster Club for printing a given number of programs, p.
   b. Graph the equation on a grid like the one shown.



7. Red Baron Printing will print the programs for \$1.82 each.
   a. If attendance for the football games averages 1,500 fans per game, which printing company should the Booster Club choose? Show your work and justify your response.
   b. Which printing company contract is an example of a direct variation? Justify your response.

Linear Equations and Rates of Change

WHO IS THAT?

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3a-d, 4a-c, 5a-c, 6a-b, 7a-b)	• Precise and accurate understanding of linear equations, linear graphs, slope, intercept, and proportional relationships.	• Understanding of linear equations, linear graphs, slope, intercept, and proportional relationships.	• Some understanding of linear equations, linear graphs, slope, intercept, and proportional relationships, but with errors.	• Incorrect or incomplete understanding of linear equations, linear graphs, slope, intercept, and proportional relationships.
Problem Solving (Items 7a-b)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1, 3c, 4a, 5a, 6a-b)	• Clear and accurate representation of linear relationships with graphs and equations.	• Representing linear relationships with graphs and equations.	• Errors in representing linear relationships with graphs and equations.	• Inaccurate or incomplete representation of linear relationships with graphs and equations.
Reasoning and Communication (Items 2, 3a-b, 3d, 4b-c, 5b-c, 7a-b)	• Accurately and precisely describing the meaning of slope and intercept and whether a relationship is proportional.	• Describing the meaning of slope and intercept and whether a relationship is proportional.	• Errors in describing the meaning of slope and intercept and whether a relationship is proportional.	• An incomplete or inaccurate description of the meaning of slope and intercept and whether a relationship is proportional.

Graphing Systems of Linear Equations

ACTIVITY 14

Systems of Trees

Lesson 14-1 Understanding Solutions to Linear Systems

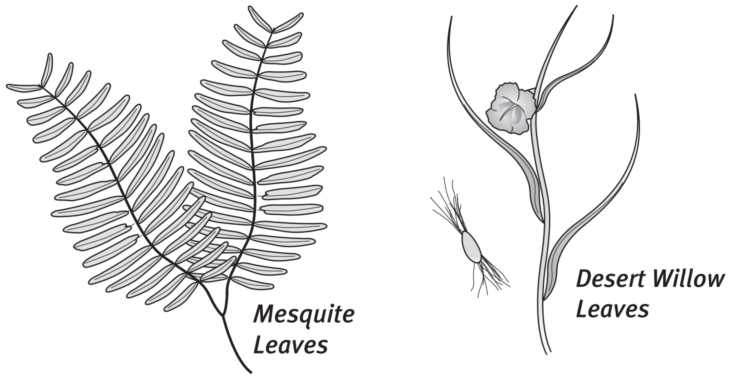
My Notes

Learning Targets:

- Understand that solutions to systems of linear equations correspond to the points of intersection of their graphs.
- Solve systems of linear equations numerically and by graphing.
- Use systems of linear equations to solve real-world and mathematical problems.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Work Backwards, Graphic Organizer, Create Representations

Chip decided to upgrade the landscaping in his yard. He bought a 10-gallon mesquite tree and a 50-gallon desert willow and planted them. After one year he was shocked at the growth of both trees, so he measured their heights. The mesquite was 5 feet tall and the desert willow was 8 feet tall. The next year he measured again and found the mesquite was 6 feet, 6 inches tall and the desert willow was 8 feet, 8 inches tall.



1. List all the numerical information associated with each tree.

2. If the trees grew at a constant rate the first two years, how tall were they when Chip planted them?

3. Use the table below to help explain how the height of the mesquite tree compares to the height of the willow over time.

Year	Mesquite (in inches)	Willow (in inches)
0		
1		
2		
3		
4		
5		
6		
7		

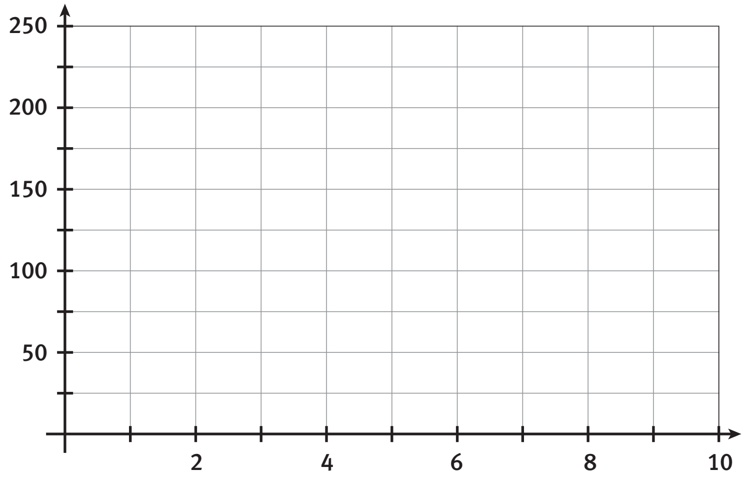
My Notes

4. Let M be the height of the mesquite tree in inches. Write a linear equation that could be used to determine the height of the tree in a given year, t.

5. Write a linear equation that could be used to determine the height, W, in inches of the desert willow in a given year, t.

6. **Model with mathematics.** Is it possible to use the equations you wrote to describe the growth of the two trees to predict the height of the trees at 1.5 years? Explain your reasoning.

7. Graph each of the equations on the grid below and use the graphs to determine in what year the mesquite will reach the same height as the desert willow. Be sure to label the axes.



8. When the mesquite tree and the desert willow are the same height, what is true about the values of W and M?

9. Write and solve an equation to determine the value of t when the mesquite tree and the desert willow are the same height.

10. Describe the meaning of the value of t determined in Item 9.

11. How does the solution to the equation in Item 9 relate to the table in Item 3 and the graph in Item 7?

Equations M and W can together be described as a ***system of linear equations***. The ***solution to a system of linear equations*** will always be the point or set of points where the two lines intersect.

Systems of linear equations can also be solved numerically.

12. Create a table of values to determine the solution to the following system of equations.

$$\begin{cases} y = -x - 2 \\ y = \frac{2}{3}x + 3 \end{cases}$$

x	y_1	y_2
−6		
−5		
−4		
−3		
−2		
−1		
0		
1		

My Notes

MATH TERMS

A **system of linear equations** is a collection of equations all considered simultaneously.

The word *linear* indicates that there will only be equations of lines in this collection.

A point, or set of points, is the **solution to a system of equations** in two variables when it makes both equations true.

READING MATH

y_1 is read "y sub 1" and refers to the y values from the first equation. y_2 refers to the y values from the second equation.

My Notes

13. Determine which ordered pair in the set {(2, 2), (2, 3), (2, 4), (3,3)} is the solution to the system of linear equations.

$$\begin{cases} y = -x + 5 \\ y = x + 1 \end{cases}$$

Check Your Understanding

14. Determine which of the points {(1, −2), (−1, 2), (1, 2), (−1, −2)} are solutions to the system of equations.

$$\begin{cases} y = 3x - 5 \\ y = -\frac{1}{4}x - \frac{7}{4} \end{cases}$$

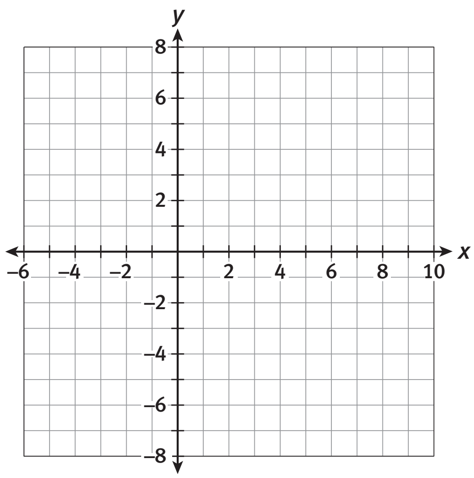
15. Create a table of values to determine the solution to the following system of equations.

$$\begin{cases} y = 5x + 4 \\ y = 2x + 1 \end{cases}$$

16. Complete the following table for each equation. Then graph each equation.

$$y = \frac{1}{2}x - 6 \text{ and } y = -\frac{5}{3}x + \frac{8}{3}$$

x	y_1	y_2
−4		
−3		
−2		
−1		
0		
1		
2		
3		
4		



17. Use the table in Item 16 to determine at which value of x the values of y_1 and y_2 are the same.

18. Use the graph in Item 16 to determine at which value of x the values of y_1 and y_2 are the same.

LESSON 14-1 PRACTICE

My Notes

19. Create a table of values to determine the solution to the following system of equations.

$$\begin{cases} y = -x - 2 \\ y = \frac{2}{3}x + 3 \end{cases}$$

20. Determine which of the points {(1, −2), (−1, 2), (1, 2), (−1, −2)}, if any, are solutions to the system of equations.

$$\begin{cases} x = \frac{4}{5}y - 7 \\ 2x + y = -1 \end{cases}$$

21. Graph each of the equations below and use your graph to determine at which value of x the values of y are the same.

$$\begin{cases} 2x - 12y = -3 \\ x + 4y = -4 \end{cases}$$

The table below represents deposits to Joselyn's savings account and to Ben's savings account over the course of 5 weeks.

Week (W)	J's Account (in dollars)	B's Account (in dollars)
0	5	12
1	10	16
2	15	20
3	20	24

22. **a.** Let J be the amount of money in Joselyn's savings account. Write a linear equation that could be used to determine the amount of money in the account in a given week, W.

b. **Reason abstractly.** Let B be the amount of money in Ben's savings account. Write a linear equation that could be used to determine the amount of money in the account in a given week, W.

c. In what week will Joselyn and Ben have the same amount of money in their accounts? Explain your reasoning.

My Notes

Learning Targets:

- Convert linear equations into slope-intercept form.
- Solve systems of linear equations by graphing.
- Solve simple systems of linear equations by inspection.

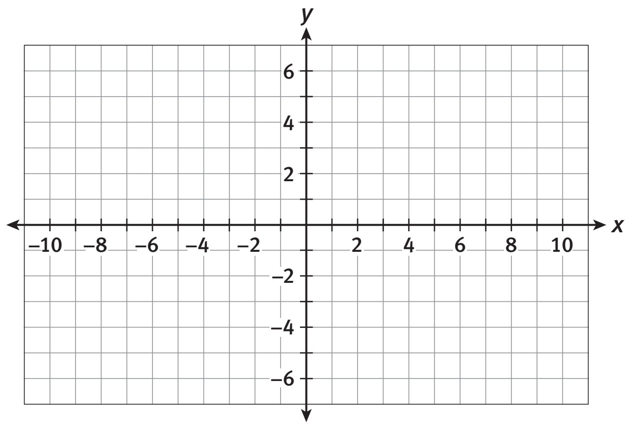
SUGGESTED LEARNING STRATEGIES: Marking the Text, Create a Plan, Identify a Subtask, Discussion Group, Construct an Argument

The solution to a system of two linear equations will always be the point or points where the two lines intersect. So it is possible to solve systems of linear equations by graphing. To solve by graphing you must:

- Graph each line on the same coordinate plane.
- Determine the coordinates of the point where the lines cross.

1. Determine the solution to the following system of equations by graphing.

$$\begin{cases} y = 2x - 4 \\ y = -\frac{1}{2}x + 1 \end{cases}$$



TECHNOLOGY TIP

Often it is helpful to use a graphing calculator when solving systems of equations by graphing. You can graph equations on the calculator and use the built-in commands to determine the intersection point.

When graphing, it is helpful to have the equation in slope-intercept form: $y = mx + b$, where m is the slope and b is the y-intercept. When graphing an equation in this form, first plot the y-intercept $(0, b)$ and use the slope to move from the y-intercept and plot other points on the line. However, not all equations are written in slope-intercept form.

My Notes

MATH TIP

To *solve for y* it is necessary to use inverse operations to isolate *y* on one side of the equal sign.

Example A

Solve $3x - 4y = 16$ for y.

Step 1: Use inverse operations. Subtract $3x$ from both sides.

$$\begin{array}{rcl} 3x - 4y &=& 16 \\ \underline{-3x \qquad} && \underline{-3x \qquad} \\ -4y &=& -3x + 16 \end{array}$$

Step 2: Use inverse operations. Divide both sides by -4.

$$\frac{-4y}{-4} = \frac{-3x}{-4} + \frac{16}{-4}$$

Solution: $y = \frac{3}{4}x - 4$

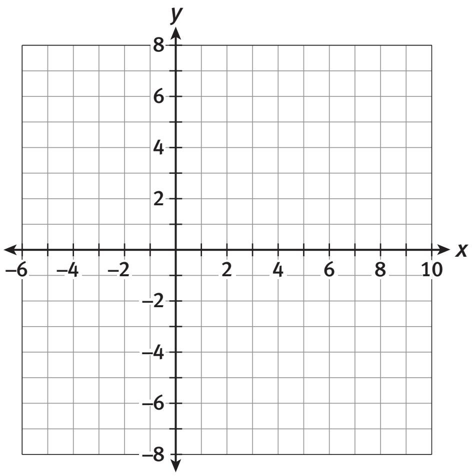
Try These A

Solve each of the following equations for y.

a. $x + y = 8$
b. $2x + y = 6$
c. $3y = -6x - 15$
d. $3x + 2y = 14$
e. $4x - 2y = 4$

2. Determine the solution to the following system of equations by graphing them on the grid provided.

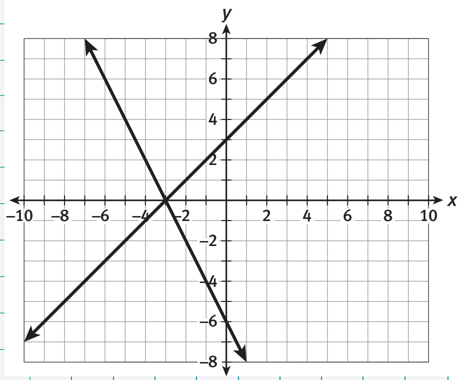
$$\begin{cases} x + y = 4 \\ 4x - y = 1 \end{cases}$$



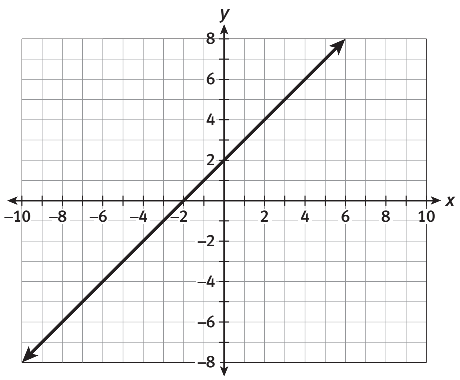
My Notes

3. Each of the three figures below is a graph of a system of equations. Identify each system of equations as having *one solution, no solution,* or *infinitely many solutions*. Justify your choices.

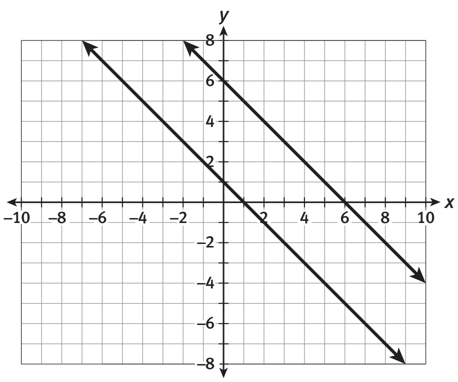
Graph 1



Graph 2



Graph 3



MATH TIP

Summarize in your own words how to use the slope and *y*-intercept of a pair of lines to determine if they intersect, coincide, or are parallel.

Often, it is possible to determine the solution to a system of equations by examining the equations carefully and *without* using a table or graphing.

4. For each system of equations shown below, carefully examine the system and make a conjecture as to the solution. Explain your reasoning.

a. $\begin{cases} 5x + 4y = 8 \\ 5x + 4y = -3 \end{cases}$

b. $\begin{cases} 3x + 2y = 4 \\ 6x + 4y = 8 \end{cases}$

c. Discuss in your groups and make a conjecture about the slopes and *y*-intercepts of lines that intersect.

ACADEMIC VOCABULARY

Coincide means to occur at the same time or to occupy the same place in space or time.

d. Discuss in your groups and make a conjecture about the slopes and *y*-intercepts of lines that **coincide**.

e. Discuss in your groups and make a conjecture about the slopes and *y*-intercepts of lines that are parallel.

5. **Construct viable arguments.** Compare and contrast the advantages and disadvantages of using the numerical method and the graphing method to solve a system of equations. Given the option, which method would you choose? Why?

My Notes

DISCUSSION GROUP TIPS

When discussing or presenting mathematical concepts with a partner or in a group, be sure to use precise terminology.

Check Your Understanding

Solve each of the following systems of equations by graphing.

6. $\begin{cases} y = -x - 1 \\ y = -5x - 17 \end{cases}$

7. $\begin{cases} y = \frac{1}{2}x + 4 \\ y = -\frac{3}{2}x - 4 \end{cases}$

8. Solve each of the following systems of equations without graphing. State your solution and the reason for the solution.

   a. $\begin{cases} x + y = 8 \\ x + y = 4 \end{cases}$

   b. $\begin{cases} 2x - 3y = -1 \\ y = \frac{2}{3}x + \frac{1}{3} \end{cases}$

9. Determine the solution to the following system of equations by graphing. Remember to solve each equation for y, if needed, before you begin.

$$\begin{cases} -3x + y = -4 \\ y = -x - 3 \end{cases}$$

My Notes

LESSON 14-2 PRACTICE

10. Solve by graphing.

$$\begin{cases} 2y = x + 8 \\ 2x - 4y = -16 \end{cases}$$

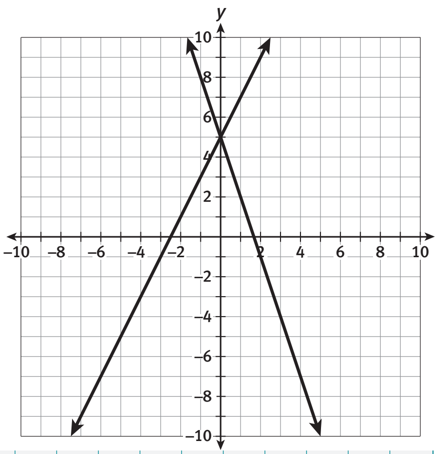
11. Explain (without graphing) why the system of equations below would not have infinitely many solutions.

$$\begin{cases} 2x - 4y = -1 \\ y = x - 1 \end{cases}$$

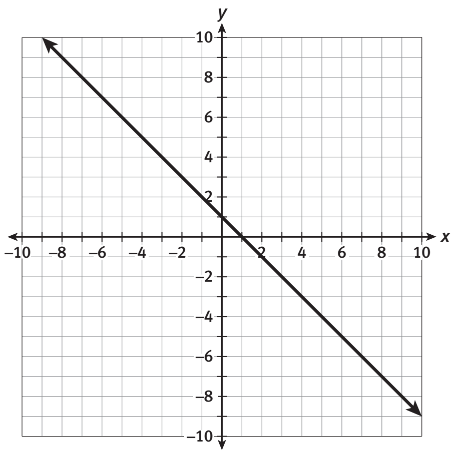
12. Graph the system of equations in Item 11 and verify that there is one unique solution.

13. Each of the three figures below is a graph of a system of equations. Identify each system of equations as having one *solution*, *no solution*, or *infinitely many solutions*. Justify your choices.

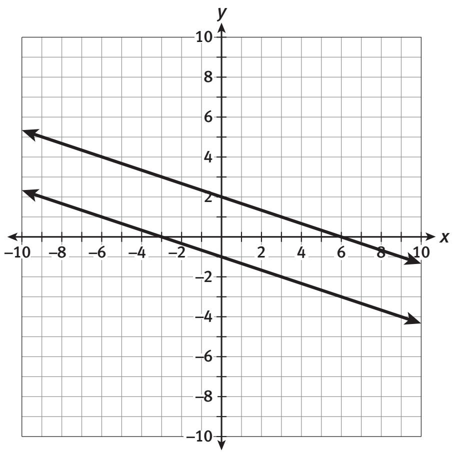
Graph 1



Graph 2



Graph 3



14. Use appropriate tools strategically. Create a table of values to determine the solution to the following system of equations.

$$\begin{cases} y = 5x - 3 \\ y = 2x - 6 \end{cases}$$

x	y_1	y_2
−4		
−3		
−2		
−1		
0		
1		
2		
3		

ACTIVITY 14 PRACTICE

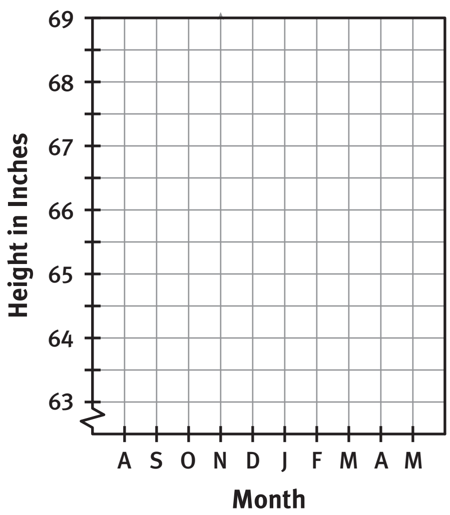
Write your answers on notebook paper.
Show your work.

Lesson 14-1

Itmar was 63 inches tall in August at the start of eighth grade. His best friend, Megan, was 65 inches tall at that time. Itmar grew an average of one-half inch each month through May. Megan grew one-fourth of an inch each month through May.

1. Write two equations, one to show Itmar's height at any time during the school year and one to show Megan's. Use h for height and m to represent the number of months since August.

2. Graph each student's height from August to May.



3. Will Itmar be taller than Megan by the end of eighth grade in May? Is there a time when their heights will be the same? Justify your reasoning.

4. Does the line through the points (2, 2) and (3, 4) intersect the line through the points (−1, 5) and (0, 7)? Explain your response.

5. Determine which of the following points are solutions to the system of equations.

$$\begin{cases} 3x + 2y = 5 \\ x + 2y = 7 \end{cases}$$

A. (0, 3) **B.** (−1, 4)
C. (3, 0) **D.** (4, −1)

6. Determine which of the following points are solutions to the system of equations.

$$\begin{cases} 3x - y = -4 \\ 2x - 5y = 19 \end{cases}$$

A. (−3, 5) **B.** (3, −5)
C. (3, 5) **D.** (−3, −5)

7. Create a table of values to determine the solution to the following system of equations.

$$\begin{cases} y = 5x + 4 \\ y = 2x + 1 \end{cases}$$

x	y_1	y_2
−4		
−3		
−2		
−1		
0		
1		
2		
3		

Lesson 14-2

8. Solve each system of linear equations by graphing.

a. $\begin{cases} y = 3x + 2 \\ y = -2x - 8 \end{cases}$

b. $\begin{cases} x + y = -1 \\ 2x + 2y = 4 \end{cases}$

9. Determine the value of $x - y$ in the following system of equations.

$$\begin{cases} 6x - 3y = 42 \\ 8x + 4y = -8 \end{cases}$$

A. -5 **B.** 11
C. 5 **D.** -11

10. Solve $2y = 3x + 12$ for y.

11. Solve the following system of equations without graphing. State your solution and the reason for the solution.

$$\begin{cases} x - 6y = -3 \\ 2x + 3y = 9 \end{cases}$$

12. Determine the solution to the following system of equations by graphing. Remember to solve each equation for y before you begin.

$$\begin{cases} 6x + 2y = -4 \\ 2x + y = -4 \end{cases}$$

MATHEMATICAL PRACTICES
Make Sense of Problems and Persevere in Solving Them

13. Determine the solution to the following system of equations by graphing.

$$\begin{cases} y = \frac{1}{3}x + 2 \\ y = -x - 3 \end{cases}$$

Solving Systems of Linear Equations Algebraically

What's the Point?

Lesson 15-1 Solving Linear Systems Algebraically

My Notes

Learning Targets:

- Connect solutions to systems of linear equations to the points of intersection of their graphs.
- Solve systems of linear equations algebraically.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Summarizing, Note Taking, Identify a Subtask, Discussion Group

The solution to a system of linear equations is the point where the two lines intersect. The intersection point is a solution to both equations in the system. Algebraically, this point can be determined by calculating when the two expressions for y will be equal to each other.

Example A

Solve the system of equations algebraically. $\begin{cases} y = 0.25x - 1 \\ y = -x + 4 \end{cases}$

Step 1: Set the expressions equal to each other.

$$0.25x - 1 = -x + 4$$

Step 2: Solve for x.

$$\begin{array}{r} 0.25x - 1 = -x + 4 \\ \underline{+x \qquad\qquad +x} \\ 1.25x - 1 = 4 \\ \underline{+1 \quad +1} \\ \frac{1.25x}{1.25} = \frac{5}{1.25} \\ x = 4 \end{array}$$

Step 3: Substitute x into one of the original equations, and solve for y.

$$y = 0.25(4) - 1$$
$$y = 1 - 1$$
$$y = 0$$

Step 4: Check your solution using the other equation.

$$y = -x + 4$$
$$0 = -4 + 4$$
$$0 = 0$$

Solution: Write the solution as an ordered pair.

(4, 0); The lines intersect at the point (4, 0).

My Notes

Try These A

Solve each system of equations algebraically. Show all steps. Remember to write the solution as an ordered pair.

a. $\begin{cases} y = \frac{3}{2}x + 1 \\ y = 2x + 2 \end{cases}$ **b.** $\begin{cases} y = 4x + 1 \\ y = -2x - 5 \end{cases}$ **c.** $\begin{cases} y = 5x + 1 \\ y = -4 \end{cases}$

Another method for solving systems of equations involves ***substitution*****.** This method is similar to the algebraic method you learned in the last example.

Example B

Solve the system of equations using substitution.

$$\begin{cases} x + y = 8 \\ 3x + 2y = 14 \end{cases}$$

Step 1: Solve one equation for one of the variables.

$$\begin{aligned} x + y &= 8 \\ \underline{-x \quad\quad} &\underline{\quad -x} \\ y &= -x + 8 \end{aligned}$$

Step 2: Substitute this expression into the other equation.

$$\begin{aligned} 3x + 2y &= 14 \\ 3x + 2(-x + 8) &= 14 \end{aligned}$$

Step 3: Solve the resulting equation for the remaining variable.

$$\begin{aligned} 3x + 2(-x + 8) &= 14 \\ 3x - 2x + 16 &= 14 \\ x + 16 &= 14 \\ \underline{-16 \quad} &\underline{\quad -16} \\ x &= -2 \end{aligned}$$

Step 4: Substitute into one of the original equations to find the corresponding value of the other variable.

$$\begin{aligned} -2 + y &= 8 \\ \underline{+2 \quad\quad} &\underline{\quad +2} \\ y &= 10 \end{aligned}$$

Step 5: Check the solution in both equations.

$$\begin{aligned} x + y &= 8 \\ -2 + 10 &= 8 \\ 8 &= 8 \end{aligned} \qquad \begin{aligned} 3x + 2y &= 14 \\ 3(-2) + 2(10) &= 14 \\ -6 + 20 &= 14 \\ 14 &= 14 \end{aligned}$$

Solution: Write the solution as an ordered pair.
(−2, 10); The lines intersect at the point (−2, 10).

MATH TIP

Solve for a variable with a coefficient of 1.

My Notes

Try These B

Use substitution to solve the systems.

a. $\begin{cases} y = 5x \\ x + y = -6 \end{cases}$ **b.** $\begin{cases} x + 2y = 8 \\ 3x - 4y = 4 \end{cases}$

Systems of linear equations can also be solved using *elimination*. In the elimination method, one variable is *eliminated*, creating an equation in one variable that can be solved using inverse operations.

MATH TIP

The *elimination method* is sometimes called *linear combination*.

Example C

Solve the system of equations using elimination. $\begin{cases} 7x + 5y = -1 \\ 4x - y = -16 \end{cases}$

Step 1: Decide which variable to eliminate. Then, use multiplication to create equations in which at least two variable terms are opposites. Select y to eliminate, and multiply the second equation by 5 so that the y terms are opposites.

$$\begin{cases} 7x + 5y = -1 \\ 4x - y = -16 \end{cases} \rightarrow \begin{cases} 7x + 5y = -1 \\ 5(4x - y) = 5(-16) \end{cases} \rightarrow \begin{cases} 7x + 5y = -1 \\ 20x - 5y = -80 \end{cases}$$

MATH TIP

Keep the equation balanced by multiplying the factor with each term in the equation.

Step 2: Add the two equations and solve for the remaining variable.

$$\begin{array}{r} 7x + 5y = -1 \\ \underline{20x - 5y = -80} \\ 27x + 0y = -81 \end{array}$$

Step 3: Solve the resulting equation.

$$\begin{aligned} 27x + 0y &= -81 \\ \frac{27x}{27} &= \frac{-81}{27} \\ x &= -3 \end{aligned}$$

Step 4: Substitute into one of the original equations to determine the value of the second variable.

$$\begin{aligned} 7x + 5y &= -1 \\ 7(-3) + 5y &= -1 \\ -21 + 5y &= -1 \\ \underline{+21 \qquad} &\underline{\quad +21} \\ \frac{5y}{5} &= \frac{20}{5} \\ y &= 4 \end{aligned}$$

Step 5: Check the solution in both equations.

$$\begin{aligned} 7x + 5y &= -1 \\ 7(-3) + 5(4) &= -1 \\ -21 + 20 &= -1 \\ -1 &= -1 \end{aligned} \qquad \begin{aligned} 4x - y &= -16 \\ 4(-3) - 4 &= -16 \\ -12 - 4 &= -16 \\ -16 &= -16 \end{aligned}$$

Solution: Write the solution as an ordered pair.
(−3, 4); The lines intersect at the point (−3, 4).

My Notes

Try These C

Use elimination/linear combination to solve the following.

a. $\begin{cases} 3x + 4y = 17 \\ 5x - 4y = 7 \end{cases}$

b. $\begin{cases} 3x - 4y = 12 \\ 5x + 8y = 20 \end{cases}$

c. $\begin{cases} 18x + 36y = -42 \\ 11x + 9y = -30 \end{cases}$

Check Your Understanding

Solve each system. Explain why you chose a particular solution method.

1. $\begin{cases} y = 3x - 4 \\ 3x + y = -4 \end{cases}$

2. $\begin{cases} 4x + 3y = 19 \\ 3x - 4y = 8 \end{cases}$

3. Sara solved a system of equations algebraically. Her result was $7 = -4$. All of her work was correct. Describe the graph of the system of equations. Explain your reasoning.

My Notes

LESSON 15-1 PRACTICE

4. Solve by elimination/linear combination:

$$\begin{cases} 3x + 7y = -1 \\ 4x - 3y = 11 \end{cases}$$

5. Solve by substitution:

$$\begin{cases} y = -3x \\ 2x - 3y = 22 \end{cases}$$

6. Solve using the method of your choice (numerical, graphing, substitution, elimination/linear combination):

$$\begin{cases} y = \frac{1}{2}x - 8 \\ 2x + 5y = -13 \end{cases}$$

7. Write a system of linear equations with no solution. Describe the appearance of the graph.

8. Reason abstractly. Write a system of linear equations that has a solution of (2, 7). Explain how you determined such a system.

My Notes

Learning Targets:

- Write linear systems to solve real-world and mathematical problems.
- Solve systems of linear equations algebraically.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Close Reading, Marking the Text, Summarizing, Note Taking, Identify a Subtask, Discussion Group

Systems of linear equations can also be used to solve real-world and mathematical problems.

Example A

A group of 6 youths and 5 adults sign up to take a karate class for a total cost of \$147. Another group of 4 youths and 4 adults pay a total cost of \$108. What was the cost for 1 adult and the cost for 1 youth to take the class?

Step 1: Define your variables.
Let x = cost for 1 youth; y = cost for 1 adult

Step 2: Write one equation to represent the first karate class.

$$6x + 5y = 147$$

Write another equation to represent the second karate class.

$$4x + 4y = 108$$

Step 3: Solve the system. Select y to eliminate. The LCM of the coefficients of the y terms is 20, so multiply the first equation by -4 and the second equation by 5.

$$\begin{cases} 6x + 5y = 147 \\ 4x + 4y = 108 \end{cases} \rightarrow \begin{cases} -4(6x + 5y) = -4(147) \\ 5(4x + 4y) = 5(108) \end{cases} \rightarrow \begin{cases} -24x - 20y = -588 \\ 20x + 20y = 540 \end{cases}$$

$$\begin{aligned} -24x - 20y &= -588 \\ \underline{20x + 20y} &= \underline{540} \\ \frac{-4x}{-4} &= \frac{-48}{-4} \\ x &= 12 \end{aligned} \qquad \begin{aligned} 6x + 5y &= 147 \\ 6(12) + 5y &= 147 \\ 72 + 5y &= 147 \\ \underline{-72} &\quad \underline{-72} \\ \frac{5y}{5} &= \frac{75}{5} \\ y &= 15 \end{aligned}$$

Step 4: Check the solution in both equations.

$$\begin{aligned} 6x + 5y &= 147 \\ 6(12) + 5(15) &= 147 \\ 72 + 75 &= 147 \\ 147 &= 147 \end{aligned} \qquad \begin{aligned} 4x + 4y &= 108 \\ 4(12) + 4(15) &= 108 \\ 48 + 60 &= 108 \\ 108 &= 108 \end{aligned}$$

Solution: Interpret the solution in the context of the problem. It costs \$12 for a youth to take the class and \$15 for an adult to take the class.

My Notes

Try These A

Use a system of linear equations to solve the following.

a. During a sale on winter clothing, Jody bought 2 scarves and 4 pairs of gloves for $43. Amanda bought 2 scarves and 2 pairs of gloves for $30. How much does one scarf cost? How much for one pair of gloves?

b. Peter placed an order with an online nursery for 6 apple trees and 5 azaleas and the order came to $147. The next order for 3 apple trees and 4 azaleas came to $96. What was the cost for each apple tree and for each azalea?

c. At the end of the 2010 baseball season, the Cincinnati Reds and the New York Yankees together had won a total of 32 World Series. At the end of that season, the Yankees had won 5.4 times as many World Series as the Reds. How many World Series did each team win?

d. Complementary angles are two angles whose measures have a sum of 90 degrees. Angles x and y are complementary. The measure of angle x is 24 degrees greater than the measure of angle y. Determine the measures of angles x and y.

Check Your Understanding

1. Mara bought 8 boxes of pencils and 3 packages of pens for $24. At the same store, Elaine bought 4 boxes of pencils and 6 packages of pens for $30. How much does one box of pencils cost? How much does one package of pens cost?

2. The difference of two numbers is 18. The sum of the numbers is 84. What is the larger number? What is the smaller number?

My Notes

LESSON 15-2 PRACTICE

Use a system of linear equations to solve each problem.

3. A group of 3 adults and 5 children pay a total of \$52 for movie tickets. A group of 2 adults and 4 children pay a total of \$38 for tickets. What is the cost of one adult ticket? What is the cost of one child ticket?

4. The sum of two angle measures is 130 degrees. The difference in the two angle measures is 12 degrees. What are the two angle measures?

5. Mandy buys 5 reams of paper and 3 ink cartridges for \$131. Kevin buys 2 reams of paper and 5 ink cartridges for \$174. How much does one ream of paper cost? How much does one ink cartridge cost?

6. Carlisle thinks of two numbers. One number is 14 less than twice the other number. The two numbers sum to 160. What is the smaller number?

7. **Critique the reasoning of others.** Jacob is asked to solve the problem below:

Kayla is 9 years older than twice Hannah's age. If the difference in their ages is 16, how old is Kayla? How old is Hannah?

Jacob finds Hannah's age to be 16 years. Is he correct? Explain.

ACTIVITY 15 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 15-1

1. Solve by substitution.

a. $\begin{cases} y = -\frac{1}{2}x + 5 \\ 3x - y = 2 \end{cases}$

b. $\begin{cases} y = -3x + 6 \\ 3x + y = 5 \end{cases}$

c. $\begin{cases} 2x + y = 8 \\ 2x - 3y = 24 \end{cases}$

2. Solve by elimination/linear combination.

a. $\begin{cases} 3x + 4y = 17 \\ 5x - 4y = 7 \end{cases}$

b. $\begin{cases} 8x - 2y = 2 \\ 6x + y = -6 \end{cases}$

c. $\begin{cases} 2x + 2y = 14 \\ x - 3y = -1 \end{cases}$

3. Solve using the method of your choice.

a. $\begin{cases} 3x + 4y = 8 \\ y = -\frac{3}{4}x + 2 \end{cases}$

b. $\begin{cases} y = -\frac{1}{2}x + 5 \\ 3x - y = 2 \end{cases}$

c. $\begin{cases} 4x - 7y = 10 \\ 3x + 2y = -7 \end{cases}$

4. Describe when a linear system is easiest to solve by each method.

a. setting expressions equal to each other

b. the substitution method

c. elimination/linear combination

5. The system below is not a linear system, but it can still be solved using the methods from this activity.

$$\begin{cases} \frac{3}{x} + \frac{1}{y} = 4 \\ \frac{6}{x} - \frac{2}{y} = -2 \end{cases}$$

a. Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$. Rewrite the system above using the variables u and v. Is the resulting system linear?

b. Solve the system from part a for u and v.

c. What is the solution to the original system? Explain how you determined the solution.

6. The solution to a linear system is (3, 0). One of the equations of the system is $x - y = 3$. Which could be the other equation?

A. $y = x + 3$

B. $3x - y = 6$

C. $x + y = 3$

D. $-y = 3 - x$

Lesson 15-2

7. The Adventure Shoe Company manufactures two types of shoes, athletic shoes and hiking shoes. The cost of manufacturing 20 pairs of athletic shoes and 10 pairs of hiking shoes is $750. If 25 pairs of athletic shoes and 20 pairs of hiking shoes were manufactured, the cost would be $1,200. How much does it cost to manufacture each type of shoe?

A. $20 for each pair of athletic shoes and $35 for each pair of hiking shoes
B. $25 for each pair of athletic shoes and $30 for each pair of hiking shoes
C. $24 for each pair of athletic shoes and $27 for each pair of hiking shoes
D. $35 for each pair of athletic shoes and $20 for each pair of hiking shoes

8. The sum of two numbers is 24. The difference between the greater number and twice the smaller number is 18. Find the *greater* number.

9. Michael buys 3 notebooks and 4 packages of highlighters for $12. Amy buys 6 notebooks and 2 packages of highlighters for $15. What is the cost of one notebook? What is the cost of one package of highlighters?

10. Karen buys 5 cups of coffee and 2 bagels for $26. Kenneth buys 3 cups of coffee and 3 bagels for $21. How much does one cup of coffee cost? How much does one bagel cost?

11. Judy pays $29 for 8 gallons of gas and 2 bottles of water. Carmen pays $45 for 12 gallons of gas and 4 bottles of water. How much does one gallon of gas cost? How much does one bottle of water cost?

12. Christie's average monthly expenses are $450 less than half of her monthly income. If the sum of her average monthly expenses and monthly income is $3,600, what is her monthly income?

13. Today, Hector is 22 years older than his sister. In 5 years, he will be 3 times as old as she will be. How old are Hector and his sister today?

14. A jar is full of 100 coins. All of the coins are either nickels or dimes.

a. The value of the coins is $6.35. Write a linear system representing this situation.
b. Solve the system to determine how many of each type of coin is in the jar.
c. Suppose the value of the coins was $6.30. Explain how you can determine the number of each type of coin without solving another system of equations.
d. Suppose the value of the coins was $6.40. Explain how you can determine the number of each type of coin without solving another system of equations.

15. Consider the jar of 100 coins from Item 14. Suppose that the coins were all dimes and quarters, with a value of $15.75.

a. Write a system to represent this situation.
b. Solve the system. Explain how the solution of the system shows that the coins cannot have the value as described.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

16. Explain how a linear system that represents a real-world situation can have a solution, but the corresponding situation has no solution. Give an example and describe why the numerical solution does not make sense in the context of the situation.

Solving Systems of Linear Equations
SUPPLY AND DEMAND

When something is popular and many people need or want to buy it, the price goes up. Items that no one wants are marked down to a lower price.

The change in an item's price and the amount available to buy are the basis of the concept of *supply and demand* in economics. Demand is the number of things people are willing to buy at a particular price. Supply refers to the number of things the manufacturer is willing to make at a particular price. The price that the customer pays is based on both supply and demand.

Suppose that during a six-month period, the supply and demand for a popular gaming system in a certain city has been tracked and approximated by the following system of equations, where x represents the number of gaming systems (in thousands) and y represents the price per gaming system.

- Demand equation: $0.7x + y = 340$
- Supply equation: $-1.5x + y = 320$

1. Solve the system algebraically to determine the balance point between supply and demand.
2. At about what price do the supply and demand balance?
3. About how many gaming systems is the manufacturer willing to supply at that price?

During the same 6-month period in a different city, the supply and demand for the same gaming system is represented by the following system of equations.

- Demand equation: $1.4x + 2y = 800$
- Supply equation: $-1.5x + y = 320$

4. Solve the system by graphing to determine the balance point between the supply and demand in this city.
5. At about what price do the supply and demand balance?
6. About how many gaming systems is the manufacturer willing to supply at that price?

The manufacturer will supply gaming systems to a third city based on the supply equation shown below.

- Supply equation for first two cities: $-1.5x + y = 320$
- Supply equation for third city: $3x - 2y = 370$

7. At what price will the manufacturer supply all three cities with the same number of gaming systems? You may solve either algebraically or graphically. Show all work and explain your response.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 5, 6, 7)	• Precise and accurate understanding of solving systems of linear equations algebraically or by graphing.	• An understanding of solving systems of linear equations algebraically or by graphing.	• Some understanding of solving systems of linear equations algebraically or by graphing.	• Incorrect or incomplete understanding of solving systems of linear equations algebraically or by graphing.
Problem Solving (Items 1, 4)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 4, 7)	• Clear and accurate representation of systems of linear equations with graphs and equations.	• Representing systems of linear equations with graphs and equations.	• Errors in representing systems of linear equations with graphs and equations.	• Inaccurate or incomplete representation of systems of linear equations.
Reasoning and Communication (Items 2, 3, 5, 6, 7)	• Accurately and precisely communicating the numerical or graphical solution as a real-world result.	• Describing the numerical or graphical solution as a real-world result.	• Errors in describing the numerical or graphical solution as a real-world result.	• An incomplete or inaccurate description of the numerical or graphical solution as a real-world result.