Unit 4 Planning the Unit

In this unit students study a variety of topics from geometry including angles, triangles, polygons and circles. They investigate similarity, discover and use formulas to calculate area and volume of 2- and 3-dimensional figures and apply their learning to real-world problems.

Vocabulary Development

The key terms for this unit can be found on the Unit Opener page. These terms are divided into Academic Vocabulary and Math Terms. Academic Vocabulary includes terms that have additional meaning outside of math. These terms are listed separately to help students transition from their current understanding of a term to its meaning as a mathematics term. To help students learn new vocabulary:

- Have students discuss meaning and use graphic organizers to record their understanding of new words.
- Remind students to place their graphic organizers in their math notebooks and revisit their notes as their understanding of vocabulary grows.
- As needed, pronounce new words and place pronunciation guides and definitions on the class Word Wall.

Embedded Assessments

Embedded Assessments allow students to do the following:

- Demonstrate their understanding of new concepts.
- Integrate previous and new knowledge by solving real-world problems presented in new settings.

They also provide formative information to help you adjust instruction to meet your students' learning needs.

Prior to beginning instruction, have students unpack the first Embedded Assessment in the unit to identify the skills and knowledge necessary for successful completion of that assessment. Help students create a visual display of the unpacked assessment and post it in your class. As students learn new knowledge and skills, remind them that they will be expected to apply that knowledge to the assessment. After students complete each Embedded Assessment, turn to the next one in the unit and repeat the process of unpacking that assessment with students.

CollegeBoard

Algebra / AP / College Readiness

This unit focuses on skills and knowledge that improve students' understanding of geometric concepts by:

- Using patterns and manipulatives to recognize structure, develop understanding and comprehend formulas.
- Providing opportunities to analyze mathematical relationships to connect ideas and concepts.
- Asking students to use appropriate tools and precision when compiling and analyzing information and solving problems.
- Providing opportunities to communicate by allowing students to share their methods and conclusions both verbally and in writing.

Unpacking the Embedded Assessments

The following are the key skills and knowledge students will need to know for each assessment.

Embedded Assessment 1

Angles and Triangles, *Pool Angles*

- Adjacent, vertical, complementary, and supplementary angles
- Angles of a triangle

Embedded Assessment 2

Circumference and Area, *In the Paint*

- Area of rectangles and circles
- Area of composite plane shapes

Embedded Assessment 3

Surface Area and Volume, *Under the Sea*

- Nets for a prism
- Surface area of a prism
- Cross section of a solid

Suggested Pacing

The following table provides suggestions for pacing using a 45-minute class period. Space is left for you to write your own pacing guidelines based on your experiences in using the materials.

	45-Minute Period	Your Comments on Pacing
Unit Overview/Getting Ready	1	
Activity 13	3	
Activity 14	4	
Embedded Assessment 1	1	
Activity 15	3	
Activity 16	3	
Activity 17	3	
Embedded Assessment 2	1	
Activity 18	7	
Activity 19	4	
Embedded Assessment 3	1	
Total 45-Minute Periods	**31**	

Additional Resources

Additional resources that you may find helpful for your instruction include the following, which may be found in the eBook Teacher Resources.

- Unit Practice (additional problems for each activity)
- Getting Ready Practice (additional lessons and practice problems for the prerequisite skills)

Geometry

4

Unit Overview

In this unit you will extend your knowledge of two- and three-dimensional figures as you solve real-world problems involving angle measures, area, and volume. You will also study composite figures.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- unique
- orientation
- decompose

Math Terms

- angle
- complementary angles
- adjacent angles
- vertical angles
- included angle
- similar figures
- corresponding parts
- plane
- circumference
- radius
- semicircle
- prism
- pyramid
- lateral face
- lateral area
- slant height
- complex solid
- vertex
- supplementary angles
- conjecture
- included side
- congruent
- circle
- center
- diameter
- composite figure
- inscribed figure
- net
- cross section
- right prism
- surface area
- regular polygon
- volume

ESSENTIAL QUESTIONS

Why is it important to understand properties of angles and figures to solve problems?

Why is it important to be able to relate two-dimensional drawings with three-dimensional figures?

EMBEDDED ASSESSMENTS

These assessments, following Activities 14, 17, and 19, will give you an opportunity to demonstrate how you can use your understanding of two- and three-dimensional figures to solve mathematical and real-world problems involving area and volume.

Developing Math Language

As this unit progresses, help students make the transition from general words they may already know (the Academic Vocabulary) to the meanings of those words in mathematics. You may want students to work in pairs or small groups to facilitate discussion and to build confidence and fluency as they internalize new language. Ask students to discuss new academic and mathematics terms as they are introduced, identifying meaning as well as pronunciation and common usage. Remind students to use their math notebooks to record their understanding of new terms and concepts.

As needed, pronounce new terms clearly and monitor students' use of words in their discussions to ensure that they are using terms correctly. Encourage students to practice fluency with new words as they gain greater understanding of mathematical and other terms.

Unit Overview

Ask students to read the unit overview and mark the text to identify key phrases that indicate what they will learn in this unit.

Materials

- dot paper
- grid paper
- index cards
- model prisms
- model pyramids
- metric ruler
- protractor
- scissors
- straws
- string
- tape
- unit cubes
- prisms
- model prisms
- metric measuring tape
- coins
- paper plates
- cups
- lids

Key Terms

As students encounter new terms in this unit, help them to choose an appropriate note taking technique such as a graphic organizer for their word study. Encourage students to make notes to help them remember the meaning of new words. Refer students to the Glossary to review translations of key terms as needed. Have students place their notes in their math notebooks and revisit as needed as they gain additional knowledge about each word or concept.

Essential Questions

Read the essential questions with students and ask them to share possible answers. As students complete the unit, revisit the essential questions to help them adjust their initial answers as needed.

Unpacking Embedded Assessments

Prior to beginning the first activity in this unit, turn to Embedded Assessment 1 and have students unpack the assessment by identifying the skills and knowledge they will need to complete the assessments successfully. Guide students through a close reading of the assessment, and use a graphic organizer or other means to capture their identification of the skills and knowledge. Repeat the process for each Embedded Assessment in the unit.

Use some or all of these exercises for formative evaluation of students' readiness for Unit 4 topics.

Prerequisite Skills

- Understand ratios (Item 1) 6.RP.A.3
- Solve equations (Item 2) 7.EE.B.3, 7.EE.B.4
- Classify geometric figures (Items 3, 6, 7, 8) 2.G.A.1, 3.G.A.1, 4.G.A.1, 4.G.A.2, 7.G.A.2
- Find area of figures (Items 4, 5) 6.G.A.1, 7.G.B.4

Answer Key

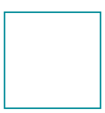
1. $\frac{2}{3}, \frac{8}{12}, \frac{12}{18}$

2. a. $5\frac{2}{3}$

b. -31

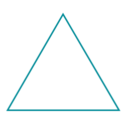
c. 25.5

3. a.



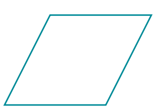
Square

b.



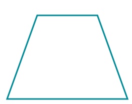
Triangle

c.



Parallelogram

d.

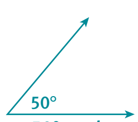


Trapezoid

e.

Right Triangle

f.



50° angle

4. a. Circle $= \pi r^2$

b. Trapezoid $= \frac{1}{2}h(b_1 + b_2)$

c. Parallelogram $= bh$

d. Triangle $= \frac{bh}{2}$

5. a. 78.5 in.^{2}

b. 14 in.^{2}

c. 60 in.^{2}

d. 60 in.^{2}

e. 20 square units

6. Answers may vary. Both complementary and supplementary angles are a pair of angles, but complementary angles have an angle sum of 90° and supplementary angles have an angle sum of 180°.

7. a. scalene, isosceles, equilateral

b. acute, right, obtuse, equiangular

8. Answers may vary. Students will likely name four from this list: triangle (3), quadrilateral, square, rectangle, parallelogram (4), pentagon (5), hexagon (6), octagon (8), decagon (10), dodecagon (12).

UNIT 4

Getting Ready

Write your answers on notebook paper. Show your work.

1. Write three ratios that are equivalent to $\frac{4}{6}$.

2. Solve each of the following equations.

a. $3x + 4 = 21$

b. $2x - 13 = 3x + 18$

c. $\frac{6}{51} = \frac{3}{x}$

3. Sketch each of the following figures.

a. square

b. triangle

c. parallelogram

d. trapezoid

e. right triangle

f. 50° angle

4. Write an expression that can be used to determine the area of each figure.

a. circle

b. trapezoid

c. parallelogram

d. triangle

5. Determine the area of each plane figure described or pictured below.

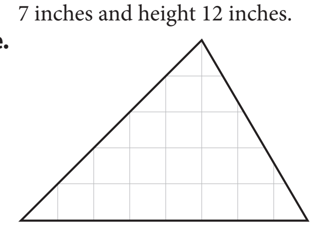
a. Circle with radius 5 inches. Round your answer to the nearest tenth.

b. Right triangle with leg lengths 4 inches and 7 inches.

c. Rectangle with length 6 inches and width 10 inches.

d. Trapezoid with base lengths 3 inches and 7 inches and height 12 inches.

e.



6. Compare and contrast the terms *complementary* and *supplementary* when referring to angles.

7. Think about triangles.

a. List three ways to classify triangles by side length.

b. List four ways to classify triangles by angle measure.

8. Polygons are named by the number of sides they have. Give the names of four different polygons and tell the number of sides each has.

Getting Ready Practice

For students who may need additional instruction on one or more of the prerequisite skills for this unit, Getting Ready practice pages are available in the eBook Teacher Resources. These practice pages include worked-out examples as well as multiple opportunities for students to apply concepts learned.

Angle Pairs

Some of the Angles

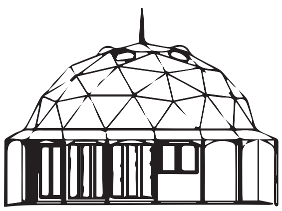
Lesson 13-1 Complementary, Supplementary, and Adjacent Angles

Learning Targets:

- Use facts about complementary, supplementary, and adjacent angles to write equations.
- Solve simple equations for an unknown angle in a figure.

SUGGESTED LEARNING STRATEGIES: Close Reading, Think Aloud, Create Representations, Marking the Text, Critique Reasoning, Sharing and Responding, Look for a Pattern

Architects think about angles, their measure, and special angle relationships when designing a building.



Two rays with a common endpoint form an ***angle***. The common endpoint is called the ***vertex***.

1. Angles are measured in degrees and can be classified by their relationship to the angle measures of 0°, 90°, and 180°. What angle measures characterize an acute angle, a right angle, an obtuse angle, and a straight angle?

An acute angle measures between 0° and 90°, a right angle measures 90°, an obtuse angle measures between 90° and 180°, and a straight angle measures 180°.

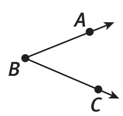
Some angle relationships have special names. Two angles are ***complementary*** if the sum of their measures is 90°. Two angles are ***supplementary*** if the sum of their measures is 180°.

2. Compare and contrast the definitions of complementary and supplementary angles.

Sample answer: Complementary angles have a sum of 90°, but supplementary angles have a sum of 180°. Two angles are needed to form both kinds of angle pairs. Complementary angles will form a right angle if they are placed next to each other, while supplementary angles form a straight angle when they are placed next to each other.

My Notes

READING MATH



To read this angle, say "angle *ABC*," "angle *CBA*," or "angle *B*."

READING MATH

A small square at the vertex of an angle denotes a right angle.

Common Core State Standards for Activity 13

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

ACTIVITY 13

Guided

Activity Standards Focus

In previous grades, students learned that an angle is a figure formed by two rays meeting at a common endpoint. They classify angles by their measure and distinguish them from related geometric figures such as triangles and polygons. In Activity 13, students begin to distinguish among various types of angles and classify them by their relationships with other angles.

Lesson 13-1

PLAN

Pacing: 1–2 class periods

Chunking the Lesson

#1–3 #4–7 #8–11

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to think silently about things they have learned about angles in previous grades. Have each student make a list of the terms and characteristics of angles that they remember, draw two different angles that they think have different characteristics, and write a definition of *angle*. Have students exchange sheets with another student and in pairs discuss the list and definitions they wrote. Randomly call on students to share definitions and characteristics charting responses.

1–3 Activating Prior Knowledge, Close Reading, Think Aloud, Word Wall

Students review the four types of angles-acute, right, obtuse, and straight and are introduced to complementary and supplementary angles. It's important for students to recognize that these classifications refer to single angles, while the terms complementary and supplementary, the primary focus of this lesson, refer to pairs of angles.

Remind students to refer to the English-Spanish glossary to aid them in comprehending new vocabulary words in this activity.

Developing Math Language

Call students' attention to the Math Terms signal boxes. As students respond to questions, monitor their use of these new terms and descriptions of applying math concepts to ensure their understanding and ability to use language correctly and precisely.

4–7 Create Representations, Sharing and Responding, Self Revision/Peer Revision Students demonstrate their understanding of the numerical and visual definitions of complementary and supplementary Be sure students understand why an angle measuring 113° (Item 4b) cannot have a complement. Ask students to describe the category of angles that do not have complements (all angles measuring 90° or greater). When completing Item 7 students should be encourage to read and discuss the signal box with a partner. Have several students share their response to Item 7 and have students revise their justification.

ELL Support

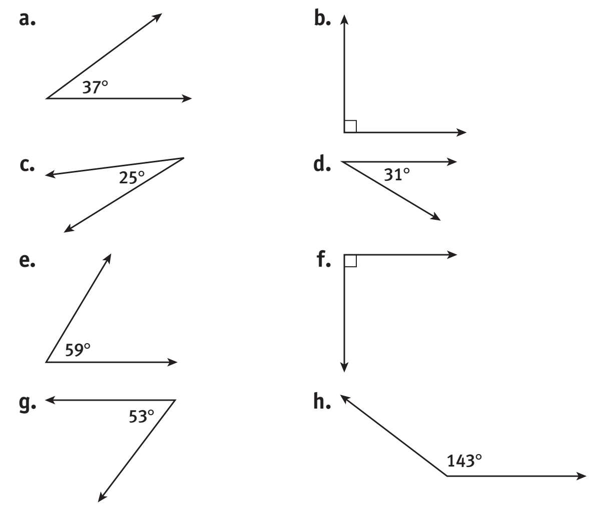
To support students' language acquisition, monitor group discussions to listen to pronunciation of terms and how students use them to describe mathematical concepts. For students whose first language is not English, monitor understanding and use of new language structures. To support students in group discussions, suggest that they make notes about what they want to say, reviewing their notes to ensure that they are using the correct language structures. Encourage students to ask questions about the meaning of new expressions they hear as a part of your classroom discussion or during their group discussions.

Teacher to Teacher

Students sometimes make the assumption that complementary and supplementary angle pairs must be adjacent to one another. Before students begin Item 3, have them discuss that while such pairs are often adjacent, they do not need to be. By having students find complementary and supplementary pairs that are clearly not adjacent, Item 3 addresses this misconception.

My Notes

3. Name pairs of angles that form complementary or supplementary angles. Justify your choices.



Complementary: angles in a and g because 37° + 53° = 90°; angles in d and e because 31° + 59° = 90°; Supplementary: angles in b and f because 90° + 90° = 180°; angles in a and h because 37° + 143° = 180°

MATH TERMS

In a pair of complementary angles, each angle is the **complement** of the other.

In a pair of supplementary angles, each angle is the **supplement** of the other.

4. Find the ***complement*** and/or ***supplement*** of each angle or explain why it is not possible.

a. 32° complement = 58°; supplement = 148°

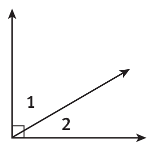
b. 113°
complement = not possible; 113° > 90°, because the sum of complementary angles is 90°; only angles that measure less than 90° can have a complement; supplement = 67°

c. 68.9° complement = 21.1°; supplement = 111.1°

Lesson 13-1
Complementary, Supplementary, and Adjacent Angles

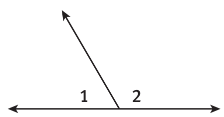
My Notes

5. Why are angles 1 and 2 in this diagram complementary?



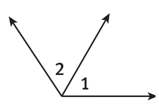
Sample answer: A right angle that measures 90° is formed by ∠1 and ∠2, so ∠1 and ∠2 must be complementary.

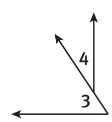
6. Why are angles 1 and 2 in this diagram supplementary?



Sample answer: A straight angle (180°) is formed by ∠1 and ∠2, so ∠1 and ∠2 must be supplementary.

7. Which of the following is a pair of ***adjacent angles***? Justify your answer.





Angles 1 and 2 are adjacent because they have the same vertex and share a side but do not overlap. Angles 3 and 4 do not have the same vertex.

8. Angle A measures 32°.

a. Angle A and $\angle B$ are complementary. Find $m\angle B$. **58°**

b. Write an equation that illustrates the relationship between the measures of $\angle A$ and $\angle B$. **$32° + m\angle B = 90°$**

c. Solve your equation from Part b to verify your answer in Part a. **$m\angle B = 58°$**

MATH TERMS

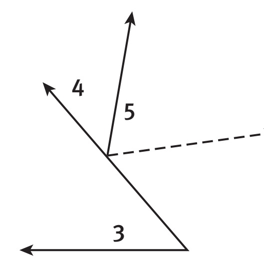
Adjacent angles have a common side and vertex but no common interior points.

READING MATH

Read $m\angle B$ as "the measure of $\angle B$." This form indicates the size of the angle.

4–7 (continued) Items 5 and 6 require students to make two important connections: (1) A right angle measures 90° AND the measures of a pair of complementary angles have a sum of 90°; (2) A straight angle measures 180° AND the measures of a pair of supplementary angles have a sum of 180°. Understanding these connections will help students answer Items 5 and 6, and will enable them to apply the connections when they see them in later work with geometric figures.

To be sure that students understand why the second pairs of angles in Item 7 are not adjacent, ask them to draw an ∠5 that is adjacent to ∠4. Sample answer:



8–11 Activating Prior Knowledge, Create Representation In Items 8–11, students use their knowledge of equations to write equations based on what they have learned about angle relationships. You may wish to review how verbal sentences can be translated into mathematical symbols. One way to do this is to have students come up with simple real-world problems that translate into equations just as Item 8 does. For example:

- The sum of Bill and Phil's ages is 90. Bill is 32. Write and solve an equation to find Phil's age. $32 + x = 90$; $x = 58$

Students can discuss how this is analogous to Item 8:

- The sum of the measures of $\angle A$ and $\angle B$ is 90°. $\angle A$ measures 32°. Write and solve an equation to find $m\angle B$. $32 + m\angle B = 90$; $m\angle B = 58$

8–11 (continued) Items 9–11 require students to analyze increasingly difficult situations involving complementary and supplementary angles, to draw diagrams illustrating the situations (Items 9 and 11), to write equations modeling the situations, and to solve the equations. Students should work in pairs or groups to solve these problems. Encourage students to check their answers to each problem by going back to the beginning and seeing if the answer makes sense. For Item 9, for example, they should check to see that the sum of 48°, 21°, and 21° is indeed 90°.

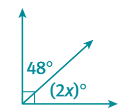
Developing Math Language

Ask students why it is important to include the word *measure* when describing the size of an angle: "The measure of ∠*A* is 56 degrees" rather than "∠*A* is 56 degrees." The latter sentence is incorrect because ∠*A* is two rays that meet at vertex *A*. The number 56, on the other hand, represents the *size* of ∠*A*—its measure—not the angle itself.

9. Model with mathematics. Two angles are complementary. One measures $(2x)°$ and the other measures 48°.

a. Draw a pair of adjacent, complementary angles and label them using the given information.

Sample diagram:



b. Write an equation to show the relationship between the two angles and solve for the value of *x*.

$2x + 48 = 90$

$x = 21$

c. Find the measure of both angles. Show your work.

first angle = 2(21°) = 42°; second angle = 48°

10. Angle *C* measures 32°.

a. Angle *C* and ∠*D* are supplementary. Find *m*∠*D*.

180° − 32° = 148°

b. Write an equation you could use to find the measure of ∠*D*.

$180° - m\angle C = m\angle D$

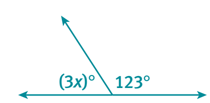
c. Solve your equation from Part b to verify your answer in Part a.

$m\angle D = 148°$

11. Make use of structure. Two angles are supplementary. One angle measures $(3x)°$ and the other measures 123°.

a. Draw a pair of adjacent supplementary angles and label them using the given information.

Sample diagram:



b. Write an equation to show the relationship between the two angles. Solve the equation for *x*.

$3x + 123 = 180$

$x = 19$

c. Find the measure of both angles. Show your work.

first angle = 3(19°) = 57°; second angle = 123°

Lesson 13-1
Complementary, Supplementary, and Adjacent Angles

My Notes

Check Your Understanding

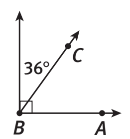
12. Explain how to find the complement and the supplement of an angle that measures 42°.

13. **Reason quantitatively.** What angle is its own complement? What angle is its own supplement? Explain.

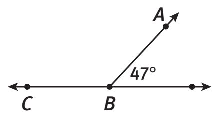
LESSON 13-1 PRACTICE

Find $m\angle ABC$ in each diagram.

14.



15.



Find the complement and/or supplement of each angle. If it is not possible, explain.

	Angle	Complement	Supplement
16.	14°	$90° - 14° = 76°$	$180° - 14° = 166°$
17.	98°	Not possible. $98° > 90°$, and complementary angles have a sum of 90°	$180° - 98° = 82°$
18.	53.4°	$90° - 53.4° = 36.6°$	$180° - 53.4° = 126.6°$

19. $\angle P$ and $\angle Q$ are complementary. $m\angle P = 52°$ and $m\angle Q = (3x + 2)°$. Find the value of x. Show your work.

20. $\angle TUV$ and $\angle MNO$ are supplementary. $m\angle TUV = 75°$ and $m\angle MNO = (5x)°$. Find the value of x and the measure of $\angle MNO$. Show your work.

21. $\angle ABC$ and $\angle TMI$ are complementary. $m\angle ABC = 32°$ and $m\angle TMI = (29x)°$. Find the measure of $\angle TMI$.

22. **Make use of structure.** $\angle ZTS$ and $\angle NRQ$ are supplementary. $m\angle ZTS = (5x - 3)°$ and $m\angle NRQ = (2x + 1)°$. Find the measure of each angle.

23. **Model with mathematics.** The supplement of an angle has a measure that is three times the angle. Write and solve an equation to find the measure of the angle and the measure of its supplement.

ACTIVITY 13 Continued

Check Your Understanding

Debrief students' answers to these items to be sure they understand the difference between the complement and the supplement of an angle.

Answers

12. Complement: $90° - 42° = 48°$; Supplement: $180° - 42° = 138°$

13. Complement: 45° because $45° + 45° = 90°$; Supplement: 90° because $90° + 90° = 180°$.

ASSESS

Students' answers to lesson practice will provide you with a formative assessment of student understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity

LESSON 13-1 PRACTICE

14. 54°

15. 133°

16. complement: 76°; supplement: 166°

17. complement: not possible; supplement: 82°

18. complement: 36.6°; supplement: 126.6°

19. $52° + (3x + 2)° = 90°$; $x = 12$

20. $x = 21$, $m\angle MNO = 105°$; $75° + 5x = 180$; $5x = 180° - 75° = 105°$; $x = 105° \div 5 = 21°$.

21. $m\angle TMI = 58°$

22. $x = 26$; $m\angle ZTS = 5(26) - 3 = 127°$, $m\angle NRQ = 2(26) + 1 = 53°$

23. $x + 3x = 180°$; 45° and 135°

ADAPT

Check students' answers to the Lesson Practice to be sure they understand how to use equations to solve problems involving angle measures, a skill they will use throughout this unit, and in their future work in geometry. If students continue to struggle solving equations use Bell-Ringer activities as an opportunity to provide practice. Have students work in pairs explaining and justifying to each other the steps used in solving multi-step equations.

Lesson 13-2

PLAN

Pacing: 1–2 class periods

Chunking the Lesson

#1–5 #6–7

Check Your Understanding

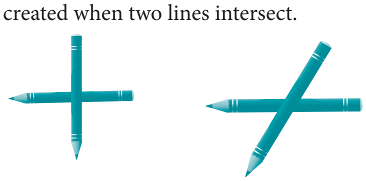
Lesson Practice

Activity Practice

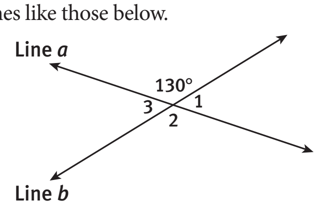
TEACH

Bell-Ringer Activity

Ask each student to place two pencils at right angles to one another, intersecting at their centers. Have students describe the relationship of the four angles that are formed. (They all measure 90°.) Then have students rotate one of the pencils slightly, still leaving the pencils intersecting at their centers. Ask what happens to the four angles. (Two grow larger and two grow smaller. Students may suggest that the angles in each pair appear to have the same measure.) Point out that in this lesson they will look closely at the angle relationships that are created when two lines intersect.



1–5 Activating Prior Knowledge, Predict and Confirm, Think-Pair-Share, Word Wall Help students see the logic behind the conjecture that the measures of a pair of vertical angles are equal by showing them two intersecting lines like those below.



Point out that ∠1 and 130° are supplementary, so ∠1 = 50°. But ∠1 and ∠2 are also supplementary, so ∠2 = 130°. So, ∠1 and ∠3 each measure 130°. Help students to see that the same argument could be made no matter what the measure of the angle you begin with. The fact that two pairs of supplementary angles are formed when two lines intersect guarantees that the measures of the vertical angle pairs will be equal.

Lesson 13-2
Vertical Angles and Angle Relationships in a Triangle

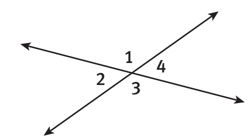
My Notes

Learning Targets:

- Write and solve equations using geometry concepts.
- Solve problems involving the sum of the angles in a triangle.
- Solve equations involving angle relationships.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think-Pair-Share, Create Representations, Identify a Subtask

Vertical angles are pairs of nonadjacent angles formed when two lines intersect. They share a vertex but have no common rays. ∠1 and ∠3 are vertical angles, as are ∠2 and ∠4.



MATH TERMS

A **conjecture** is a statement that seems to be true but has not been proven to be either true or false.

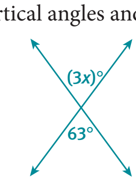
1. Visually inspect the diagram above. Then make a ***conjecture*** about the measures of a pair of vertical angles.
   Sample answer: The measure of a pair of vertical angles is equal.

2. Find $m\angle 1$ and $m\angle 3$ above using your protractor. **130°**

3. Find the measure of ∠2 and ∠4 using geometry concepts.
   $m\angle 2$ **and** $m\angle 4 = 180° - 130° = 50°$

4. **Construct viable arguments.** Do your answers to Items 2 and 3 support your conjecture? Support your reasoning.
   Sample answer: Yes. $m\angle 1$ **and** $\angle 3 = 130°$ **and** $m\angle 2$ **and** $\angle 4 = 50°$**. This supports the conjecture that the measure of vertical angles is equal.**

5. **Reason quantitatively.** Two angles are vertical angles. One angle measures $(3x)°$ and the other measures 63°.
   a. Draw the pair of vertical angles and label them using the given information.



   b. Write an equation to show the relationship between the two angles. Then solve the equation for x.
   $3x = 63; x = 21$

Developing Math Language

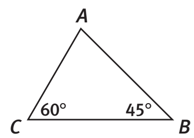
Direct students' attention to the Math Terms signal box. Have several students paraphrase to assess their understanding of *conjecture*, an important concept because students make conjectures throughout their study of mathematics.

Lesson 13-2
Vertical Angles and Angle Relationships in a Triangle

My Notes

A *triangle* is a closed figure made of three line segments that meet only at their endpoints. The sum of the angle measures of a triangle is 180°.

6. Triangle *ABC* is shown.

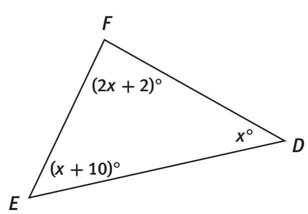


a. Find the measure of $\angle A$. Explain your answer.
75°. Sample explanation: $180° - 60° - 45° = 75°$

b. Write an equation that illustrates the relationship between the measures of $\angle A$, $\angle B$, and $\angle C$.
$m\angle A + m\angle B + m\angle C = 180°$
$m\angle A + 45° + 60° = 180°$

c. Solve your equation from Part b to verify your answer in Part a.
$m\angle A = 75°$

7. Reason quantitatively. Triangle *DEF* is shown.



a. Write an equation that illustrates the relationship between the measures of $\angle D$, $\angle E$, and $\angle F$.
$m\angle D + m\angle E + m\angle F = 180°$
$x + (x + 10)° + (2x + 2)° = 180°$

b. Solve the equation to find the value of *x*.
$x = 42°$

c. Find the measure of all three angles of $\triangle DEF$.
$m\angle D = 42°$, $m\angle E = 52°$, $m\angle F = 86°$

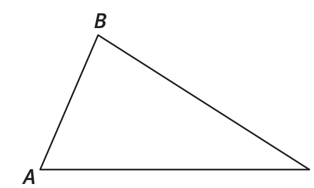
ACTIVITY 13 Continued

6–7 Create Representations, Identify a Subtask Encourage students to check their answers to each problem by going back to the beginning and seeing if the answer makes sense. After solving Item 7, for example, they should check to see that the sum of 42°, 52°, and 86° is 180°.

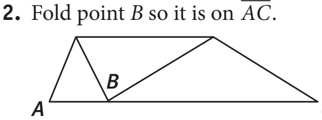
ELL Support

To help students understand why the sum of the measures of the angles of a triangle is 180°, have them perform this activity.

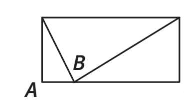
1. Draw and cut out a triangle. Label the angles *A*, *B*, and *C*.



2. Fold point *B* so it is on $\overline{AC}$.



3. Fold $\angle A$ and $\angle C$ to fit beside $\angle B$.



4. Look at the three angles that are formed at point *B*. What is the sum of the measures of the angles? (180°) Why? (The angles form a straight angle.)

5. So, $m\angle A + m\angle B + m\angle C = 180°$.

Check Your Understanding

Debrief students' answers to these items to be sure they understand the relationships among the measures of the four angles formed when two lines intersect, and the relationships among the measures of the angles of a triangle.

Answers

8. The angles are vertical angles so $(4x + 15)° = 79°$ and $x = 16°$.

9. Yes. Since the right angle measures 90°, the third angle measure is 180° − 90° − the measure of the non-right angle.

ASSESS

Use the Lesson Practice to assess your students' understanding of how to find missing angle measures when two lines intersect and when information about the measures of the angles of a triangle is given. Pay particular attention to Items 8 and 13–18, which require students to solve equations.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 13-2 PRACTICE

10. 73° **11.** 107°

12. 107° **13.** 23°

14. 135° **15.** 45°

16. $x = 31°$. The angles measure 123°, 29°, and 28°. Check students' work.

17. $m\angle K = 180° - m\angle G - m\angle J = 80°$

18. Answers may vary. Sample answer: One measure is 90° because it is a right triangle. Measure of third angle = 180° − 90° − 29° = 61°

19. 36° and 108°; Answers may vary. Sample answer: Since the base angles are equal, the other base angle also measures 36°. The measure of the third angle is $180 - 2(36°) = 108°$.

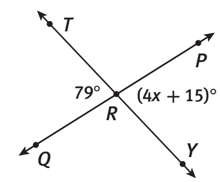
ADAPT

Check students' answers to the Lesson Practice to be sure they understand how to use equations to solve problems involving vertical angles and the angles of a triangle. Encourage students to create visual representation and label all angles with pertinent information . Assist them in seeing how their diagram will help them write equations they will use to solve problems.

My Notes

Check Your Understanding

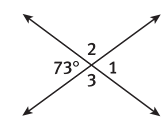
8. Explain how to find the value of x in the diagram shown.



9. Construct viable arguments. If you know the measure of one non-right angle of a right triangle, can you always find the measure of the third angle? Explain.

LESSON 13-2 PRACTICE

Use the diagram for Items 10–12.

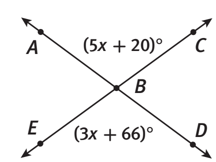


10. Find the measure of ∠1.

11. Find the measure of ∠2.

12. Find the measure of ∠3.

Use the diagram for Items 13–15.

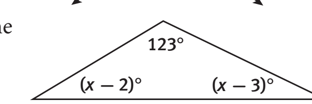


13. Find x.

14. Find the measure of ∠ABC.

15. Find the measure of ∠ABE.

16. Reason quantitatively. Find the measure of each of the angles in the triangle shown.



17. In triangle GKJ, $m\angle G = 72°$ and $m\angle J = 28°$. Write and solve an equation to find the measure of ∠K.

18. In a right triangle, one of the angles measures 29°. What are the measures of the other angles in the triangle? Explain.

19. Reason quantitatively. In an isosceles triangle, the two base angles are congruent. One of the base angles measures 36°. What are the measures of the other two angles in the triangle? Support your answer with words and equations.

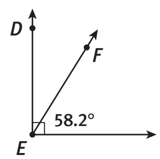
ACTIVITY 13 PRACTICE

Write your answers on notebook paper. Show your work.

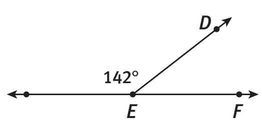
Lesson 13-1

Find the measure of angle *DEF* in each diagram.

1.



2.



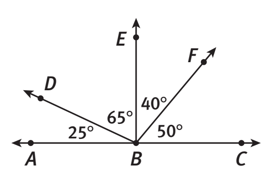
3. $\angle JKL$ and $\angle RST$ are complementary. $m\angle JKL = 36°$ and $m\angle RST = (x + 15)°$. Find the value of x and the measure of $\angle RST$.

4. $\angle SUN$ and $\angle CAT$ are supplementary. $m\angle SUN = (2x)°$ and $m\angle CAT = 142°$. Find the value of x and the measure of $\angle SUN$.

5. $\angle P$ and $\angle Q$ are supplementary. $m\angle P = (5x + 3)°$ and $m\angle Q = (x + 3)°$. What is the measure of $\angle Q$?

A. 17° **B.** 26°
C. 29° **D.** 32°

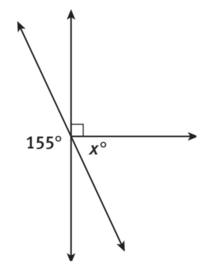
6. In the diagram shown, which angle pairs form complementary angles?



A. $\angle ABD$ and $\angle CBF$ **B.** $\angle DBE$ and $\angle FBE$
C. $\angle ABD$ and $\angle DBC$ **D.** $\angle EBF$ and $\angle FBC$

Lesson 13-2

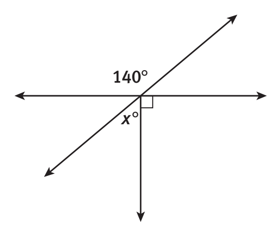
7. Find the measures of the vertical angles in the diagram. Then find x in the diagram.



8. $\angle B$ and $\angle D$ are vertical angles. $m\angle B = (2x + 1)°$ and $m\angle D = (x + 36)°$.

Find the measure of each angle.

9. Find x in the diagram shown.



10. In right triangle *TWZ*, $\angle W$ is a right angle and $m\angle Z = 41°$. Find $m\angle T$.

ACTIVITY 13 Continued

ACTIVITY PRACTICE

1. 31.8°
2. 38°
3. $x = 39$, $m\angle RST = 54°$
4. $x = 19$, $m\angle SUN = 38°$
5. D
6. D
7. 25°; $x = 65°$
8. $m\angle B = m\angle D = 71°$
9. $x = 50°$
10. $m\angle T = 49°$

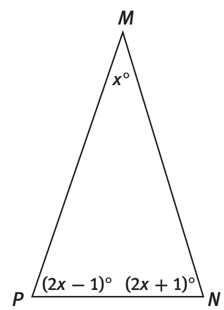
11. $m\angle Q = m\angle S = 61°$
12. $x = 36$; $m\angle M = 36°$, $m\angle P = 71°$, $m\angle N = 73$
13. No. $\angle 2$ and $\angle 3$ are not vertical angles. They are supplementary, so $m\angle 3 = 94°$.
14. $\triangle ABC$: $m\angle A = 54°$, $m\angle B = 85°$, $m\angle C = 41°$; $\triangle CDE$: $m\angle C = 41°$, $m\angle D = 97°$, $m\angle E = 42°$
15. 60°
16. 45°, 90°, 45°. Since it is a right triangle, $m\angle B = 90°$. Since it is an isosceles triangle, the measures of the other two angles are equal. $m\angle A$ and $C = (180° - 90°) \div 2$.
17. Sample answer: No. Two angles cannot be both complementary and supplementary, nor can they be both adjacent and vertical.

ADDITIONAL PRACTICE

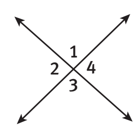
If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

11. In triangle *QRS*, $\angle Q$ and $\angle S$ have the same measure. If $m\angle R = 58°$, find the measures of $\angle Q$ and $\angle S$.

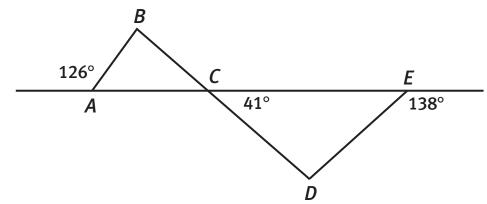
12. For triangle *MNP* shown, find the value of *x* and the measure of each of the three angles.



13. In the diagram, $m\angle 2 = 86°$. Erica says that $m\angle 3 = 86°$ because the diagram shows vertical angles and all vertical angles are congruent. Is her statement reasonable? Explain.



14. Use the diagram shown to find the measures of the each of the angles of $\triangle ABC$ and $\triangle CDE$.



15. The angles of an equilateral triangle are congruent. What are the measures of the angles?

16. In isosceles triangle *ABC*, $\angle B$ is a right angle. What are the measures of angles *A*, *B*, and *C*? Justify your answer.

MATHEMATICAL PRACTICES
Reason Abstractly

17. Consider the angle pair classifications from this activity: adjacent, complementary, supplementary, and vertical angles. Can two angles fit all four categories? Explain.

Triangle Measurements

ACTIVITY 14

Rigid Bridges

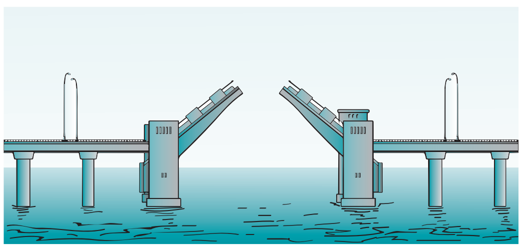
Lesson 14-1 Draw Triangles from Side Lengths

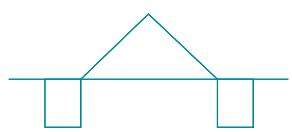
Learning Targets:

- Decide if three side lengths determine a triangle.
- Draw a triangle given measures of sides.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Use Manipulatives, Predict and Confirm, Shared Reading, Visualization

When the sides of a drawbridge are raised or lowered, the sides move at the same rate.



1. Look at the drawbridge.
   a. What will happen if the sum of the lengths of the moving sides is greater than the opening? Draw an illustration to explain your answer.
   Sample answer: The bridge could not close because the sides would meet, preventing them from going all the way down.
   
   b. What will happen if the sum of the lengths of the moving sides is equal to the length of the opening? Draw an illustration to explain your answer.
   Sample answer: The bridge will close to form a road.
   
   c. What will happen if the sum of the lengths of the moving portions of the bridge is less than the length of the opening? Draw an illustration to explain your answer.
   Sample answer: The sides of the bridge will go all the way down but will not meet.
   

My Notes

CONNECT TO ENGINEERING

A drawbridge is a bridge that can be drawn up, let down, or drawn aside, to permit or prevent ships and other watercraft from passing below it.

ACTIVITY 14
Guided

Activity Standards Focus

Until now, students' study of geometric shapes has largely been confined to lower-order knowledge levels—identifying and classifying triangles and angles, measuring, solving equations and routine multi-step problems. In Activity 14 they move beyond the routine to assess whether certain triangles are possible, and to explain why some are not.

Lesson 14-1

PLAN

Materials

- string
- metric ruler

Pacing: 2 class periods

Chunking the Lesson

#1–4 #5–6

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Show two foot-long rulers and a yardstick or meter stick. Tell students that since the class is studying triangles, you would like them to help you make a triangle with the rulers and yardstick. Have students explain why you can or cannot make the triangle. (Once the two rulers are attached to the ends of the yardstick, they will not be long enough to join each other.)

1–4 Create Representations, Use Manipulatives, The drawbridge example helps students to see that a triangle can be constructed from the three segments only if the sum of the lengths of the two shortest line segments is greater than the length of the longest segment.

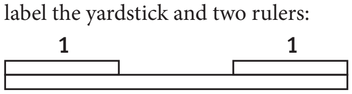
Common Core State Standards for Activity 14

7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

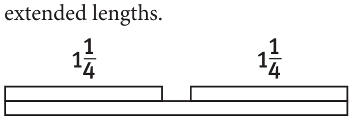
1–4 (continued) Items 2a, 2b, and 2c illustrate the three possible cases when attempts are made to construct triangles from given side lengths. In Item 2a, the sum of the lengths of each pair of sides is always *greater than* the length of the third side: $DO + OG > DG$; $DO + DG > OG$; $DG + OG > DO$. So, a triangle can be formed from these lengths. In Item 2b, the sum of the lengths of the short sides is *equal to* the length of the longest side, so a triangle cannot be formed. In Item 2c, the sum of the lengths of the short sides is *less than* the length of the longest side, so a triangle cannot be formed.

Differentiating Instruction

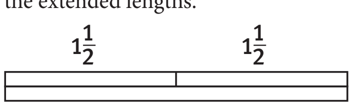
Return to the yardstick and two rulers question in the Bell-Ringer activity. Have a student draw and label the yardstick and two rulers:



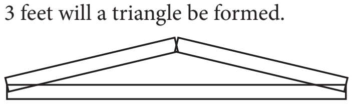
Ask: If you extend the lengths of the two rulers to $1\frac{1}{4}$ feet, can a triangle be formed? Have a student draw the extended lengths.



Discuss the fact that the total length of the two short pieces is still less than 3 feet, so a triangle cannot be formed. Ask: What if the two short lengths are extended to $1\frac{1}{2}$ feet? Draw the extended lengths.



There is still no triangle. Finally, show that only when the sum of the lengths of the two short pieces is greater than 3 feet will a triangle be formed.

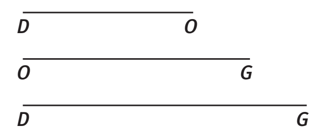


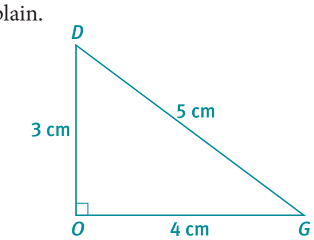
ACTIVITY 14 *continued*

My Notes

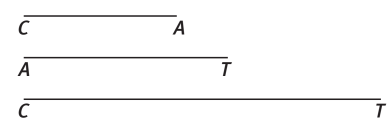
2. Work with a partner or with your group. Use paper or string to cut out segments that are the same length as the segments shown. If possible, connect the segments at the endpoints to create a triangle and draw the triangle. If it is not possible, explain.

a. $\triangle DOG$



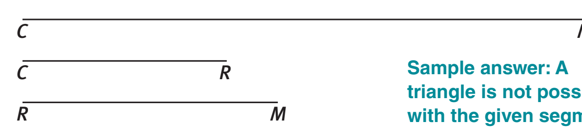


b. $\triangle CAT$



Sample answer: A triangle is not possible with the given segment lengths. The two shorter pieces are not long enough to make a triangle.

c. $\triangle RCM$



Sample answer: A triangle is not possible with the given segment lengths. The two shorter pieces are not long enough to make a triangle.

3. Use appropriate tools strategically. Measure the lengths in centimeters of the segments in Item 2. Then write an equation or inequality to compare the sum of two side lengths to the longest side length.

a, $DO = 3$ cm, $OG = 4$ cm, $DG = 5$ cm; $3 + 4 > 5$; b, $CA = 3$ cm, $AT = 4$ cm, $CT = 7$ cm; $3 + 4 = 7$; c, $CR = 4$ cm, $RM = 5$ cm, $CM = 11$ cm; $4 + 5 < 11$

4. Based on your results for Items 2 and 3, make a conjecture about what side lengths can form a triangle. As you share your ideas with your group about making the conjecture, be sure to explain your thoughts using precise language and details to help the group understand your ideas and reasoning.

Sample answer: To form a triangle, the sum of the lengths of two sides of the triangle must be greater than the length of the third side.

My Notes

5. Use a ruler to draw each triangle described below. You may want to cut out segments and use the segments to help form the triangle.

a. Draw a triangle with side lengths that each measure 3 centimeters. Can you form more than one triangle with the given side lengths? Explain.

Check students' drawings. No, the side lengths always make the same triangle.

b. Draw a triangle with side lengths that are 3 centimeters, 3 centimeters, and 5 centimeters long. Can you form more than one triangle with the given side lengths? Explain.

Check students' drawings. No, the side lengths always make the same triangle.

c. Draw a triangle with side lengths that are 3 centimeters, 4 centimeters, and 5 centimeters long. Can you form more than one triangle with the given side lengths? Explain.

Check students' drawings. No, the side lengths always make the same triangle.

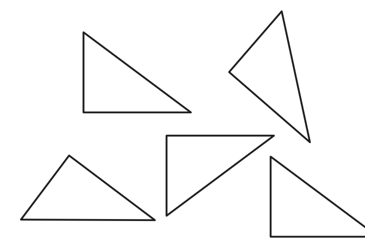
6. **Construct viable arguments.** When a triangle is formed from three given side lengths, is the triangle a ***unique*** triangle or can more than one triangle be formed using those same side lengths? Explain.

Sample answer: The triangle is unique since the side lengths always make the same triangle.

ACADEMIC VOCABULARY

The term ***unique*** means "only" or "single." In geometry, a unique triangle is a triangle that can be drawn in only one way.

5–6 Create Representations, Use Manipulatives, Visualization In Items 5 and 6, students move from deciding whether or not a triangle can be formed from given side lengths to the question of *how many* can be formed if at least one can. They discover that for any three given side lengths, one and only one triangle can be formed. Be sure students understand that a triangle that is transformed by flipping, sliding, or turning still has the same side lengths. So, the triangles below, all of which have sides measuring 3 units, 4 units, and 5 units, are transformations of the same, unique triangle.



Check Your Understanding

Debrief students' answers to these items to be sure they understand the two main conclusions of this lesson. The sum of the lengths of any two sides of a triangle must be greater than the third side. Given three side lengths that form a triangle, the triangle is unique; that is, it is the only one that can be formed from the three sides.

ASSESS

Use the lesson practice to assess your students' understanding of the conditions that must be met if a triangle is to be formed from three given side lengths and the number of such triangles than can be formed.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 14-1 PRACTICE

9. No: $2 + 5 = 7$, which is not > 7
10. Yes: $3 + 4 = 7$, which is > 6
11. No: $5 + 7 = 12$, which is < 15
12. No: $6 + 6 = 12$, which is not > 12
13. Yes: $8 + 4 = 12$, which is > 11
14. No: $9 + 16 = 25$, which is < 30
15. If the sum of the two shortest lengths is not greater than the third length, the lengths cannot form a triangle.
16. Answers may vary. Any length greater than 3 and less than 9 inches is correct.
17. Check students' drawings.
18. Check students' drawings.
19. Yes; If the side lengths form a triangle, then the sum of the two shortest lengths is greater than the third length and a unique triangle forms. However, if the sum of the two shortest lengths is not greater than the third length, the sides cannot form a triangle.

ADAPT

Check students' answers to the Lesson Practice to be sure they understand (1) how to apply inequalities in a triangle to find permissible side lengths, and (2) if a triangle can be formed from three given side lengths, it is unique. If students struggle with this understanding have them create manipulitives by cutting coffee stirrers or straws in the specified lengths to determine whether triangles can be created. Challenge them to create more than one triangle using their manipulatives.

My Notes

Check Your Understanding

7. Is it possible to draw a triangle with sides that are 4 inches, 5 inches, and 8 inches long? Justify your answer.
8. Draw a triangle with sides that are 2 inches, 2 inches, and 3 inches long. Can you form more than one triangle with the given side lengths? Explain.

Lesson 14-1 PRACTICE

Determine whether it is possible to draw a triangle with the given side lengths. Justify your answers.

9. 7 feet, 5 feet, and 2 feet
10. 3 meters, 4 meters, and 6 meters
11. 5 inches, 7 inches, and 15 inches
12. 6 yards, 12 yards, and 6 yards
13. 8 millimeters, 11 millimeters, and 4 millimeters
14. 30 feet, 9 feet, and 16 feet
15. **Express regularity in repeated reasoning.** To check that three side lengths can form a triangle, you only have to check the sum of the two shortest lengths. Explain why.
16. **Look for and make use of structure.** Two sides of a triangle measure 3 inches and 6 inches. What is one possible length for the third side of the triangle? Explain.
17. Draw a triangle with side lengths that are 6 centimeters, 8 centimeters, and 10 centimeters long.
18. Draw a triangle with side lengths that are 2 centimeters, 6 centimeters, and 7 centimeters long.
19. **Express regularity in repeated reasoning.** A triangle is formed using three given side lengths. Do these side lengths always form a unique triangle? Explain.

Answers

7. Yes: The sum of two sides' lengths is greater than the third side: $4 + 5 > 8$, $4 + 8 > 5$, and $5 + 8 > 4$.
8. Check students' drawings. No, the side lengths always make the same triangle.

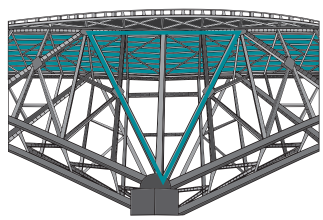
Lesson 14-2
Draw Triangles from Measures of Angles or Sides

My Notes

Learning Targets:

- Draw a triangle given measures of angles and/or sides.
- Recognize when given conditions determine a unique triangle, more than one triangle, or no triangle.

SUGGESTED LEARNING STRATEGIES: Create Representations, Graphic Organizer, Think-Pair-Share, Visualization

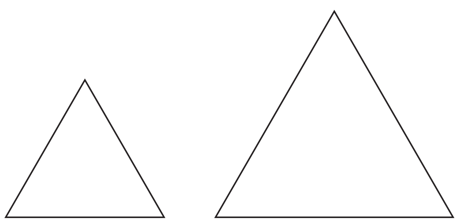


Triangles are rigid shapes. Structures, such as bridges and towers, are reinforced with triangles that give the structures added strength. Notice how triangles within structures are used to support the bridge in the picture.

In the last lesson, you learned that three given side lengths determine a unique triangle. Other conditions can also determine a unique triangle.

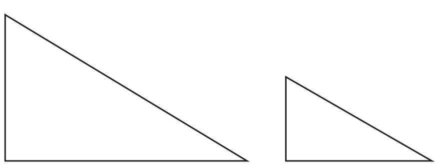
1. Use a protractor to measure each angle in the triangles.

a.



Each angle measures 60°.

b.



The angles in each triangle measure 30°, 60°, and 90°.

Lesson 14-2

PLAN

Materials

- metric ruler, protractor, straws

Pacing: 2 class periods

Chunking the Lesson

#1–4 #5–8

Check Your Understanding

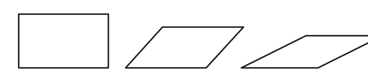
Lesson Practice

Activity Practice

TEACH

Bell-Ringer Activity

To help students understand the term *rigid* as applied to polygons, have students sketch a rectangle with sides measuring approximately 1 inch and 2 inches, Challenge students to sketch other quadrilaterals that have opposite sides of the same measure but which are not rectangles. They should realize that there are other quadrilaterals with the same side measures.



A rectangle is not *rigid*. From the last lesson, students know that a triangle *is* rigid, however, because given its three side lengths, only one triangle can be formed.

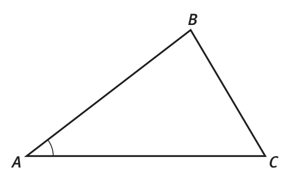
1–4 Create Representations, Visualization Students have learned that three side lengths determine a unique triangle. In Items 1–4, they explore various combinations of sides and angles to discover which combinations determine unique triangles. Items 1–2 establish that many different triangles can be formed from a given set of three angle measures. Three angle measures do not determine a unique triangle. Students have an understanding of the term included and will understand an included angle; include d sides may not be obvious to them Assess their understanding of included angles and sides to insure success with items in this lesson.

1–4 (continued) Items 3–4 establish that (i) two sides and the included angle, and (ii) two angles and the included side both determine unique triangles.

My Notes

2. Reason abstractly. When a triangle is formed from three given angle measures, is the triangle a unique triangle, or can more than one triangle be formed using those same angle measures? Explain.
More than one triangle can be formed. Knowing the three angle measures is not enough to construct a unique triangle.

READING MATH



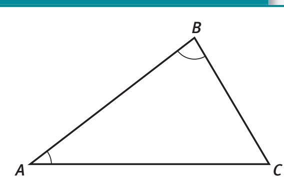
Angle *A* is the **included angle** between sides *AB* and *AC*.

Two sides of a triangle form an angle called an ***included angle***.

3. a. Use a ruler and protractor. Draw a triangle with two sides that measure 4 centimeters each and an included angle of 30°.
Check students' drawings.

b. Is there only one triangle that fits the description given in Part a? Explain.
Yes. Sample explanation: The given side lengths and the fixed angle measure determine the third side of the triangle.

READING MATH



Side *AB* is the **included side** between angles *A* and *B*.

An ***included side*** is the side between two angles.

4. a. Use a ruler and protractor. Draw a triangle with two angles that each measure 30° and an included side that measures 5 centimeters.
Check students' drawings.

b. Is there only one triangle that fits the description given in Part a? Explain.
Yes. Sample explanation: The given angle measures and the fixed side length determine the lengths of the remaining two sides of the triangle.

Lesson 14-2
Draw Triangles from Measures of Angles or Sides

My Notes

5. **Construct viable arguments.** Decide if the given conditions create a unique triangle.
   a. When a triangle is formed from two side lengths and an included angle measure, is the triangle a unique triangle, or can more than one triangle be formed? Explain.
   The triangle is unique since the side lengths always make the same triangle.

   b. When a triangle is formed using two given angle measures and an included side length, is the triangle a unique triangle, or can more than one triangle be formed? Explain.
   The triangle is unique since the side lengths always make the same triangle.

Two known angle measures and the length of a non-included side also form a unique triangle. However, two given side lengths and the measure of a non-included angle may or may not form a unique triangle.

6. Two angles of a triangle measure 40° and 110°. The side opposite the 40° angle is 6 inches long. Can more than one triangle be drawn with these conditions? Explain.
   No. Two known angle measures and the length of a nonincluded side form a unique triangle.

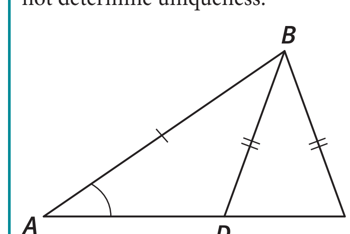
7. Two sides of a triangle are 4 inches and 7 inches long. The included angle has a measure of 35°. Can more than one triangle be drawn with these conditions? Explain.
   No. Knowing the measure of two sides and an included angle is enough information to form a unique triangle.

8. **Make use of structure.** Two angles of a triangle each measure 70° and 55°. The side adjacent to the 70° angle is 3 inches long. Can more than one triangle be drawn with these conditions? Justify your answer.
   No; the third angle of the triangle measures $180 - 55 - 70 = 55°$. Now I know the measure of two angles and an included side. The measures of two angles and their included side determine the other side lengths, so the triangle is unique.

5–8 Create Representations, Graphic Organizer, Think-Pair-Share, Visualization In these items students apply what they learned earlier in the lesson to the question of whether or not a unique triangle can be formed given two sides and the included angle or two angles and the included side. Item 8 presents a new set of conditions, two angles and a side not included between the angles. They see that these conditions, too, guarantee that a triangle will be unique.

Differentiating Instruction

In their study of geometry in advanced classes, students will use the letters A and S to abbreviate the conclusions of this lesson. For example, the fact that two sides and the included angle are sufficient to determine a unique triangle is abbreviated SAS (side-angle-side). Similarly, ASA and AAS both determine unique triangles. However, as students discovered in Items 1–2, AAA does not establish uniqueness. The text before Item 6 mentions that SSA is not sufficient to establish uniqueness but does not explain why. You may want to show why this is true. In the figure below, $\triangle ABD$ and $\triangle ABC$ have a side, a side, and a non-included angle, all with equal measures. Since two different triangles can be formed, SSA does not determine uniqueness.



Check Your Understanding

Debrief students' answers to these items to be sure they understand that three angles are not sufficient to determine a unique triangle but that two sides and an included angle are.

Answers

9. No, a side length is also needed to determine a unique triangle.
10. Yes, two sides and an included angle determine a unique triangle.

ASSESS

Use the lesson practice to assess your students' understanding of the criteria necessary to determine that a triangle is unique. Pay particular attention to Items 13 and 15, which give conditions that are *not* sufficient to establish uniqueness.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 14-2 PRACTICE

11. unique; two angles and an included side form a unique triangle.
12. unique; two angles and a side form a unique triangle.
13. more than one; a side length is also needed to determine a unique triangle.
14. unique; three side lengths determine a unique triangle.
15. more than one; two sides and a non-included angle do not form a unique triangle.
16. unique; two angles and a side form a unique triangle.
17. unique; two sides and an included angle form a unique triangle; the right angle is the included angle.
18. Yes, two angles and a side are sufficient to determine a unique triangle.

ADAPT

Check students' answers to the Lesson Practice to be sure they understand the conditions relating to the side and angle measures of a triangle that are sufficient to guarantee uniqueness and the conditions that are not. Students can make graphic organizers summarizing the number of triangles that can be created given certain conditions.

My Notes

DISCUSSION GROUP TIPS

As your group explores and discusses the relationship between the sides and angles of triangles in the Check Your Understanding problems, demonstrate listening comprehension of what each group member says by taking notes on their contributions. Ask and answer questions to clearly aid understanding of all group members' ideas.

Check Your Understanding

9. Is it possible to draw a unique triangle with angle measures of 35°, 65°, and 100°? Explain.
10. Is it possible to draw a unique triangle with two sides that are each 5 centimeters long and an included angle that measures 40°? Explain.

LESSON 14-2 PRACTICE

Determine whether the given conditions determine a unique triangle or more than one triangle. Justify your answers.

11. Two angles of a triangle measure 40° and 75°. The side between the angles is 3 feet long.
12. Two angles of a triangle each measure 55°. The side opposite one of the 55° angles is 2 meters long.
13. The angles of a triangle measure 40°, 60°, and 80°.
14. The sides of a triangle are 5 inches, 12 inches, and 13 inches long.
15. Two sides of a triangle are 10 centimeters and 13 centimeters long. One of the nonincluded angles measures 95°.
16. Two angles of a triangle measure 61° and 48°. One of the sides formed by the 48° angle is 15 millimeters long.
17. The two sides that form the right angle of a right triangle are 9 centimeters and 12 centimeters long.
18. **Look for and make use of structure.** If the measures of the angles of a triangle are known, is the length of one side of the triangle sufficient to determine if the triangle formed is a unique triangle? Explain.

Triangle Measurements
Rigid Bridges

ACTIVITY 14 PRACTICE

Write your answers on a separate piece of paper. Show your work.

Lesson 14-1

For 1–6, determine whether it is possible to draw a triangle with the given side lengths. Justify your answers.

1. 8 feet, 5 feet, and 9 feet
2. 3 centimeters, 2 centimeters, and 7 centimeters
3. 14 inches, 6 inches, and 10 inches
4. 3 yards, 2 yards, and 5 yards
5. 1.5 meters, 1.1 meters, and 2 meters
6. 42 feet, 18 feet, and 23 feet
7. Draw a triangle with side lengths that are 3 inches, 5 inches, and 6 inches long. Is this the only triangle that you can draw using these side lengths? Explain.
8. Multiple Choice: Which of the following cannot be the side lengths of a triangle?
   A. 4 inches, 4 inches, and 4 inches
   B. 3 inches, 3 inches, and 5 inches
   C. 15 centimeters, 16 centimeters, and 17 centimeters
   D. 2 centimeters, 10 centimeters, and 20 centimeters
9. Multiple Choice: Which of the following could be the length of the third side of a triangle with side lengths 2 feet and 10 feet?
   A. 12 feet
   B. 20 feet
   C. 11 feet
   D. 8 feet
10. **Express regularity in repeated reasoning.** Explain how to determine whether a triangle can be formed from three given segment lengths.

Lesson 14-2

Determine whether the given conditions determine a unique triangle or more than one triangle. Justify your answers.

11. Two angles of a triangle measure 36° and 102°. One of the sides formed by the 36° angle is 9 inches long.
12. The angles of a triangle measure 25°, 73°, and 82°.
13. Two angles of a triangle measure 86° and 67°. The side between the angles is 2.5 meters long.
14. Two sides of a triangle are 3.6 meters and 5.2 meters long. One of the non-included angles measures 48°.
15. The two sides that form the right angle of a right triangle are 6 inches and 4 inches long.
16. The sides of a triangle are 10 centimeters, 12 centimeters, and 14 centimeters long.
17. Two angles of a triangle each measure 62°. The side opposite one of the 62° angles is 34 inches long.

MATHEMATICAL PRACTICES
Look for and Make Use of Structure

18. **a.** If the lengths of two sides of a triangle are known, is the measure of one of the angles of the triangle enough to determine a unique triangle? Explain.
    b. If the measures of two angles of a triangle are known, is the length of one side of the triangle sufficient to determine if the triangle formed is a unique triangle? Explain.

ACTIVITY 14 Continued

ACTIVITY PRACTICE

1. Yes: $8 + 5 > 9$
2. No: $3 + 2 < 7$
3. Yes: $10 + 6 > 14$
4. No: $3 + 2 = 5$
5. Yes: $1.5 + 1.1 > 2$
6. No: $18 + 23 < 42$
7. Check students' drawings. Yes, a unique triangle is formed from three given side lengths.
8. D
9. C
10. Sample explanation: The sum of any two sides of a triangle must be greater than the third side.
11. Unique: Two angles and a side form a unique triangle.
12. More than one: A side length is also needed to determine a unique triangle.
13. Unique: Two angles and an included side form a unique triangle.
14. More than one: Two sides and a nonincluded angle do not form a unique triangle.
15. Unique: Two sides and an included angle form a unique triangle; the right angle is the included angle.
16. Unique: Three side lengths determine a unique triangle.
17. Unique: Two angles and a side form a unique triangle.
18. **a.** No; the angle must be the included angle to determine a unique triangle.
    b. Yes; two angles and a side are sufficient to determine a unique triangle.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Embedded Assessment 1

Assessment Focus

- Adjacent, vertical, complementary, and supplementary angles
- Angles of a triangle

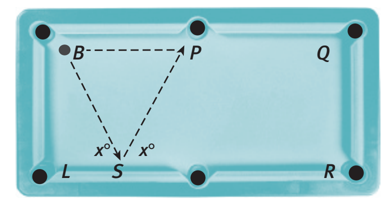
Answer Key

1. Answers may vary. There are three correct pairs: ∠QPS and ∠BPS, ∠PSR and ∠PSL, ∠LSB and ∠BSR
2. **a.** $x = 56°$
   b. $m\angle PSL = 124°$
3. Answers may vary. Since the angles of a triangle add up to 180°,
   $m\angle BLS + m\angle LSB + m\angle LBS = 180°$
   $90° + m\angle LSB + m\angle LBS = 180°$
   $m\angle LSB + m\angle LBS = 90°$, so the angles are complementary.
4. **a.** 68°
   b. △*BPS* is a unique triangle. Sample explanation: Knowing the measure of two sides, *BS* and *PS*, and their included angle, ∠*BSP*, gives enough information to form a unique triangle.

Embedded Assessment 1
Use after Activity 14

Angles and Triangles
POOL ANGLES

Pool is a game that requires talent and a knowledge of angles to play well. Bank and kick shots involve hitting a ball (B) into a rail of a rectangular pool table, and then into a pocket, somewhere on the other side of the table. As shown below, the angle at which the ball hits the side is equal to the angle at which it leaves the side.



1. Name a pair of adjacent, supplementary angles in the diagram.
2. Angle *QPS* is supplementary to ∠*PSR*. Also, $m\angle QPS = (2x + 12)°$ and $m\angle PSR$ is $x°$. Answer each question below and show your work.
   a. Find the value of x.
   b. Find $m\angle PSL$.
3. The measure of ∠*BLS* is 90°. Explain why ∠*LSB* and ∠*LBS* must be complementary.
4. The measures of segment *BS* and segment *PS* are both 4.5 feet and $m\angle PBS = 56°$.
   a. Find the measure of ∠*BSP*. Item 2 says that "Angle *QPS* is supplementary to ∠*PSR*."
   b. Is △*BPS* a unique triangle, or can more than one triangle be formed using the given segment lengths and angle measures? Explain.

Common Core State Standards for Embedded Assessment 1

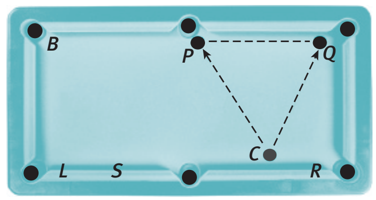
7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Angles and Triangles
POOL ANGLES

Embedded Assessment 1
Use after Activity 14

Another type of pool shot involves aiming the ball at point C directly at a pocket, as shown below.



5. The measure of $\angle RQC = (x + 12)^\circ$ and $m\angle PQC = (2x)^\circ$. Set up and solve an equation to find the value of x.

6. In $\triangle QPC$, $m\angle C = (x + 11)^\circ$, $m\angle Q = (2x + 6)^\circ$, and $m\angle P = 70^\circ$.
 a. Set up and solve an equation to find the value of x.
 b. Use the value you found for x to find $m\angle C$ and $m\angle Q$. Show your work.
 c. Is $\triangle QPC$ a unique triangle, or can more than one triangle be formed using the three angle measures? Justify your answer.

7. Is it possible for $\triangle QPC$ to have side lengths of 4 feet, 1.5 feet, and 2 feet? Justify your answer.

8. The measures of two side lengths of a triangle are 6 centimeters and 8 centimeters, and the measure of one angle is 35°.
 a. Use a ruler and a protractor to draw a triangle or triangles that meet these conditions
 b. Attend to precision. Is there only one triangle or more than one triangle that meets these conditions? Explain.

Embedded Assessment 1

5. $x + 12^\circ + 2x = 90^\circ$
$x = 26^\circ$

6. a. $x + 11^\circ + 2x + 6^\circ + 70^\circ = 180^\circ$
$x = 31^\circ$
 b. $m\angle C = (x + 11)^\circ = 31^\circ + 11^\circ = 42^\circ$, $m\angle Q = (2x + 6)^\circ = 2(31^\circ) + 6^\circ = 62^\circ + 6^\circ = 68^\circ$.
 c. No. Sample answer: The triangle is not unique because three angle measures alone are not sufficient to form a unique triangle. At least one side length also is needed to ensure a unique triangle.

7. No. Sample answer: It is not possible because the sum of the two shorter sides must be greater than the longest side: $1.5 + 2 = 3.5$, which is not > 4.

8. a. Check student's drawings.
 b. More than one; Because the conditions do not say that the 35° angle is an included angle or which side is opposite the 35° angle, the only time there will be only one triangle that meets the conditions is if the 35° angle is the included angle between the 6-and 8-centimeter side lengths.

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Unpacking Embedded Assessment 2

Once students have completed this Embedded Assessment, turn to Embedded Assessment 2 and unpack it with students. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 2.

Embedded Assessment 1
Use after Activity 14

Angles and Triangles
POOL ANGLES

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2a-b, 3, 4a-b, 5, 6a-c, 7, 8b)	• Clear and accurate understanding of adjacent angle relationships and angle relationships in a triangle.	• An understanding of adjacent angle relationships and angle relationships in a triangle.	• Partial understanding of adjacent angle relationships and angle relationships in a triangle.	• Incorrect or incomplete understanding of adjacent angle relationships and angle relationships in a triangle.
Problem Solving (Items 2a-b, 4a, 5, 6a-b)	• An accurate interpretation of a problem in order to find missing angle measurements.	• A somewhat accurate interpretation of a problem to find missing angle measurements.	• Difficulty interpreting a problem to find missing angle measurements	• Incorrect or incomplete interpretation of a problem.
Mathematical Modeling / Representations (Items 4b, 6c, 8a-b)	• An accurate drawing of a triangle given information on the side lengths and angles.	• A drawing of a triangle given information on the side lengths and angles.	• Difficulty in drawing a triangle given information on the side lengths and angles.	• An incorrect drawing of a triangle given information on the side lengths and angles.
Reasoning and Communication (Items 1, 3, 4b, 6c, 7, 8b)	• Precise use of appropriate terms to describe angle relationships and triangles.	• Use of appropriate terms to describe angle relationships and triangles.	• A partially correct use of terms to describe angle relationships and triangles.	• An incomplete or inaccurate use of terms to describe angle relationships and triangles.

Similar Figures

The Same but Different

ACTIVITY 15

Lesson 15-1 Identify Similar Figures and Find Missing Lengths

My Notes

Learning Targets:

- Identify whether or not polygons are similar.
- Find a common ratio for corresponding side lengths of similar polygons.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Summarize/Paraphrase/Retell, Visualization, Create Representations, Identify a Subtask

The Pentagon, the headquarters of the United States Department of Defense, is located in Arlington County, Virginia. This building is named for its shape.

1. Study these two aerial photos of the Pentagon.

Photo 1

Photo 2

a. How are the photos alike?

Sample answer: They are of the same building from the same angle. Both show the building in the shape of a pentagon.

b. How are the photos different?

Sample answer: The buildings and other objects in the two photos are

Common Core State Standards for Activity 15

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

ACTIVITY 15

Guided

Activity Standards Focus

Students have learned that a ratio is a comparison of two numbers and that a proportion is an equation equating two ratios. Ratios are useful in finding rates and unit rates and, especially, when they can be used to write a proportion which can be used to find a missing variable. In this activity, students apply ratios and proportions to learn whether figures are similar and, if they are, to calculate the measures of missing angles and sides.

Lesson 15-1

PLAN

Materials

- metric ruler
- protractor

Pacing: 1–2 class periods

Chunking the Lesson

#1–2 #3–4 #5–7

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to think about the Statue of Liberty and a souvenir model of the Statue of Liberty. Ask:

- How are the actual statue and the model alike? [*Sample answer: They are the same shape.*]
- How are they different? [*Sample answer: They are different sizes.*]

Conclude by telling the class that in this lesson, they will use ratios and proportions to analyze relationships between figures that are the same shape but different sizes.

1–2 Activating Prior Knowledge, Marking the Text, Summarizing, Visualization After students read the opening paragraph have them make connections to previous learning by discussing the statement " This building is named for its shape" and characteristics of pentagons. Also have the, explain connections between the two pictures and between the pictures and the actual building. Pentagon in the two photos is the same shape but is shown at different sizes. The same is true of the real Pentagon: it is the same shape as the Pentagon in the photos but a different size.

1–2 (continued) Item 2 provides an opportunity for students to practice using a protractor and ruler. Depending of students' abilities it may not be necessary for each student to make every measure. You may want to assign different groups or different students within groups different segments and angles to measure. After students have completed the measuring have them share their measures and discuss the fact that in making their measurements of the segment lengths and the angle measures of the Pentagon in the two photographs, students may find slightly different values than the actual values. This may lead them to conclude that the two figures are *almost* the same shape but not exactly the same shape. Help them to see that the discrepancies are due to their inability to make exact measurements, not to a difference in the shapes of the figures, and that if they could make exact measurements, they would find that the shapes were equivalent.

CONNECT TO AP

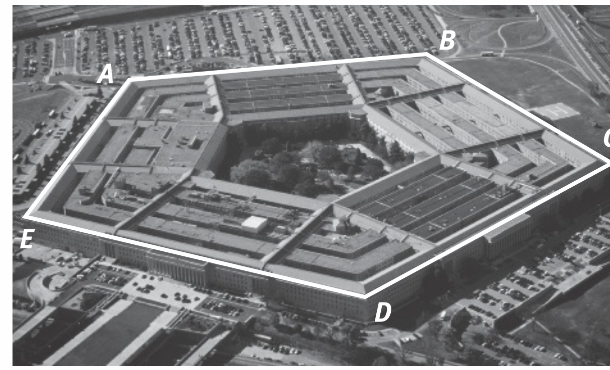
Many objects in nature exhibit *self*-similarity, meaning that as you view then at smaller and smaller scales, all the way down to microscopic size, the views appear approximately alike. Clouds, coastlines, fern leaves, mountain goat horns, and lightning bolts, among countless other examples, all have this property. Self-similarity is central to the study of *fractals*, a fascinating and complex branch of mathematics.

My Notes

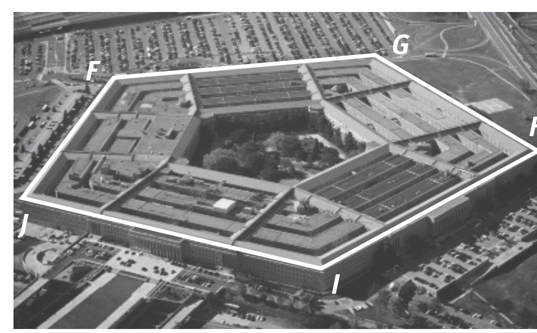
MATH TIP

The shape of the Pentagon building is actually a regular pentagon, but the perspective in this picture makes the lengths appear to be different.

2. Use a protractor and ruler to measure the line segments and angles in both photos below. Measure segments to the nearest millimeter and angles to the nearest degree.



a. $AB =$ 40 mm **b.** $m\angle A =$ 131°
c. $BC =$ 29 mm **d.** $m\angle B =$ 144°
e. $CD =$ 37 mm **f.** $m\angle C =$ 60°
g. $DE =$ 47 mm **h.** $m\angle D =$ 138°
i. $EA =$ 23 mm **j.** $m\angle E =$ 67°



k. $FG =$ 36 mm **l.** $m\angle F =$ 131°
m. $GH =$ 27 mm **n.** $m\angle G =$ 144°
o. $HI =$ 35 mm **p.** $m\angle H =$ 60°
q. $IJ =$ 43 mm **r.** $m\angle I =$ 138°
s. $JF =$ 21 mm **t.** $m\angle J =$ 67°

Lesson 15-1
Identify Similar Figures and Find Missing Lengths

My Notes

3. Use the measurements from Item 2 to find the following ratios to the nearest tenth.

a. $\frac{AB}{FG} = $ **1.1**   **b.** $\frac{BC}{GH} = $ **1.2**   **c.** $\frac{CD}{HI} = $ **1.2**

d. $\frac{DE}{IJ} = $ **1.2**   **e.** $\frac{EA}{JF} = $ **1.2**

4. What can you conclude about the ratio of the lengths of the segments and the measures of the angles in the photos?

The ratios are almost equal, and the measures of the angles in the same position are the same.

Similar figures are figures in which the lengths of the corresponding sides are in proportion and the corresponding angles are ***congruent***. ***Corresponding parts*** of similar figures are the sides and angles that are in the same relative positions in the figures.

5. Construct viable arguments. Are the two photographs of the Pentagon *similar*? Justify your reasoning.

Sample answer: Yes. The corresponding angles in the photos have the same measure, so they are congruent. The ratios of the corresponding side lengths are very close, so the corresponding sides are almost in proportion.

In a *similarity statement*, such as $\triangle ABC \sim \triangle DEF$, the order of the vertices shows which angles correspond. So, $\triangle ABC \sim \triangle DEF$ means that $\angle A$ corresponds to $\angle D$, $\angle B$ corresponds to $\angle E$, and $\angle C$ corresponds to $\angle F$. The corresponding sides follow from the corresponding angles. They are AB and DE, BC and EF, and CA and FD.

6. The lengths of the sides of quadrilateral $ABCD$ are 4, 6, 6, and 8 inches. The lengths of the sides of a similar quadrilateral $JKLM$ are 6, 9, 9, and 12 inches.

a. Write the ratios for the corresponding sides of the quadrilaterals.

$\frac{4}{6}, \frac{6}{9}, \frac{6}{9}, \frac{8}{12}$

b. What do you notice about the ratios of the sides of the similar quadrilaterals?

They are all equivalent to $\frac{2}{3}$.

MATH TERMS

Figures that are **congruent** have exactly the same size and the same shape

READING MATH

The symbol ~ means "is similar to." Read the similarity statement $\triangle ABC \sim \triangle DEF$ as "Triangle *ABC* is similar to triangle *DEF*."

3–4 Sharing and Responding Students should share their work and conjectures. Before moving on revisit reasons measures and ratios were varied. It will be important moving forward for students to understand that for figures to be similar corresponding sides are in proportion and angle measures are equal.

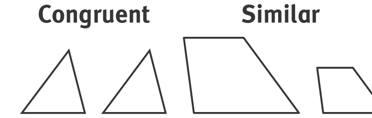
ELL Support

To support students' language acquisition, monitor their listening skills and understanding as they participate in group discussions. Carefully group students to ensure that all group members participate in and learn from collaboration and discussion.

5–7 Create Representations, Marking the Text Assess student understanding of corresponding parts of figures. They may be familiar with congruent figures and should compare and contrast congruence and similarity. It is important that students understand that for figures to be similar the ratios of corresponding sides must be proportional and the angle measures must be equal.

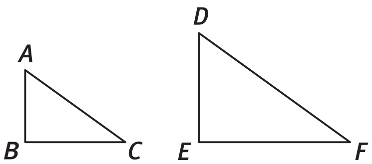
You may wish to show two pairs of figures like the following, where one pair is congruent and the other similar.

Congruent   **Similar**



There are two important points to bring out when discussing congruence and similarity with students. One is that congruence is part of the idea of similarity: the corresponding angles of *similar* figures are *congruent*. The other is that, even though similarity is usually discussed in relation to figures of different sizes, congruent figures are similar, too. That is because they are the same shape (that is, their corresponding angles are congruent), and their corresponding sides are in proportion (the ratio of their corresponding sides is 1 because they are congruent).

5–7 (continued) Be sure students understand the importance of order in listing correspondences between similar figures. Consider these similar triangles:



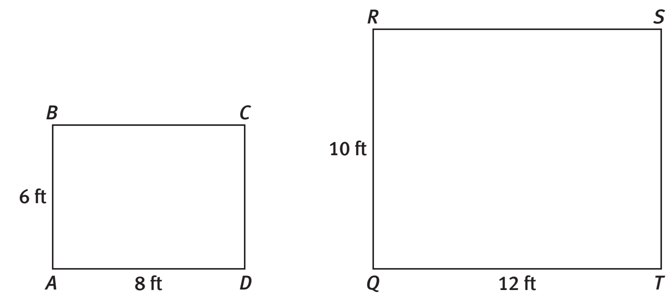
It is correct to write $\triangle ACB \sim \triangle DFE$, $\frac{AB}{BC} = \frac{DE}{EF}$, and $\angle C$ is congruent to $\angle F$. It is not correct to write $\triangle ACB \sim \triangle DEF$, $\frac{AB}{BC} = \frac{EF}{DE}$, or $\angle C$ is congruent to $\angle D$. When tackling a problem in similarity, students may benefit from listing the letters of the figures in correct corresponding order before they begin. So, for the figures above, they might write $\frac{ABC}{DEF}$. They can refer to the list as they work the problems and use it to check their answers.

You may wish to review writing fractions in simplest form so that students can easily compare ratios of corresponding side lengths of figures. Deciding whether $\frac{9}{21}$ and $\frac{15}{35}$ are equal is much easier when both are written in simplest form, $\frac{3}{7}$.

My Notes

If two figures have the same shape with corresponding angles congruent and proportional corresponding sides, then the two figures are similar. When the corresponding sides of similar figures are in proportion, they are equivalent to a common ratio.

7. Rectangle *ABCD* is 6 feet by 8 feet. Rectangle *QRST* is 10 feet by 12 feet.



a. Name the corresponding angles.
$\angle A$ corresponds to $\angle Q$, $\angle B$ corresponds to $\angle R$, $\angle C$ corresponds to $\angle S$, $\angle D$ corresponds to $\angle T$

b. Are the corresponding angles congruent? Explain.
Yes; all the angles of a rectangle are right angles. So the corresponding angles are congruent.

c. Name the corresponding sides.
$\overline{AB}$ corresponds to $\overline{QR}$, $\overline{BC}$ corresponds to $\overline{RS}$, $\overline{CD}$ corresponds to $\overline{ST}$, $\overline{AD}$ corresponds to $\overline{QT}$.

d. Write the ratios of the corresponding widths and lengths of the rectangles.
$\frac{AB}{QR} = \frac{6}{10}$, $\frac{AD}{QT} = \frac{8}{12}$

e. Are the corresponding sides in proportion? Explain.
No; $\frac{6}{10} = \frac{3}{5}$, $\frac{8}{12} = \frac{2}{3}$, and $\frac{3}{5} \neq \frac{2}{3}$. The ratios of the corresponding sides are not equal to a common ratio, so they are not in proportion.

f. Is rectangle *ABCD* similar to rectangle *QRST*? Explain.
No. Although the corresponding angles are congruent, the corresponding sides are not in proportion.

Lesson 15-1
Identify Similar Figures and Find Missing Lengths

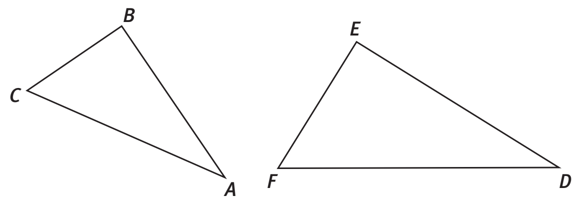
Check Your Understanding

8. Are two congruent figures similar? Explain.

9. When are two polygons with the same number of sides not similar?

LESSON 15-1 PRACTICE

Use a protractor and a ruler to measure the angles and sides of triangles *ABC* and *DEF*.



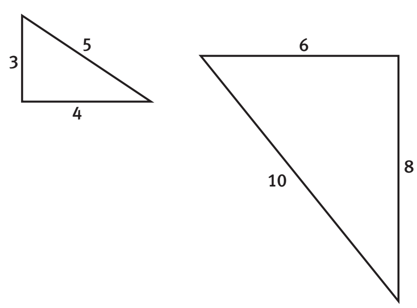
10. Are the corresponding angles congruent? Explain.

11. Are the corresponding sides in proportion? Explain.

12. Are the triangles similar? If so, explain why and write a similarity statement. If not, explain why not.

13. **Model with mathematics.** Sketch two similar rectangles. Explain why they are similar.

14. Are the ratios of the corresponding sides of the right triangles shown equal? Explain.



15. **Make sense of problems.** Rectangle *A* is 3 meters wide and 5 meters long. Rectangle *B* is 2.5 meters wide and 4.5 meters long. Rectangle *C* is 10 meters wide and 18 meters long. Are any of the rectangles similar? Explain.

16. **Reason abstractly.** Are all squares similar? Explain.

My Notes

MATH TIP

When two figures have a different ***orientation***, it can be tricky to identify corresponding sides or angles. Try turning and/or flipping one of the figures until its shape looks like the other figure.

In Item 14, turn the larger triangle to the right until the 8-unit side is the base. Then flip the triangle over an imaginary vertical line. This can help you identify the corresponding sides.

ACADEMIC VOCABULARY

The ***orientation*** of a figure is the way in which the figure is positioned.

Answers

8. Yes. In congruent figures, corresponding angles have the same measure and corresponding sides have in proportion.

9. When corresponding angles have different measures or corresponding sides are not in proportion.

ACTIVITY 15 Continued

Check Your Understanding

Debrief students' answers to these items to be sure they understand the central points of this lesson: Similar figures are the same shape but not necessarily the same size. Corresponding angles are congruent and corresponding sides are in proportion.

ASSESS

Use the Lesson Practice to assess your students' understanding of the relationship between congruent and similar figures. Before students begin, draw their attention to Item 14. Point out that similar figures do not have to appear oriented the same way. Here they can determine the correct side correspondences by matching short side with short side, medium with medium, and long with long.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 15-1 PRACTICE

10. Yes. $m\angle A = m\angle D = 32°$, $m\angle B = m\angle E = 90°$, $m\angle C = m\angle F = 58°$

11. Yes, the common ratio is about 1.3.

12. Yes, corresponding angles are congruent and corresponding sides are in proportion. $\triangle ABC \sim \triangle DEF$.

13. Check students' rectangles. Sample answer: Corresponding angles are congruent right angles and corresponding sides are proportional

14. Yes. They are in the ratio 1:2.

15. Yes. Rectangle $B \sim$ Rectangle C. The angles are congruent and the sides are proportional.

16. Yes. All angles are congruent, and all sides are proportional.

ADAPT

Check students' answers to the Lesson Practice to be sure they understand that for similar figures, the corresponding sides are proportional and the corresponding angles are congruent. For congruent figures, the corresponding sides are congruent and the corresponding angles are congruent. Have student us colored pencils to mark corresponding sides and angles. Students may benefit from creating a graphic organizer to compare and contrast congruent and similar figures in which they include definitions and mathematical symbols.

Lesson 15-2

PLAN

Materials

- metric ruler
- protractor

Pacing: 1–2 class periods

Chunking the Lesson

#1–2 #3

Check Your Understanding

Lesson Practice

TEACH

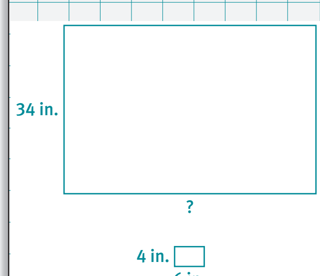
Bell-Ringer Activity

Help students understand that number sense can often help them solve problems involving proportions by presenting this situation: A pancake recipe calls for 4 cups of water and 6 cups of pancake mix. Ask students:

- How many cups of water should you use if you only have 3 cups of pancake mix? Why? *2; You have only half the normal amount of pancake mix, so you should use only half the normal amount of water.*
- How many cups of pancake should you use if you plan to use 8 cups of water? Why? *12; You plan to use twice the normal amount of water, so you should use twice the normal amount of pancake mix.*

1–2 Visualization, Discussion Groups, Create Representations In the last lesson, students were given information about pairs of geometric figures and were asked to decide whether the figures were similar. This prepared them to use proportions to find missing side lengths of figures already known to be similar. Item 1 introduces the method. Before students begin, ask them to look at the triangles to identify corresponding parts and describe the relationship between the side lengths. Students should see that the lengths of the sides of the larger triangle are 4 times those of the smaller triangle, and that they can use this information to find the length of the missing side. Encourage students to take this common sense approach to problems like this, noting relationships among numbers and using them to guide and check the solution.

My Notes



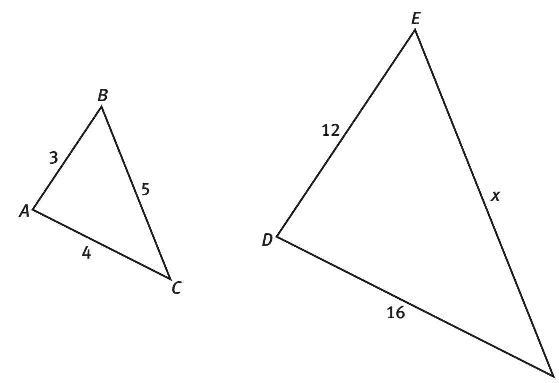
Learning Targets:

- Apply properties of similar figures to determine missing lengths.
- Solve problems using similar figures.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, KWL Chart, Think Aloud, Visualization, Discussion Groups, Create Representations

The corresponding sides of similar figures are in proportion and form a common ratio. This relationship can be used to find missing lengths.

1. $\triangle ABC \sim \triangle DEF$



a. Write the ratios of the corresponding sides.

$\frac{AB}{DE} = \frac{3}{12}, \frac{AC}{DF} = \frac{4}{16}, \frac{BC}{EF} = \frac{5}{x}$

b. What is the common ratio of the corresponding sides with known lengths?

$\frac{1}{4}$

c. Use the common ratio to write a proportion to find the value of x, the missing side length. Solve the proportion.

$\frac{1}{4} = \frac{5}{x}; x = 20$

2. Model with mathematics. A rectangular postcard of a painting is similar to the original painting. The postcard is 4 inches wide and 6 inches long. The original painting is 34 inches wide.

a. Draw similar rectangles to model the problem.

b. Write and solve a proportion for the situation. Let x represent the length of the original painting.

$\frac{34}{4} = \frac{x}{60}; x = 51$

Lesson 15-2
Indirect Measurement

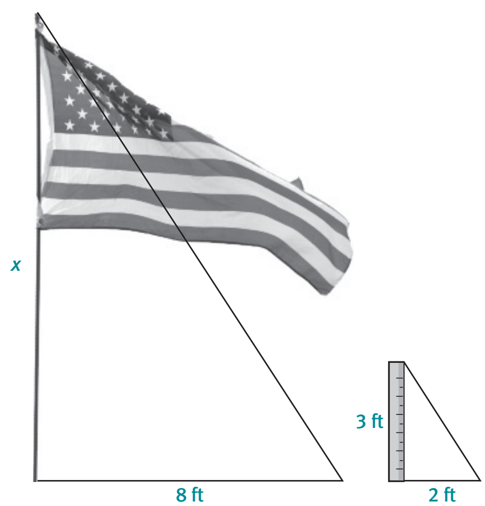
My Notes

c. How long is the original painting?
51 inches

d. Is your answer reasonable? Explain.
Yes. Sample answer: Both the width and the length of the painting are a little more than 8 times the width and length of the post card.

Some objects may be too tall to measure with rulers. Similar triangles can be used to indirectly measure the heights of these objects in real life.

3. A flagpole casts a shadow 8 feet long. At the same time, a yardstick 3 feet tall casts a shadow 2 feet long. The drawing shows how similar triangles can be used to model the situation.



a. Label the picture with the appropriate lengths. Let x represent the height of the flagpole. **See students' work.**

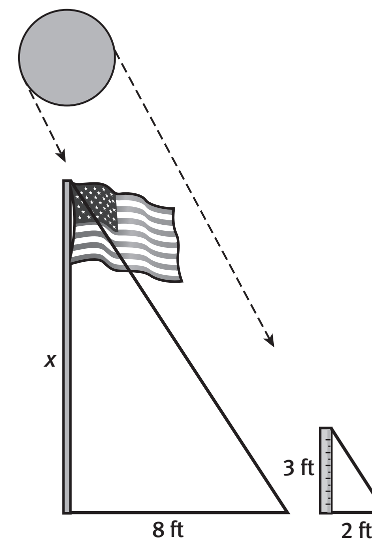
b. Use the corresponding sides of similar triangles to write and solve a proportion for the situation.
$\frac{x}{3} = \frac{8}{2}$; $x = 12$

c. How tall is the flagpole?
12 feet

d. Is your answer reasonable? Explain.
Yes. Sample answer: Both the shadow and height of the flagpole are 4 times the shadow and height of the yardstick.

e. Lars claims that he can solve the flagpole problem using measures within each figure and writes $\frac{x}{8} = \frac{3}{2}$. Is Lars correct? Explain.
yes; the solution of this proportion is also 12

3 Discussion Groups, Create Representations Before students solve Item 3, ask them why the shadows of the flagpole and the yardstick create similar triangles. The reason is that the sun's rays are parallel when they approach both objects, causing them to strike the objects at the same angle and to create congruent acute angle pairs in each triangle. Because the corresponding angles of the triangles are congruent, the triangles are similar.



Check Your Understanding

Debrief students' answers to these items to be sure they understand how to use proportions to find missing side lengths in triangles.

Answers

4. No. You can use corresponding sides to find an unknown side length only if the rectangles are similar. The problem must tell you the rectangles are similar or you must be able to prove that they are similar.
5. No. You need to know three side measures, two for the corresponding sides and the third for the side that corresponds to the missing length.

ASSESS

Use the lesson practice to assess the students' understanding of the method used to find missing side lengths in similar triangles. Make sure students understand the drawing for Item 11, which shows the entire scene: Lena is at the far left. Twenty-four inches in front of her is a $\frac{7}{8}$-inch-tall section of a ruler. One thousand meters farther to the right is a ship. Because the similar triangles overlap, you may want to show them separately, to simplify the writing of the ratios.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 15-2 PRACTICE

6. $\frac{15}{6} = \frac{5}{2}$
7. 10
8. 30
9. 18
10. 16 m
11. The proportion is $\frac{\frac{7}{8}\text{ in.}}{24\text{ in.}} = \frac{h}{1000\text{ m}}$; $24h = (0.875)(1{,}000)$; $24h = 875$; $h = 36.5$ (rounded to tenths); $h \approx 36$ m (rounded to the nearest whole number)

ADAPT

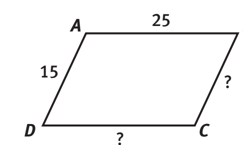
Check students' answers to the Lesson Practice to be sure they understand how to use drawings of similar triangles to write and solve proportions that will allow them to find missing side lengths.

Check Your Understanding

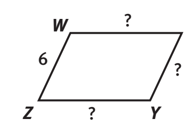
4. If you know the length and width of one rectangle and the length of a second rectangle, can you always use corresponding sides to find the width of the second rectangle? Support your answer.
5. Do you need to know the lengths of all the sides of one triangle to find a missing length of a similar triangle? Explain.

LESSON 15-2 PRACTICE

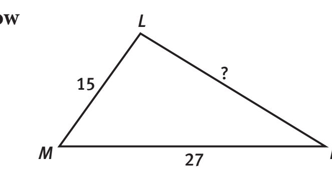
parallelogram *ABCD* ~ parallelogram *WXYZ*



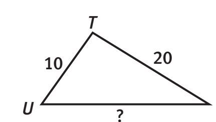
6. What is the common ratio of side *AD* to side *WZ*?
7. Find the length of segment *WX*.



△*LMN* ~ △*TUV*. Use what you know about common ratios to answer Items 8–9.

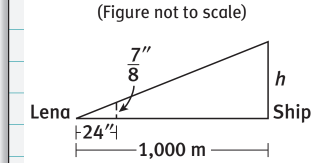


8. How long is segment *LN*?
9. How long is segment *UV*?



10. A 4-meter-tall flagpole casts a 6-meter shadow at the same time that a nearby building casts a 24-meter shadow. What is the height of the building? Solve this problem two different ways. First, set up and solve a proportion in which each ratio compares corresponding side lengths in the two figures. Then set up and solve a proportion in which each ratio compares side lengths within each figure.
11. **Make sense of problems.** Lena is standing on the beach when she sees a tall sailing ship pass by 1,000 meters offshore. She holds a ruler vertically 24 inches in front of her eyes, and the ship appears to be $\frac{7}{8}$ inch high. The figure in the My Notes column represents the situation as two similar right triangles. Find the approximate height (h) of the sailing ship above the water. Explain your answer.

(Figure not to scale)

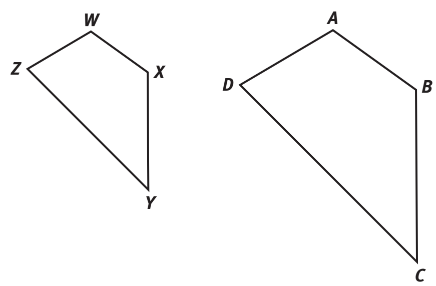


ACTIVITY 15 PRACTICE

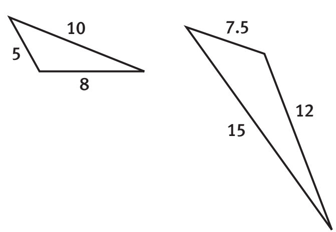
Write your answers on a separate piece of paper. Show your work.

Lesson 15-1

1. Use a ruler and protractor to decide if the figures are similar. Justify your decision.



2. **a.** Identify the corresponding sides on the figures below.
   b. Are the ratios of the corresponding sides of the triangles equal? Explain.



3. $\triangle ABC \sim \triangle EFG$, $m\angle A = 25°$, and $m\angle F = 100°$. What is $m\angle C$?
   A. 125°
   B. 100°
   C. 55°
   D. 25°
4. Rectangle J is 6 feet wide and 9 feet long. Rectangle K is 9 feet wide and 12 feet long. Rectangle L is 15 feet wide and 22.5 feet long. Are any of the rectangles similar? Explain.
5. **a.** Are all equilateral triangles similar? Explain.
   b. Are all right triangles similar? Explain.
6. How is the mathematical meaning of the word "similar" the same as or different from the word "similar" in everyday conversation?

ACTIVITY 15 Continued

ACTIVITY PRACTICE

1. Yes. The corresponding angles are the same measure and the corresponding sides are in proportion.
2. **a.** 5 and 7.5, 10 and 15, 8 and 12
   b. Yes, they are all equal to $\frac{2}{3}$.
3. C
4. Yes. Rectangle $J \sim$ Rectangle L. Angles are congruent and sides are proportional.
5. **a.** Yes. The angles are all 60°, and since all of the sides are equal they will be in proportion.
   b. No. A right triangle can be isosceles or scalene. A scalene triangle will not be similar to an isosceles triangle.
6. In real life, the word "similar" is used to describe things that may look somewhat alike. In mathematics, similar figures must have exactly the same angle measures and sides that are proportional.

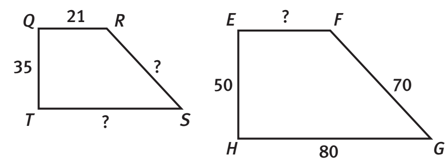
7. $RS = 49$, $TS = 56$, $EF = 30$
8. $TV = 9$, $MN = 8$
9. $QR = 48$, $FG = 9\frac{3}{8}$
10. 23 in.
11. C
12. 147 in. or 12 ft 3 in.
13. 10.92 meters

ADDITIONAL PRACTICE

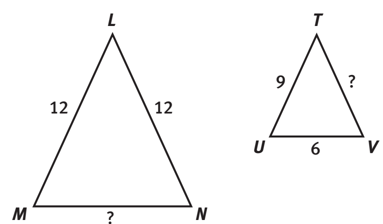
If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Lesson 15-2

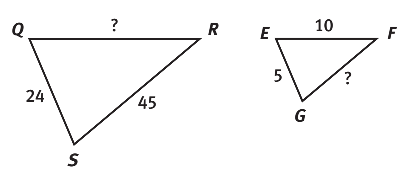
7. Trapezoid $QRST \sim$ trapezoid $EFGH$. Find the measures of the missing sides.



8. $\triangle LMN \sim \triangle TUV$. Find the measures of the missing sides.

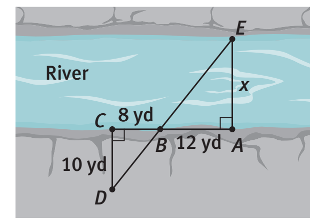


9. $\triangle QRS \sim \triangle EFG$. Find the measures of the missing sides.



10. A rectangular room is 42 feet wide and 69 feet long. On a blueprint, the room is 14 inches wide. How long is the room on the blueprint?

11. John wants to find the width of a river. He marks distances as shown in the diagram. Which of the following ratios can be used to find the width of the river?



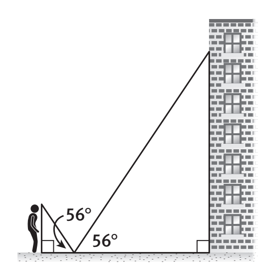
A. $\frac{10}{8} = \frac{12}{x}$ **B.** $\frac{10}{12} = \frac{8}{x}$

C. $\frac{8}{10} = \frac{12}{x}$ **D.** $\frac{x}{12} = \frac{8}{10}$

12. Miguel is 5 feet 10 inches tall. On a sunny day he casts a shadow 4 feet 2 inches long. At the same time, a nearby electric tower casts a shadow 8 feet 9 inches long. How tall is the tower?

MATHEMATICAL PRACTICES
Make Sense of Problems

13. Sam wants to find the height of a window in a nearby building but it is a cloudy day with no shadows. Sam puts a mirror on the ground between himself and the building. He tilts it toward him so that when he is standing up, he sees the reflection of the window. The base of the mirror is 1.22 meters from his feet and 7.32 meters from the base of the building. Sam's eye is 1.82 meters above the ground. How high up on the building is the window?



Circles: Circumference and Area

ACTIVITY 16

Gardens Galore

Lesson 16-1 Circumference of a Circle

My Notes

Learning Targets:

- Investigate the ratio of the circumference of a circle to its diameter.
- Apply the formula to find the circumference of a circle.

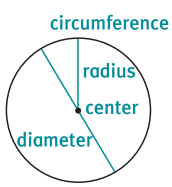
SUGGESTED LEARNING STRATEGIES: Think Aloud, Create Representations, Discussion Groups, Summarizing, Paraphrasing

Rose wants to create several circular gardens in her yard. She needs to find the distance around and the area of each garden.

A ***circle*** is the set of points in the same ***plane*** that are an equal distance from a given point, called the ***center***. The distance around a circle is called the ***circumference***.

A line segment through the center of a circle with both endpoints on the circle is called the ***diameter***. A line segment with one endpoint on the center and the other on the circle is called the ***radius***.

1. A circle is shown.



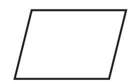
a. Use the information given above to label the center and the circumference of the circle

b. Draw and label a diameter and a radius in the circle.

2. What is the relationship between the length of the diameter and the length of the radius of a circle?
The diameter is twice the length of the radius.

MATH TERMS

A **plane** is a flat surface that extends in all directions. A parallelogram is usually used to model a plane.



Common Core State Standards for Activity 16

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

ACTIVITY 16

Investigative

Activity Standards Focus

In earlier grades, students learned basic facts about plane figures–how to classify them, distinguish them from one another, and, in certain cases, find their perimeters and areas. In this unit they examined more challenging topics: What conditions determine a unique triangle? How can you find a missing side of a triangle if it is similar to a triangle whose sides you know? In this activity, students learn how to find the circumference and area of a circle, the first figure with curved sides they have dealt with. This leads to the introduction of the number π, whose digits, students are informed, "never end or repeat."

Lesson 16-1

PLAN

Materials

- string
- metric ruler
- metric measuring tape
- circular objects such as coins, paper plates, cups, lids

Pacing: 1–2 class periods

Chunking the Lesson

#1–2 #3–6 # 7–8 #9
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

See that each student has a circular object, e.g., a coin, a ring, a can lid, a CD, or a plate. Ask students to estimate the distance around the outside of the object, the distance across it (from edge to edge through the center), and the ratio of the circumference to the diameter. Have them write their estimates on the board and have the students estimate the average of the guesses. Reintroduce the average later, after students have learned the value of π.

1–2 Marking the Text, Word Wall Create Representations Students sometimes see a drawing of a diameter or a radius and assume it is the only one in the circle. Emphasize that *any* line segment satisfying the definition of a diameter is a diameter, and that any segment satisfying the definition of a radius is a radius.

3–6 Use Manipulatives, Create Representations, Discussion Groups

For the table in Item 3, have students measure the objects they used in the Bell-Ringer activity. To measure the circumference, students can use a measuring tape or they can use a string and then measure the part of the string that represents the circumference.

Students should round the ratios in the bottom row to the nearest hundredth. They are likely to find that the ratios are close to 3.14 but not equal to it. From this they could conclude that the ratio of the circumference of a circle to its diameter varies. To convince them otherwise, have several students or groups measure the same circular object and calculate the circumference-diameter ratio independently. Their results are likely to vary. Help students to see that the inexactness of the measurements must have caused the discrepancies, not variations in the ratios. Emphasize that the value 3.14 in this context is a approximate value.

Teacher to Teacher

The following steps show how the formula $C = \pi d$ can be rewritten as $C = 2\pi r$.

$C = \pi d$	Formula for the circumference of a circle
$= \pi(2r)$	diameter $= 2 \times$ radius
$= (\pi \times 2)r$	Associative Property of Multiplication
$= (2 \times \pi)r$	Commutive Pripperty of Multiplication
$= 2\pi r$	Simplify.

My Notes

There is also a relationship between the circumference and the diameter of a circle. Work with your group to complete the activity below to find the relationship.

3. Measure the circumference and diameter of the circular objects provided by your teacher. Use the table to record the data. Then calculate the ratios.
Sample answers shown in the table.

Description of object	Object 1	Object 2	Object 3	Object 4	Object 5
	Soup can	Coffee can	Trash can		
Circumference	23.6 cm	31.4 cm	175.8		
Diameter	7.5 cm	10 cm	56 cm		
Ratio of circumference to diameter (as a fraction)	$\frac{23.6}{7.5}$	$\frac{31.4}{10}$	$\frac{175.8}{56}$		
Ratio of circumference to diameter (as a decimal)	3.15	3.14	3.14		

The ratio of the circumference to the diameter of a circle is called **pi**, denoted by the Greek letter π. A commonly used approximation for *pi* is $\pi \approx 3.14$.

Writing Math

The digits following the decimal point in the decimal representation for *pi* never end or repeat, so all values found using *pi* are approximations. The symbol $\approx$ means "approximately equal to."

4. Which measurement tools used by your class gave the most accurate approximation of *pi*? Why do you think this is true?
Sample answer: I think measuring tapes gave the best measurements. It is too hard to place beans (or other items) around a circle and not have spaces or overlaps. The tapes are flexible and come very close to going exactly around the circles.

Connect to Language

The Greek letter π is the first letter in the Greek words for perimeter and periphery.

5. Express regularity in repeated reasoning. Using the data on your table, write the equation that relates the circumference of a circle, *C*, to *pi* (π) and the diameter, *d*, of a circle.
$C = \pi d$,= $\pi = \frac{C}{d}$, or any other equivalent expression

6. Rewrite the equation from Item 5 to show the relationship of the circumference of a circle, *C*, to its radius (r) and π.
$C = 2r\pi$

The equations you wrote for Items 5 and 6 above are the formulas for finding the circumference of a circle given its diameter or radius.

7. Sometimes $\frac{22}{7}$ is used as an approximation of π. Why is this fraction a good approximation?
After you divide to express the fraction as a decimal, the quotient rounded to the nearest hundredth is 3.14.

8. Attend to precision. Should the circumference of a circle be labeled in units or in square units? Explain.
Units; since circumference is the distance around a circle, it is a linear measurement.

Now you have an equation you can use to find the circumference of a circle. Use what you know to help Rose find the distance around one of her gardens.

9. One of the circular gardens Rose wants to make has a diameter of 6 feet.

a. Use a circumference formula to find the amount of decorative fencing that Rose needs to enclose this garden. Use 3.14 or $\frac{22}{7}$ for π. Show your work. Tell which value you used for *pi*.
$\approx$ 18.84 (using $\pi \approx 3.14$) or $\approx$ 18.86 (using $\pi \approx \frac{22}{7}$) feet

b. Decorative fencing is sold in packages of 12-foot sections. How many packages must Rose buy to enclose this garden? Explain your reasoning and show your work.
2 packages; Since she needs 18.84 feet of fencing, and fencing is sold in 12-ft packages, she will need 2 packages, which will give her 24 feet.

ACTIVITY 16 Continued

7–8 Discussion Groups Items 7 and 8 provide an opportunity to stress the importance of precise mathematical language. As with 3.14 emphasize that there is not an exact value of π. You may wish to introduce the fact that π is not a rational number, that it is called an irrational number. Ask students to find the circumference of two circles with diameters of 34 cm and 35 cm. Have them discuss whether it matters if they use 3.14 or $\frac{22}{7}$ for π. Students may discover that $\frac{22}{7}$ is especially useful as an approximation of π when the radius or diameter of a circle is a multiple of 7.

diameter: 35 cm

$$\begin{aligned} C &= \pi d \\ &\approx \frac{22}{7} \cdot \frac{35}{1} \\ &\approx \frac{22}{\cancel{7}_1} \cdot \frac{^5\cancel{35}}{1} \\ &\approx 22 \cdot 5 \\ &\approx 110 \end{aligned}$$

The advantages diminish in other instances.

diameter: 34 cm

$$\begin{aligned} C &= \pi d \\ &\approx \frac{22}{7} \cdot \frac{34}{1} \\ &\approx \frac{22}{7} \cdot \frac{34}{1} \\ &\approx \frac{748}{7} \\ &\approx 106\frac{6}{7} \end{aligned}$$

9 Create Representations, Discussion Groups Again, point out that while 3.14 and $\frac{22}{7}$ are good approximations of π and give answers that are close to the actual answers, the only way to find *exact* answers to problems involving circumferences of circles is to use π for π! The *exact* circumference of a circle with a diameter of 10 cm is $\pi d = \pi(10) = 10\pi$ cm. Introducing students to this idea will help them to begin thinking of π as an actual number, one that is as much of a number as is 28 or 5.29 or $\frac{7}{8}$.

Check Your Understanding

Debrief students' answers to these items to be sure they understand the relationship of π to the circumference of a circle, as well as the relative advantages of using 3.14 or $\frac{22}{7}$ as an approximation of π.

Answers

10. Sample answer: Use $\frac{22}{7}$ if the radius or the diameter is a multiple of 7 or if it is a fraction.

11. Sample answer: *Pi* is the ratio of the circumference of a circle to its diameter. The circumference of a circle is the product of its diameter and *pi*.

ASSESS

Use the Lesson Practice to assess your students' understanding of meaning of π, the relationship of the radius of a circle to its diameter, and the relationships of the radius and the diameter to the circumference.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 16-1 PRACTICE

12. **a.** 43.96 cm
b. 18.84 in.
c. 50.24 ft
d. 78.50 m

13. 220 m

14. 47.1 cm

15. 113.04 in.

16. 40 cm; 20 cm

17. about 74 revolutions;
500 ft = 6,000 in.;
$C \approx 81.64$ in.;
$\frac{6,000}{81.64} \approx 73.5$

ADAPT

Check students' answers to the Lesson Practice to be sure they understand how to find the circumference of a circle given either the radius or the diameter, and how to find the radius and the diameter of a circle given the circumference. Have students use colored pencils to visualize the relationships of the formula for circumference and the parts of a circle. Have them write the formula using one color for *C*, another for π, and a third for *d* (or *r*). Then have them draw a circle using corresponding colors.

My Notes

Check Your Understanding

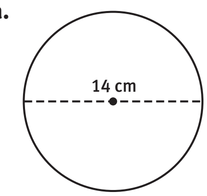
10. Explain how you could decide which approximation of π—3.14 or $\frac{22}{7}$—to use to compute the circumference of a circle.

11. Explain how the circumference of a circle and the definition of *pi* are related.

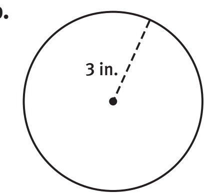
LESSON 16-1 PRACTICE

12. For Items a–d, find the circumference of each circle expressed as a decimal.

a.



b.



c. a circle with a radius of 8 ft

d. a circle with a diameter of 25 m

13. Find the circumference of a circular dog pen that has a radius of 35 meters. Use $\frac{22}{7}$ for π.

14. A window shaped like a circle has a diameter of 15 centimeters. What is the circumference of the window? Use 3.14 for π.

15. A circular tablecloth has a radius of 18 inches. What is the circumference of the tablecloth? Use 3.14 for π.

16. **Make use of structure.** A circle has a circumference of 125.6 centimeters. What is the diameter of the circle to the nearest centimeter? What is the radius to the nearest centimeter?

17. **Make sense of problems.** The diameter of a bicycle wheel is 26 inches. About how many revolutions does the wheel make during a ride of 500 feet? Use 3.14 for π. Explain your answer.

Lesson 16-2
Area of a Circle

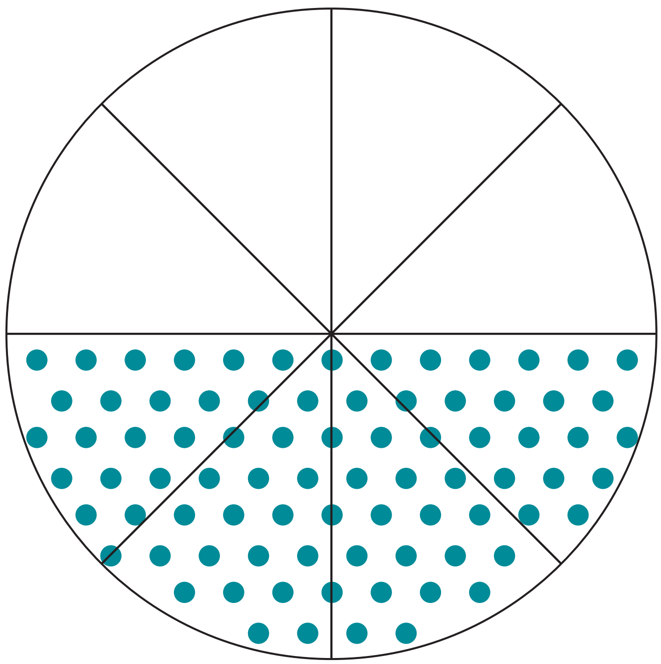
My Notes

Learning Targets:
- Approximate the area of a circle.
- Apply the formula to find the area of a circle.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think Aloud, Think-Pair-Share, Use Manipulatives

Rose also needs to know the area of each of the circular gardens.

1. On a separate sheet of paper, trace the circle shown. Shade half of the circle. Then cut your circle into eight congruent pie-shaped pieces.



Lesson 16-2

PLAN

Materials
- scissors

Pacing: 1–2 class periods

Chunking the Lesson

#1–8 #9–10

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Show a sheet of $8\frac{1}{2}$ in. $\times$ 11 in. paper and tell students the dimensions. Have students find the area of the sheet (93.5 in.^{2}). Now have them cut the sheet into two or three pieces and rearrange them into a different shape than the original. Ask: What is the area of the new shape? (93.5 in.^{2}) Why? (Sample answer: The new shape consists of the same paper that composed the original shape, so the area must be the same.)

1–8 Create Representations, Paraphrasing, Use Manipulatives

Students carry out an activity to find a formula for the area of a circle. With each step have students paraphrase the steps they will be completing. First, they cut a circle into eight pieces and rearrange them to form a figure that resembles a parallelogram. Then they use their knowledge of quadrilaterals to find the area of the parallelogram. Students should conclude that the circle must have the same area as the parallelogram, since it and the parallelogram are made from the same piece of paper.

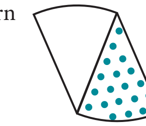
1–8 (continued) Some students may argue that the figure they construct from the eight pieces of the circle isn't really a parallelogram, since the top and bottom each consists of four curves rather than a straight line. You can agree that they are right and that the activity produced only an approximation of a parallelogram. Go on to say that the more pieces the original circle is cut into, the closer the rearranged figure will resemble a parallelogram. Furthermore, no matter how many pieces are used, the resulting formula for the area of a circle always turns out to be $A = \pi r^2$. With a million pieces, no one would be able to tell that the figure wasn't a parallelogram, and the resulting formula would again be $A = \pi r^2$.

CONNECT TO AP

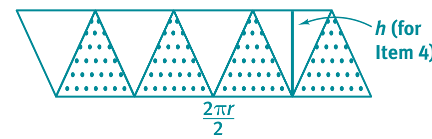
Calculus is needed to prove that the area of a circle of radius r is $A = \pi r^2$. Middle School students must be content with the argument that all approximations of a circle's area yield the formula $A = \pi r^2$.

My Notes

2. Arrange the eight pieces using the alternating pattern shown.



3. Sketch the shape you made with the circle pieces. **Sample sketch:**



a. What geometric shape does the shape resemble?
parallelogram

b. What do you know about the area of the circle and the area of the figure you make? **The area is the same for both figures.**

4. On your sketch, draw and label the height of the figure. What part of the circle does the height represent?
See *h* on anno above; the height of the parallelogram corresponds to the radius of the circle.

5. What other measure of the circle do you need to know to determine the area of the shape you sketched? Label it on your sketch and explain your reasoning.
Sample answer: One-half the circumference of the circle; see label on anno for Item 3. To find the area of the parallelogram, I need to know the length of its base, which is half of the circumference of the circle.

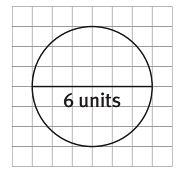
6. Model with mathematics. Use words, symbols, or both to describe how you can now calculate the area of the circle. Start with the formula $A = b \times h$ and substitute into the formula. Refer to your labeled sketch as needed.
Sample answer: Using the area formula for a parallelogram $A = b \times h$, half the circumference of the circle can be substituted for b, and the radius of the circle can be substituted for h: $\left(\frac{1}{2} \times 2\pi r\right)(r)$.

7. Use your answer from Item 6 to write the formula for the area of a circle A in terms of its radius r and π. Explain how you found the formula.
A of a circle $= \pi r^2$; I simplified the equation I wrote: $A = b \times h = \left(\frac{1}{2} \times 2\pi r\right)(r) = \left(\frac{2\pi r}{2}\right)\left(\frac{r}{1}\right) = (\pi r)(r) = \pi r^2$.

8. Should the area of a circle be labeled in units or in square units? Explain.
Sample answer: square units, because to find area you multiply units by units which gives units squared.

My Notes

9. A circle with a radius of 3 units is graphed.



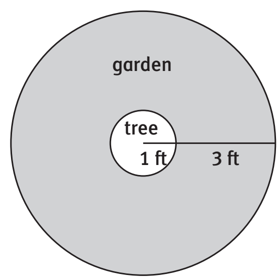
a. Estimate the area by counting the number of enclosed squares. Show the calculations that lead to your estimation.

Sample answer: 28 square units

b. Use the formula you wrote in Item 7 to find the area of the circle. Use 3.14 for π.

$A = \pi r^2 = \pi \times 3^2 = 3.14 \times 9 \approx 28.26$

10. Make sense of problems. One of Rose's gardens will have a tree in the center as shown. The radius of the large outer circle will be 4 feet while the radius of the inner circle for the tree will be 1 foot. Find the area of the garden without the tree. Show your work.



≈ 47.1 ft^2; A larger circle $= \pi 4^2 \approx (3.14)(16) \approx 50.24$ ft^2; A smaller circle $= \pi 1^2 \approx 3.14$ ft^2, $50.24 - 3.14 = 47.1$ ft^2

Check Your Understanding

11. Is the circumference of a circle enough information to determine the area of the circle? Explain.

12. What is the formula for the area of a circle in terms of its diameter, d?

9–10 Create Representations, Think-Pair-Share, Debriefing Students apply the formula for the area of a circle. In Item 9, they first estimate the area by counting squares on a grid and then calculate the area using the formula. Students should use their estimate as a check on their calculations. In Item 10 they must see that the area without the tree can be found by subtracting the area of the tree circle (radius = 1 ft) from the area of the big circle (radius = 3 ft + 1 ft = 4 ft).

Using 3.14 for π:

$r = 1$ ft $\quad A = \pi r^2$
$\approx 3.14(1)^2$
$\approx 3.14(1)$
≈ 3.14

$r = 4$ ft $\quad A = \pi r^2$
$\approx 3.14(4)^2$
$\approx 3.14(16)$
≈ 50.24

$50.24 \div 3.14 \approx 16$ square feet

Ask students to explain why the answer is 16 and not 4. Elicit that in the formula, the radius is squared. $4^2 = 16$, which is 16 times 1^2.

Differentiating Instruction

You may wish to pose this problem relating to Item 10 to motivated students: How many times the area of the 1-ft-radius circle is the area of the 4-ft-radius circle? Ask students to estimate the answer before solving. Students may be surprised to find that the answer is 16, not 4.

Check Your Understanding

Debrief students' answers to these items to be sure they understand the relationships among the area, the circumference, the diameter, and the radius of a circle.

Answers

11. Sample answer: Yes. Use the circumference to find the radius, and then use the radius to find the area.

12. $r = \frac{d}{2}$; $A = \pi r^2$

$= \pi\left(\frac{d}{2}\right)^2$

$= \frac{\pi d^2}{2^2}$

$= \frac{\pi d^2}{4}$

ASSESS

Use the lesson practice to assess your students' understanding of how to find the area of a circle given its radius, diameter, or circumference, and how to the area of composite figures composed of circles.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 16-2 PRACTICE

13. **a.** 153.86 cm^2
b. 254.34 $in.^2$
c. 200.96 km^2
d. 490.63 ft^2
14. 3.14: 3,846.5 m^2 or $\frac{22}{7}$: 3,850 m^2
15. 551.27 mm^2
16. 40.82 ft^2
17. 452.16 $in.^2$
18. 78.50 cm^2
19. $\pi \approx 3.16$; $A = \pi r^2$, $8^2 = \pi(4.5)^2$, $64 = \pi(20.25)$, so $\pi \approx \frac{64}{20.25}$, or 3.16 to the nearest hundredth.

ADAPT

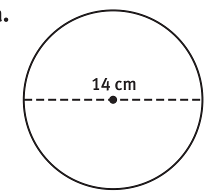
Check students' answers to the Lesson Practice to be sure they understand how to solve problems involving the areas of circles, given information about the radii, diameters, or circumferences of the circles. In a manner similar to adapting students understanding of circumference, have students use colored pencils to visualize the relationships of the formula for circumference and the parts of a circle. Have them write the formula using one color for A, another for π, and a third for d (or r). Then have them draw a circle using corresponding colors. Also, have them use colors to distinguish area and circumference.

My Notes

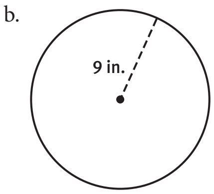
LESSON 16-2 PRACTICE

13. Find the approximate area of each circle. Use the value for π that makes the math simplest for you.

a.



b.



c. a circle with a radius of 8 km
d. a circle with a diameter of 25 ft

14. What is the approximate area of a circular pond that has a radius of 35 meters? Tell what value you used for π.

15. A penny has a diameter of about 26.5 millimeters. What is the area of a penny to the nearest hundredth? Use 3.14 for π.

16. One trampoline has a diameter of 12 feet. A larger trampoline has a diameter of 14 feet. How much greater is the area of the larger trampoline? Use 3.14 for π.

17. **Make sense of problems.** A painting shaped like a circle has a diameter of 20 inches. A circular frame extends 2 inches around the edge of the painting. How much wall space does the framed painting need? Use 3.14 for π.

18. The circumference of a circle is 31.4 centimeters. What is the area of the circle?

19. **Reason abstractly.** In ancient Egypt, a scribe equated the area of a circle with a diameter of 9 units to the area of a square with a side length of 8 units. What value of π does this method produce? Explain.

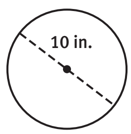
ACTIVITY 16 PRACTICE

Write your answers on a separate piece of paper. Show your work.

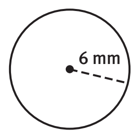
Lesson 16-1

1. Find the circumference of each circle below. Use 3.14 for π.

a.



b.



2. The diameter of a pizza is 14 inches. What is the circumference of the pizza? Tell what value you used for π.

3. The radius of a circular mirror is 4 centimeters. What is the circumference of the mirror? Tell what value you used for π.

4. The radius of a circular garden is 28 feet. What is the circumference of the garden? Tell what value you used for π.

5. Find the diameter of a circle if $C = 78.5$ feet. Use 3.14 for π.

6. Find the radius of a circle if $C = 88$ yards. Use $\frac{22}{7}$ for π.

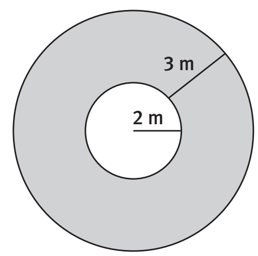
7. Multiple Choice. A standard circus ring has a radius of 6.5 meters. Which of the following is the approximate circumference of the circus ring?
A. 13 meters
B. 20.4 meters
C. 40.8 meters
D. 132.7 meters

Lesson 16-2

8. What is the area of a pizza with a diameter of 12 inches?

9. A circle has circumference 28.26 cm. What is the area of the circle? Use 3.14 for π.

10. Find the area of the shaded region. Use 3.14 for π.



11. Multiple Choice. The circular base of a traditional tepee has a diameter of about 15 feet. Which of the following is the approximate area of the base of the tepee?
A. 23.6 square feet
B. 47.1 square feet
C. 176.6 square feet
D. 706.5 square feet

ACTIVITY PRACTICE

1. a. $C = 31.4$ in.
b. $C = 37.7$ mm
2. 3.14: 43.96 in. or $\frac{22}{7}$: 44 in.
3. 3.14: 25.12 cm or $\frac{22}{7}$: 25.14 cm
4. 3.14: 175.84 ft or $\frac{22}{7}$: 176 ft
5. 25 ft
6. 14 yd
7. C
8. 113.0 or 113.1 mm^2
9. 63.6 cm^2
10. 65.94 m^2
11. C

12. About 4.8 ft^2; I found that the area of a circle with diameter $3\frac{1}{2}$ ft equals about 9.6 ft. I divided it by 2 because the window is a semicircle.
13. a. square; about 4.3 m greater
    b. square; about 5.375 m^2 greater
14. $r = 3$ units; 28.26 $units^2$
15. Yes; serving size for 12-in. quiche is about 18.8 $in.^2$, while the serving size for 10-in. quiche is about 19.6 $in.^2$, and $19.6 > 18.8$.
16. The 14-in. diameter is the better buy since you get about 9.61 $in.^2$ per $1, while you get about 5.02 $in.^2$ per $1 with the 8-in. diameter.
17. a. Circumference doubles.
    b. Area is 4 times as great.
18. Yes. When the radius is equal to 2 units, the circumference and the area will both be $4 \times \pi$.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

12. A window is shaped like a semicircle. The base of the window has a diameter of $3\frac{1}{2}$ feet. Find the area of the window to the nearest tenth of a foot. Explain how you found the answer.

13. A circle has a diameter of 5 meters and a square has a side length of 5 meters.
- **a.** Which has the greater perimeter? How much greater?
- **b.** Which has the greater area? How much greater?

14. A circle with center at $(1, -1)$ passes through the point $(1, 2)$. Find the radius and then the area of the circle. Use 3.14 for π. Make a sketch on graph paper if it is helpful.

15. A quiche with a diameter of 12 inches can feed 6 people. Can a quiche with a diameter of 10 inches feed 4 people, assuming the same serving size? Explain your thinking.

16. A pizza with a diameter of 8 inches costs $10. A pizza with a diameter of 14 inches costs $16. Which is the better buy? Explain your thinking.

17. The radius of a circle is doubled.
- **a.** How does the circumference change?
- **b.** How does the area change?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

18. Is it possible for a circle to have the same numerical value for its circumference and area? Explain your reasoning.

Composite Area

Tile Designs

Lesson 17-1 Area of Composite Figures

ACTIVITY 17

My Notes

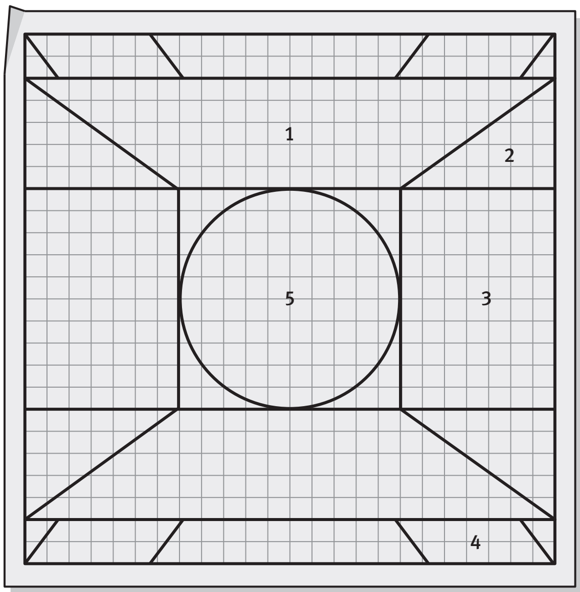
Learning Targets:

- Determine the area of geometric figures.
- Determine the area of composite figures.

SUGGESTED LEARNING STRATEGIES: Create Representations, Discussion Groups, Identify a Subtask, Think-Pair-Share, Visualization

Each year the students in Ms. Tessera's classes create a design for a stained-glass window. They draw two-dimensional figures on grid paper to create the design for their stained-glass windows.

1. This drawing shows the design for one of the projects done last year.



a. What is the area of the entire stained-glass window? Explain.

576 units^2; the design measures 24 units by 24 units, so I used the formula A square $= s^2$.

b. What is the most precise geometric name for each of the numbered shapes in the design?

Figure 1: isosceles trapezoid; Figure 2: right triangle; Figure 3: rectangle; Figure 4: parallelogram; Figure 5: circle

Common Core State Standards for Activity 17

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

ACTIVITY 17

Investigative

Activity Standards Focus

Until now, students' study of geometric shapes has largely been confined to identifying polygons by the number of sides or the measure of their angles, and then finding the areas of the polygons. In Activity 17 they move on to finding the area and perimeter (and circumference) of two-dimensional shapes that are composites of polygons.

Lesson 17-1

PLAN

Materials

- grid paper

Pacing: 1–2 class periods

Chunking the Lesson

#1 #2

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Have students draw their own composite figures and then break them apart into shapes that have area and perimeter formulas that they have studied. Then have them list each shape that makes up the composite figure and give the formula for the area and perimeter of that shape.

1 Marking the Text, Visualization

Have students label the simpler geometric figures and review the formulas to find the area of each. Encourage students to show the substitutions for the formulas as they apply them to the numbered shapes.

2 Create Representations, Identify a Subtask, Think-Pair-Share, Visualization Tell students that there is more than one way to find the area of the composite figure. Have students work in pairs and work to come up with the two different ways to divide the figure into two rectangles. Challenge students to find the area by dividing the figure into three rectangles.

Developing Math Language

Help students understand the difference between *composing* a shape out of simpler shapes (making a composite shape) and *decomposing* a shape, or dividing the shape into simpler figures.

ELL Support

Help students see that composite shapes can be divided into simpler shapes, sometimes in more than one way.

Point out that the shape should be divided so that all of the dimensions of the simpler shapes can be determined.

MATH TIP

Recall the following area formulas.

Triangle: $A = \frac{1}{2}bh$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Circle: $A = \pi r^2$

ACADEMIC VOCABULARY

In geometry, when you divide a composite figure into smaller figures, you ***decompose*** the figure.

c. Find the area of each numbered shape.

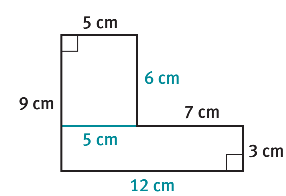
Figure 1	Figure 2	Figure 3
85 units²	**17.5 units²**	**70 units²**
Figure 4	Figure 5	
12 units²	**≈ 78.5 units²**	

d. Explain why the students might be interested in finding the areas of the different shapes used in the design of the stained-glass window.
Answers may vary. The shapes may be different colors. They will need to know the area of each shape to determine how much of each color of glass is needed to make the window.

Some geometric figures have formulas that can be used to find the area of the figure. Other shapes do not have their own formulas.

A ***composite figure*** is made up of two or more geometric figures. You can find the area of a composite figure by dividing, or ***decomposing***, it into simpler geometric shapes with known formulas.

2. The composite figure below can be divided into two rectangles.



a. Draw a line segment on the diagram to divide the figure into two rectangles. Label the length and width of each rectangle.
Sample answer is shown.

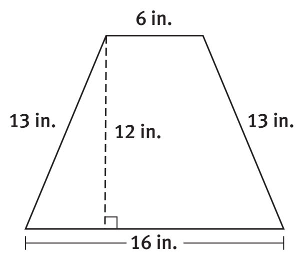
b. Find the total area of the composite figure. Show your work. Label your answer with the appropriate unit of measure.
Total area = 66 cm²; Work will vary depending on how students divide the figure. Sample: Rectangle 1 = 5 cm × 6 cm = 30 cm², Rectangle 2 = 12 cm × 3 cm = 36 cm²

c. Find the perimeter of the composite figure. Show your work. Label your answer with the appropriate unit of measure.
perimeter = 9 + 5 + 7 + 6 + 3 + 12 = 42 cm

My Notes

Check Your Understanding

3. Use the trapezoid shown.



a. Explain how to find the area of the trapezoid by dividing it into simpler geometric shapes.

b. Find the area of the trapezoid using the simpler geometric shapes you found in part a.

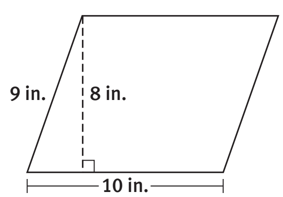
c. Use the formula for the area of a trapezoid to find the area. Compare this area to the area you found in Part b.

4. Construct viable arguments. When dividing a composite figure into simpler geometric shapes to find the area, explain why the simpler figures cannot overlap or have gaps.

LESSON 17-1 PRACTICE

For Items 5–10, find the perimeter and area of each figure.

5.



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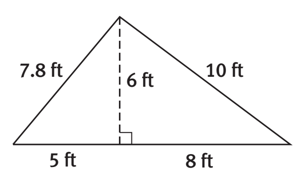

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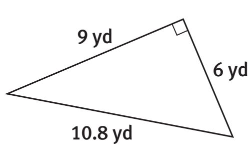

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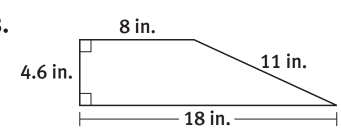

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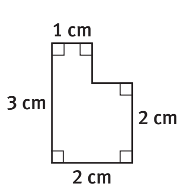
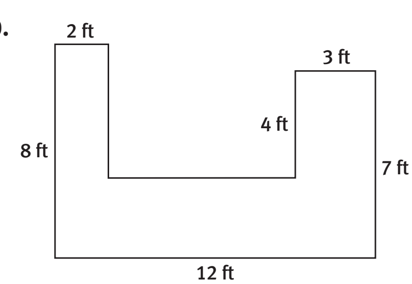


10.



Check Your Understanding

Debrief students' answers to these items to be sure they understand how to find the area of composite figures. Use Item 3 to summarize the lesson's content. Students should explain how to divide a trapezoid into simpler shape, find the area of the composite figure, and then compare it to the area of the trapezoid using a formula.

Answers

3. a. Divide the figure into two right triangles and a rectangle. Then find the area of each figure.
b. 132 $in.^2$
c. 132 $in.^2$; The areas are the same.

4. Sample answer: If there are gaps or overlaps, you will not find the total area of the original figure.

ASSESS

Use the lesson practice to assess your students' understanding of finding the total area of a composite figure. Pay particular attention to Item 12, which has students find the area of a mat by subtracting the area of a rectangle from the total area of a larger rectangle.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 17-1 PRACTICE

5. 38 in.; 80 $in.^2$
6. 30.8 ft; 39 ft^2
7. 25.8 yd; 27 yd^2
8. 41.6 in.; 59.8 $in.^2$
9. 10 cm; 5 cm^2
10. 48 ft; 58 ft^2

Teacher to Teacher

In Item 11, consider dividing the figure into simpler geometric figures in more than one way, and have students show how to find the area of each. Discuss why one way to find the area may be easier.

LESSON 17-1 PRACTICE

11. 53.5 square units; Sample answer: I divided the figure into a triangle, rectangle, and trapezoid with areas of 17.5, 24, and 12 sq units; and 17.5 + 24 + 12 = 53.5.

12. $42\frac{2}{3}$ in.^{2}

13. The areas are the same since the composite figure was formed from a parallelogram that is identical to Parallelogram 1, and the cut off triangle is placed so there are no gaps or overlap

14. 220 in.^{2}

TEACHER TO TEACHER

In Item 12, have students show a drawing of the composite figure. Explain that the area of the mat can be found by subtracting the area of the picture from the area found by multiplying the outside dimensions of the mat.

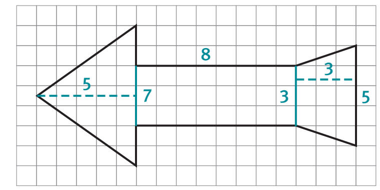
For Items 13 and 14, consider having students work in pairs or groups since the diagrams they draw may be difficult to do. Have the group discuss how to find and compare the areas they found for the composite figures.

ADAPT

Check students' answers to the Lesson Practice to be sure they understand how to make a composite figure from a word description of the figure. Encourage students show the figures they are creating as they decompose a figure. Have them label each new figure with its name and the associated area formula.

My Notes

11. What is the area of the arrow in square units? Justify your answer.



12. Make sense of problems. A 4-inch-wide by 6-inch-long picture is placed on a solid mat that forms a frame around it. The mat is 8 inches long. The mat and the picture are similar rectangles. What is the area of the mat?

13. Make use of structure. Parallelogram 1 has a base of 26 centimeters and a height of 15 centimeters. Parallelogram 2 is identical to it. A triangle with a base of 8 centimeters is cut from parallelogram 2 and placed so its base rests on the top of the original figure. How does the area of the resulting composite figure compare to the area of parallelogram 1? Explain.

14. Reason quantitatively. A kite is formed by connecting the bases of two triangular frames. The height of the top frame of the kite is 8 inches. The height of the lower section is 14 inches. The bases of the frames are 10 inches long. What is the least amount of material needed to make two kites?

Lesson 17-2
More Areas of Composite Figures

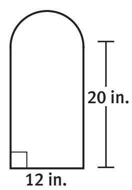
My Notes

Learning Targets:
- Determine the area of composite figures.
- Solve problems involving area.

SUGGESTED LEARNING STRATEGIES: Chunking the Activity, Group Presentation, Summarizing, Paraphrasing, Identify a Subtask, Visualization

Composite figures may contain parts of circles. To find the area of these figures, it is necessary to identify the radius or the diameter of the circle.

1. The composite figure shown can be divided into a rectangle and a semicircle.

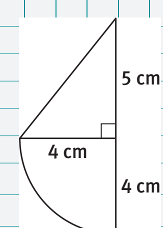


a. What is the diameter of the ***semicircle***? **12 in.**

b. Find the total area of the figure. Use $\pi = 3.14$. Show your work.
≈ 296.52 in.^{2}; A semicircle $= \pi r^2 \div 2$, $3.14 \times 6^2 \div 2 \approx 56.52$; A rectangle $= 20 \times 12 = 240$, $56.52 + 240 = 296.52$

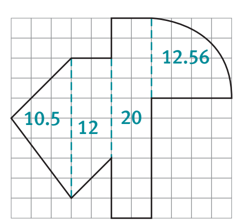
c. Find the distance around the figure. Show your work.
≈ 70.84 in.; $C = \pi d$, so the circumference of the semicircle is approximately $3.14 \times 12 \approx 37.68 \div 2 \approx 18.84$, and the perimeter of the three sides of the rectangle is $20 + 20 + 12 = 52$. $18.84 + 52 = 70.84$.

2. The figure in the My Notes column is divided into a right triangle and a quarter-circle. Find the area of the composite figure. Use $\pi = 3.14$.
22.56 cm^2



3. A student dropped a piece of stained glass. A fragment has the shape shown below.

a. Divide the fragment into smaller shapes you can use to find its total area.
Student divisions may vary. See sample above.



MATH TERMS

A **semicircle** is an arc whose measure is half of a circle. The area of a semicircle is half of the area of a circle with the same radius.

Lesson 17-2

PLAN

Pacing: 1–2 class periods

Chunking the Lesson
#1–3 #4
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Have students make a list of as many geometric figures as they can think of and list as many corresponding area formulas as they can. Have students compare lists with a partner. Create a comprehensive list by having a student scribe on the board as classmates share out.

1–3 Create a Plan, Sharing and Responding Have students work in pairs to discuss and solve each problem As they begin each item they should study the composite figure and develop a plan for decomposing it and using the smaller figures to find the area of the original figure. Pairs should come together to discuss their approach to the solving the problem and the solution they found.

Developing Math Language
The lesson contains multiple terms relating to circles. Encourage students to explain the vocabulary in their own words and list examples related to circles they draw. The terms *semi-circle, arc, radius, diameter,* and *inscribed* are important for helping students gain fluency with circles. Have students add the words to their math notebooks. As their study of geometry gets deeper, they should revisit their notes and add more properties of circles. Add the words to your Word Wall, and encourage students to use the words correctly as they discuss the lesson and practice problems.

Differentiating Instruction
Help students see that the distance around a figure is not always found by using a formula as in Item 1. Students should find the length of each part of the composite figure as they trace it out. For this figure, only three sides of the rectangle are part of the perimeter and only one half of a circle.

4 Visualization Have students brainstorm why the diameter of the circle is equal to the length of a side of the square in Item 4.

Check Your Understanding

Debrief students' answers to these items to be sure they understand how to find the area of composite figures containing circles or parts of circles. Use Item 6 to summarize how to divide a composite shape into simpler shapes with known area formulas.

Answers

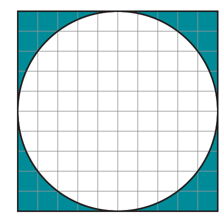
5. Subtract the area of the smaller rectangle from the area of the larger rectangle: $216\text{ cm}^2 - 12\text{ cm}^2 = 204\text{ cm}^2$.
6. Sample answer: Divide the composite figure into simpler shapes with known area formulas. Then add to find the area of the composite figure. Alternatively, find the area of a larger figure and subtract the area of a smaller figure.

b. Find the area of the fragment if each square on the grid represents 1 cm^2. Use $\pi = 3.14$. Show the calculations that led to your answer.

Student calculations will differ depending on how they divide the figure. Sample answer: Area $= 10.5 + 12 + 20 + 12.56 \approx 55.06\text{ cm}^2$

You can break a composite figure into geometric shapes and add the areas of the shapes to find the area. You can also subtract to find the area of part of a composite figure.

4. The stained-glass design on page 179 contains a circle that fits exactly in a square, as shown.



a. Shade the region that is inside the square but outside the circle.

b. Describe a method for finding the area of the shaded region.

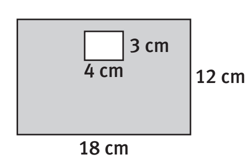
Sample answer: Find the area of the square and subtract the area of the circle.

c. Find the area of the shaded region if each square on the grid represents 1 cm^2. Use $\pi = 3.14$. Show your work.

$100 - 78.5 \approx 21.5\text{ cm}^2$

Check Your Understanding

5. What is the area of the shaded region? Explain your thinking.



6. Construct viable arguments. Explain how to find the area of a composite figure composed of simpler geometric shapes.

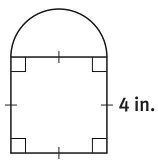
Lesson 17-2
More Areas of Composite Figures

My Notes

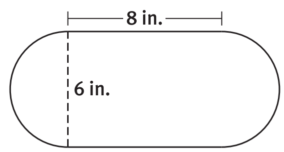
LESSON 17-2 PRACTICE

Find the area of each figure. Use $\pi = 3.14$.

7.

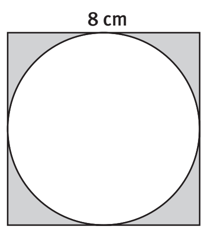


8.

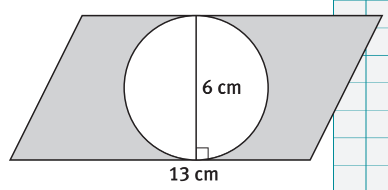


Find the area of the shaded region. Use $\pi = 3.14$.

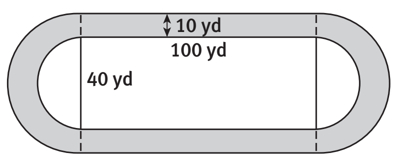
9.



10.



11. An athletic field has the shape of a 40-yard-by-100-yard rectangle with a semicircle at each end. A running track that is 10 yards wide surrounds the field. Use this information to answer the questions below. Use $\pi = 3.14$.



a. Find the area of the athletic field, without the track.
b. Find the area of the athletic field with the track included.
c. Find the area of the track, the shaded portion of the diagram.
d. Suppose a rectangular fence of 180 yards by 80 yards encloses the athletic field and running track. How much of the fenced area is not a part of the field and track?

12. Make sense of problems. A circular plate has a diameter of 6 inches. A pancake in the center of the plate has a radius of 2 inches. How much of the plate is not covered by the pancake? Use $\pi = 3.14$.

13. Reason quantitatively. A section of stained glass is made by placing a circle at the top of a triangle with a base of 10 centimeters and a height of 8 centimeters. The diameter of the circle is equal to the height of the triangle. What is the area of the section of stained glass? Show your work. Use $\pi = 3.14$.

ASSESS

Use the lesson practice to assess your students' understanding of how to find the area of the shaded part of a composite figure. Pay particular attention to Item 11, which has students find the area of a track by subtracting a composite figure from the total area of a larger composite figure.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Teacher to Teacher

In Item 10, students should notice that while the circle is not inscribed in the parallelogram, the height of the parallelogram is the same as the diameter of the circle. They still find the area of the shaded region by subtracting the area of the circle from the area of the parallelogram.

In Item 11, explain that there is no formula for finding the area of the track directly, but it can be found by subtracting the area of the inner unshaded area from the area of the larger similar shape.

For Items 12 and 13, have students compare their diagrams. Discuss how to find the areas they found for the composite figures.

LESSON 17-2 PRACTICE

7. 22.28 in.^2
8. 76.26 in.^2
9. 13.76 cm^2
10. 49.74 cm^2
11. a. 5,256 yd^2
b. 8,826 yd^2
c. 3,570 yd^2
12. about 15.7 in.^2
13. about 90.24 cm^2;
$A_{\text{triangle}} = \frac{1}{2}(10 \times 8) = 40 \text{ cm}^2$,
$A_{\text{circle}} = 3.14 \times 4^2 \approx 50.24 \text{ cm}^2$

ADAPT

Check students' answers to the Lesson Practice. Have students create a visual representation of geometric figures and write the formulas use to find the area of that figure inside the figure. Also suggest the make a list of steps to use when find the area of composite figures,

ACTIVITY PRACTICE

1. 114 cm^2
2. 64 $in.^2$
3. 99.25 $in.^2$
4. 1,285.64 cm^2
5. 51 mm^2
6. 7.74 m^2
7. 54.5 m^2
8. 6.88 ft^2

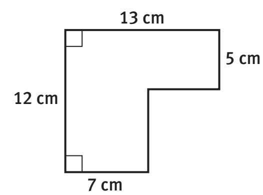
ACTIVITY 17 PRACTICE

Write your answers on a separate piece of paper. Show your work.

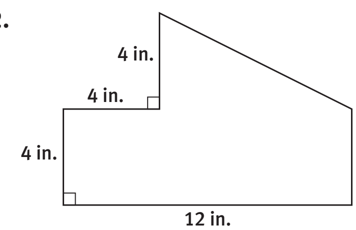
Lesson 17-1

For Items 1–4, find the area of the figure. Use $\pi = 3.14$.

1.



2.



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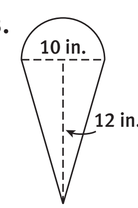

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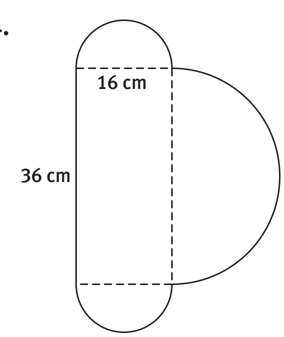
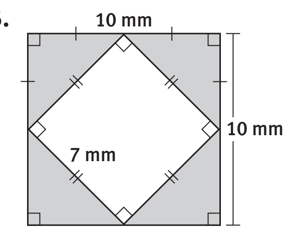


For Items 5–8, find the area of the shaded region. Use $\pi = 3.14$.

5.



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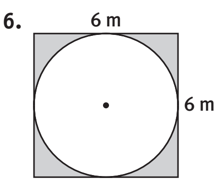

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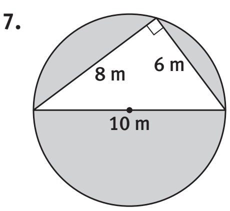

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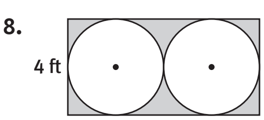
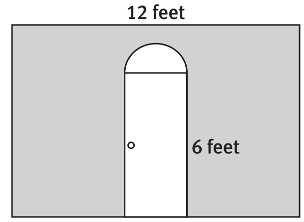


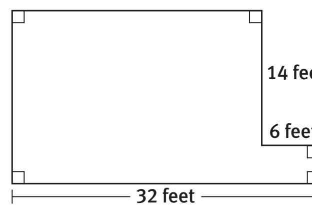
Composite Area
Tile Designs

Lesson 17-2

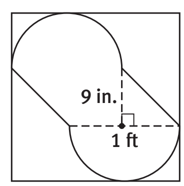
9. Sue wants to paint the wall shown. What is the area of the wall to the nearest tenth? Use $\pi = 3.14$.
A. 72.2 ft^2
B. 75.1 ft^2
C. 77.7 ft^2
D. 80.4 ft^2



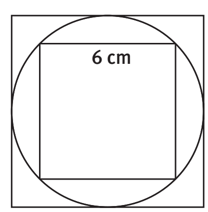
10. A room with the dimensions shown needs carpet. How much carpet is needed to cover the entire floor of the room?
A. 576 sq ft
B. 492 sq ft
C. 472 sq ft
D. 388 sq ft



11. A square blanket has a design on it as shown. Find each of the following in square inches and in square feet. Use $\pi = 3.14$.
a. area of the design
b. area of the blanket without the design



12. Each square section of a quilt has the design shown. Use $\pi = 3.14$.
a. What is the area of the circular section between the two squares?
b. What is the area of the four corner sections?



MATHEMATICAL PRACTICES
Look for and Make Use of Structure

13. How does knowing the area formulas of simple geometric shapes help you find the area of composite figures?

ACTIVITY 17 Continued

9. C
10. B
11. a. 221.04 in.^{2}; 1.535 ft^2
b. 102.96 in.^{2}; 0.715 ft^2
12. a. 14.24 cm^2
b. 13.76 cm^2
13. Answers may vary. Composite shapes can be broken apart into simple shapes for which there are formulas for finding the area.

ADDITIONAL PRACTICE
If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Assessment Focus

- Area of rectangles and circles
- Area of composite plane shapes

Answer Key

1. 2 gallons; Sample answer: Area of blue regions: area of a semicircle + area of outer blue ring + area of a semicircle = 56.52 + (113.04 − 12.56) + 56.52 = 213.52 ft^2, Since 1 gallon of paint covers 110 ft^2, 2 gallons of paint will cover 220 ft^2.

2. 284.52 ft^2

3. a. No: $\frac{12}{19} \neq \frac{50}{94}$

b. Yes, all circles are similar.

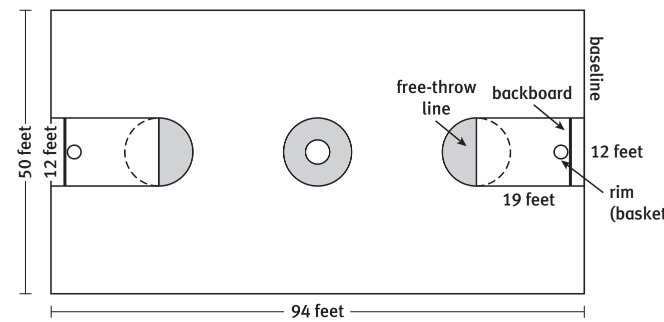
Embedded Assessment 2

Use after Activity 17

Circumference and Area

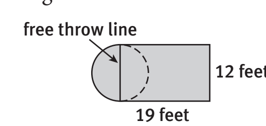
IN THE PAINT

An NBA basketball court is 94 feet long and 50 feet wide. It contains three circles, each with a diameter of 12 feet. Two of these circles are located at the free-throw lines, and the third circle is at the center of the court. Within the third circle is another circle with a radius of 2 feet.



1. One gallon of paint will cover 110 square feet. How many gallons of paint will be needed to paint the shaded regions on the court? Use $\pi \approx 3.14$. Explain your thinking.

2. The region including the circle at the free-throw line to the baseline is shown. Find the area of this region. Use $\pi \approx 3.14$.



3. The key is the rectangular region on the basketball court from the free-throw line to the backboard. The backboard is 4 feet from the baseline.

a. Is the key similar to the basketball court? Explain.

b. Is the inner circle similar to the entire circle in the center of the court? Explain.

Common Core State Standards for Embedded Assessment 2

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

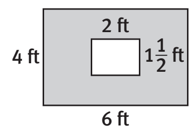
7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Circumference and Area
IN THE PAINT

Embedded Assessment 2
Use after Activity 17

A vertical *backboard* located 4 feet from the baseline supports the rim of the basketball net. The backboard measures 6 feet wide and 4 feet high. The shooter's square is a white box above the rim of the basket. It must measure $1\frac{1}{2}$ feet high and 2 feet wide, as shown at right.

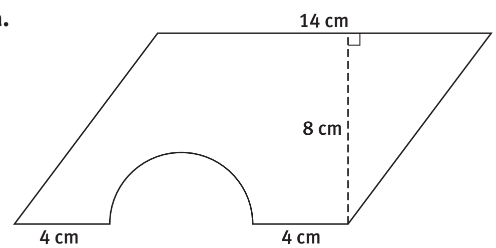


4. What is the area of the portion of the backboard that is NOT white?

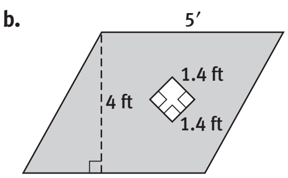
5. The rim of the basket has a radius of 9 inches.
   a. What is the approximate circumference of the basket? Use $\pi \approx 3.14$.
   b. Explain why 3.14 is used when finding the circumference of circles.

6. The design of a basketball team's logo sometimes includes geometric designs. The shapes below are from the logos of two teams. Find the area of each shape.

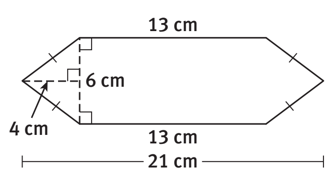
a.



b.



7. Michael claims he can find the area of the composite shape shown by inscribing it in a rectangle and subtracting. Devora claims that to find the area you need to use addition. Which student is correct? Justify your answer.



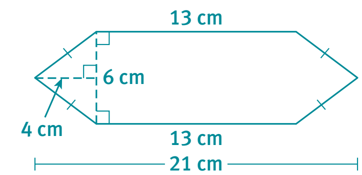
Embedded Assessment 2

4. 21 ft^2

5. **a.** 56.52 inches
   b. Answers may vary. Sample answer: 3.14 is very close to the actual value of π because it is the ratio of the circumference and the diameter of a circle.

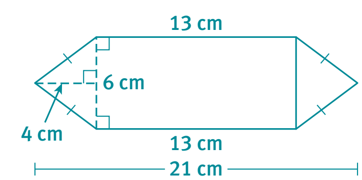
6. **a.** 97.87 cm^2
   b. 18.04 ft^2

7. Both students are correct.
Michael's solution:



Area of rectangle =
21 cm × 6 cm = 126 cm^2;
Area of each right triangle =
$\frac{1}{2} \times 3 \text{ cm} \times 4 \text{ cm} = 6 \text{ cm}^2$;
Area of composite figure =
126 − (4 × 6) = 102 cm^2

Devora's solution:



Area of rectangle =
13 cm × 6 cm = 78 cm^2;
Area of each isosceles triangle =
$\frac{1}{2} \times 6 \text{ cm} \times 4 \text{ cm} = 12 \text{ cm}^2$;
Area of composite figure =
78 + (2 × 12) = 102 cm^2

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Unpacking Embedded Assessment 3

Once students have completed this Embedded Assessment, turn to Embedded Assessment 3 and unpack it with students. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 3.

Embedded Assessment 2

Use after Activity 17

Circumference and Area

IN THE PAINT

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3a-b, 4, 5a-b, 6a-b, 7)	• Accurately and efficiently finding the circumference and area of circles and the area of composite figures.	• Finding the circumference and area of circles and the area of composite figures.	• Difficulty finding the circumference and area of circles and the area of composite figures.	• No understanding of finding the circumference and area of circles and the area of composite figures.
Problem Solving (Items 1, 2, 4, 5a, 6a-b)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 3a-b, 7)	• Clear and accurate understanding of similar figures. • Solving composite figures by adding or subtracting.	• An understanding of similar figures. • Recognizing that composite figures are made up of simpler figures.	• Difficulty recognizing similar figures. • Difficulty in working with composite figures.	• No understanding of similar figures. • No understanding of composite figures.
Reasoning and Communication (Items 1, 3a-b, 5b, 7)	• Precise use of appropriate terms to explain similar figures, finding area, and π.	• An adequate explanation of similar figures, finding area, and π.	• A partially correct explanation of similar figures, finding area, and π.	• An incomplete or inaccurate explanation of similar figures, finding area, and π.

Sketching Solids

ACTIVITY 18

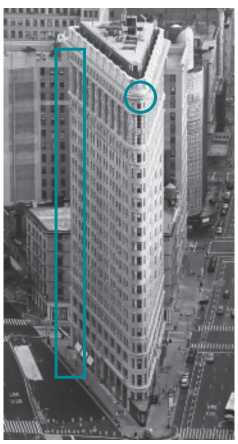
Putt-Putt Perspective
Lesson 18-1 Shapes That Result from Slicing Solids

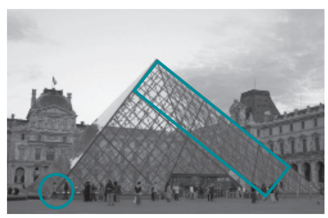
My Notes

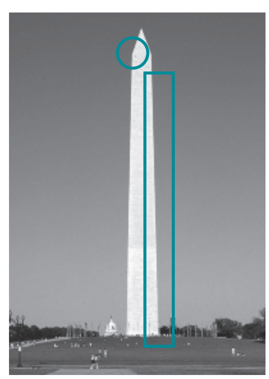
Learning Targets:
- Draw different views of three-dimensional solids.
- Identify cross sections and other views of pyramids and prisms.

SUGGESTED LEARNING STRATEGIES: Visualization, Look for a Pattern, Use Manipulatives, Create Representations

The Service Club at Park Middle School is creating a miniature golf course to raise funds for a food bank. The theme is interesting structures around the world. Buildings that will be included are the Washington Monument, the Flatiron Building, the Louvre Pyramid, and the Pentagon Building.







CONNECT TO GEOGRAPHY

The Washington Monument is located in Washington, DC. The Pentagon is nearby in Arlington County, VA. The Flatiron Building is in New York, NY. The Louvre Pyramid is the entrance to the Louvre Museum in Paris, France.

1. Compare and contrast the two- and three-dimensional shapes in these four buildings.
   Sample answer: The Flatiron Building and Pentagon are prisms; the Louvre Pyramid and the top of the Washington Monument are pyramids. The base of the Flatiron is a triangle, the bases of the Pyramid and the Monument are quadrilaterals, and the base of the Pentagon is a pentagon. The faces of the Pentagon, the Flatiron, and the long sides of the Monument are rectangles, and the faces of the Louvre Pyramid and the top of the Washington Monument are triangles.

Common Core State Standards for Activity 18

7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

ACTIVITY 18
Investigative

Activity Standards Focus
Until now, students have applied area formulas to known geometric shapes in two dimensions. In Activity 18 they move on to finding the surface area of three-dimensional shapes. They learn the terminology associated with solids, how to find the cross section of solids, and how to find the lateral area and surface area of right prisms and pyramids.

Lesson 18–1

PLAN

Materials
- dot paper
- scissors

Pacing: 2 class periods

Chunking the Lesson

#1 #2–4 #5–7

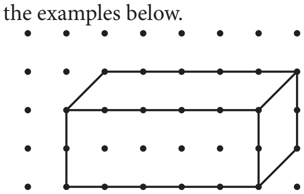
Check Your Understanding

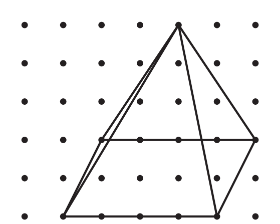
Lesson Practice

TEACH

Bell-Ringer Activity

Have students use dot paper to practice drawing prisms and pyramids as shown in the examples below.





Have them highlight the base(s) and faces of each.

Developing Math Language
Help students understand the terms related to sketching solids: *shape, view, drawing*, and *net*. Students need to know that a prism or pyramid is named using the shape of its *base*(*s*) and that all sides of a prism or pyramid are called *faces*.

1 Activating Prior Knowledge, Visualization This item allows you to assess students' knowledge of prisms and pyramids and understanding a three-dimension figure can be the composition of other three-dimensional figures. The Washington Monument is effectively a composite figure, with a pyramid on top of a rectangular prism. Students can consider only the top, or the top and the shaft separately.

2–4 Visualization, Use Manipulatives, Create Representations Students have the opportunity to see connections between nets and three-dimensional figures. Students may have trouble visualizing a slice of a prism. Help students see that when you make a slice through a prism parallel to a base, that the shape of the slice is identical to the shape of the base. You can use a deck of cards to demonstrate how a slice parallel to the base of a rectangular prism is one of the cards, and the card slice is identical to the card base.

Ask students to think of real-life objects that they may have sliced, such as a stick of butter, a loaf of bread, a block of cheese. If modeling clay is available, students can make and slice models of prisms.

My Notes

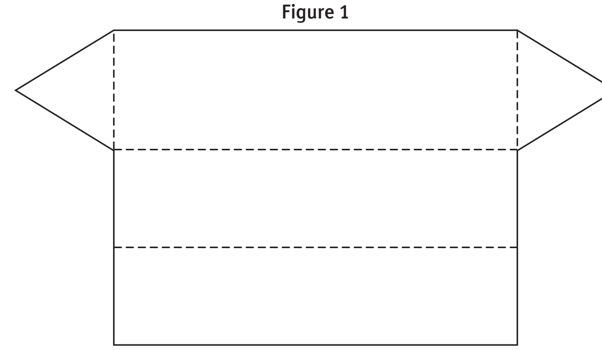
Lesson 18-1
Shapes That Result from Slicing Solids

To prepare for drawing and building the structures for the mini-golf course, the students explore relationships between shapes, views of shapes, and drawings of shapes.

A ***prism*** is a solid with two parallel congruent bases that are both polygons.

2. The ***net*** shows a two-dimensional pattern for a prism.

Figure 1



MATH TERMS

A **net** is a a two-dimensional pattern that can be folded to form a solid.

a. Cut out Figure 1 on the next page. Fold it along the dashed lines to form a prism.

b. A prism is named using the shape of its bases. Name the solid formed by the net in Figure 1.
triangular prism

c. The **faces** of a prism are the sides that are not the bases. What is the shape of the three faces?
rectangle

d. **Reason abstractly.** Imagine making a slice through the prism parallel to the bases. What is the shape of the two-dimensional slice?
triangle

e. **Reason abstractly.** Imagine making a slice through the prism perpendicular to the bases. What is the shape of the two-dimensional slice?
rectangle

Lesson 18-1
Shapes That Result from Slicing Solids

My Notes

Figure 1

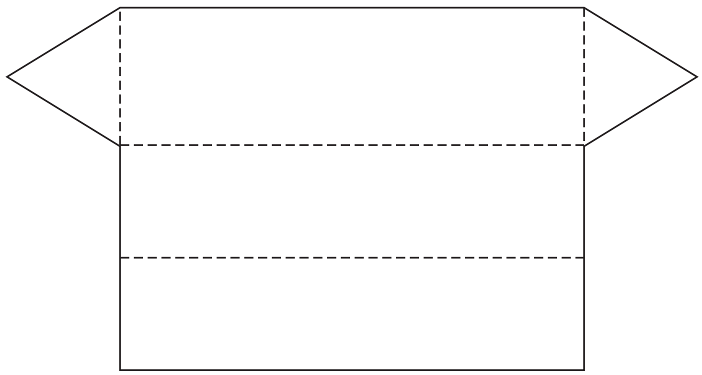
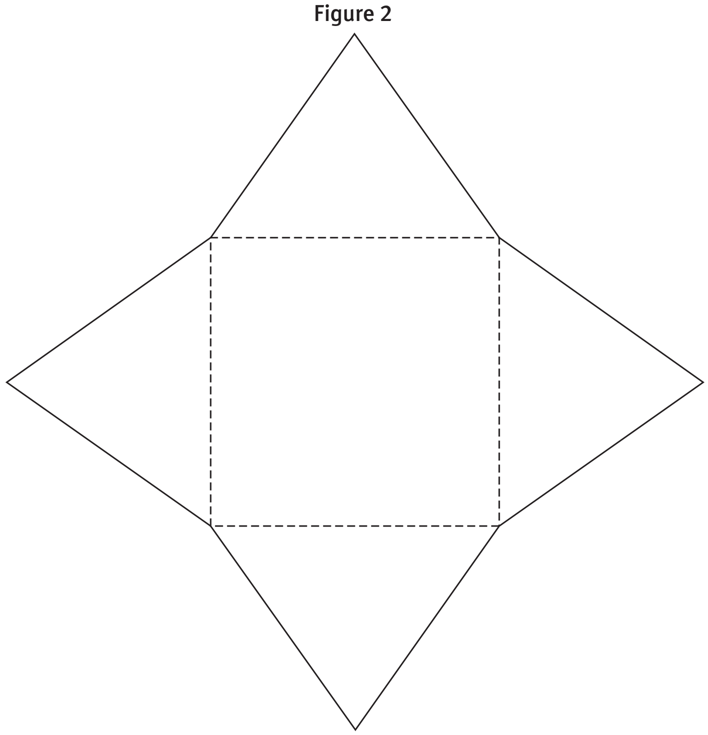


Figure 2



Teacher to Teacher

Have students cut out these figures for use in this activity.

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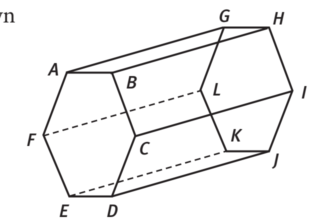
Lesson 18-1
Shapes That Result from Slicing Solids

My Notes

3. Consider the hexagonal prism shown on the right.



a. Why do you think some of the lines are dotted?

Sample answer: The dotted lines represent the edges you cannot see because they are hidden by the solid.

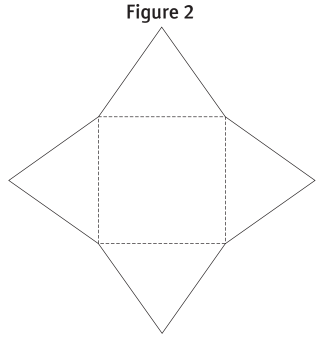
b. Imagine making a slice that goes through points *B*, *D*, *J*, and *H* of the prism. What is the shape of the two-dimensional slice?

rectangle

A ***pyramid*** is a solid that has only one base. That base is a polygon. The faces of a pyramid are all triangles.

4. The net shows a pattern for a pyramid.

Figure 2



a. Cut out Figure 2 on page 193. Fold it to form a pyramid.

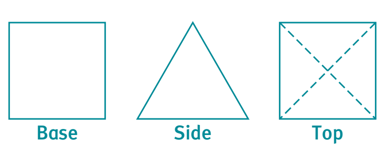
b. A pyramid is named using the shape of its base. What is the name of the solid formed by Figure 2?

square pyramid

c. **Reason abstractly.** Imagine making a slice through the pyramid parallel to the base. What is the shape of the two-dimensional slice?

square

d. **Model with mathematics.** Sketch and label the view of the base, side, and top of the pyramid.



ACTIVITY 18 Continued

2–4 (continued) Ask students how a slice through a pyramid parallel to a base is different from the base. Help students recognize that the shape of the slice is the same as the base the slice will be smaller than the base. The exception would be that a slice at the top of the pyramid parallel to the base would be a single point.

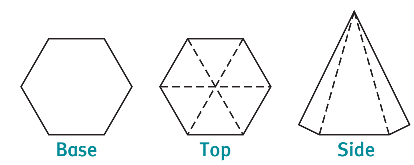
ELL Support

Encourage students to create an index card for each of the solids they study. Have them include a diagram with different views of the solid, labels for the faces and base(s), and any other attributes they learn.

5–7 Visualization, Use Manipulatives, Create Representations, Group Presentation. Students may benefit from seeing a model of a hexagonal pyramid to view as they respond to Item 5. After students complete their sketches have them compare drawings with a partner and make revision to their work. Students should understand that all cross sections that are parallel to the base are similar hexagons, and the cross sections that are perpendicular to the base are trapezoids, except for the one cross section that contains the vertex of the pyramid. The one cross section that is perpendicular to the base and is not a trapezoid is an isosceles triangle.

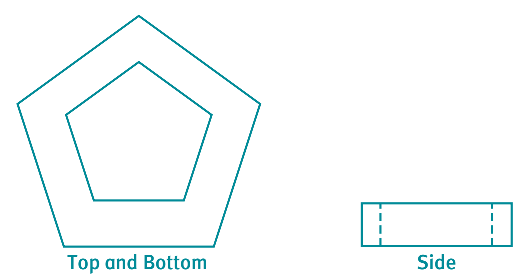
5. Three views of a hexagonal pyramid are shown.

a. Label each view as *side*, *top*, or *base*.



b. Explain the significance of the dashed segments in the second view.
They represent the edges of the pyramid.

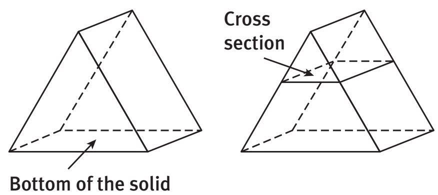
6. Model with mathematics. Sketch and label the bottom, top, and side views of the Pentagon Building.



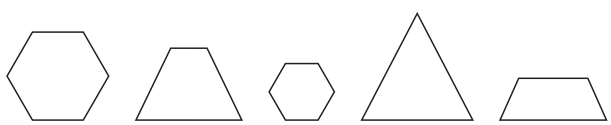
Lesson 18-1
Shapes That Result from Slicing Solids

ACTIVITY 18
continued

A ***cross section*** of a solid figure is the intersection of that figure and a plane.



7. Reason abstractly. Several cross sections of a hexagonal pyramid are shown. Label each cross section as *parallel* or *perpendicular* to the base of the pyramid.



From left to right the relationship of the cross sections to the base are: parallel, perpendicular, parallel, perpendicular, perpendicular

My Notes

MATH TIP

You can think of a cross section as a slice of a solid that is parallel or perpendicular to the base of the solid.

Check Your Understanding

For Items 8–10, consider a rectangular pyramid.

8. Describe the shape of the base and the faces of the pyramid.

9. a. What shapes are formed by cross sections parallel to the base? Explain your thinking.

b. Are all of the cross sections parallel to the base the same size?

10. Construct viable arguments. Are all of the cross sections perpendicular to the base the same shape and size? Justify your answer.

ACTIVITY 18 Continued

Check Your Understanding

Debrief students' answers to these items to ensure that they understand how to describe the bases and faces of a pyramid and can explain how to determine the shape of the cross sections of a pyramid that are parallel to a base.

Answers

8. The base is a rectangle and the faces are triangles.

9. a. rectangles

b. Sample explanation: Since all the sections are parallel to the rectangular base, all the cross sections are similar rectangles.

10. No. Sample justification: Trapezoids and a triangle are formed. Moving across from one side of the base to the other, the perpendicular slices are trapezoids that get larger until the center slice, which forms a triangle, and then the trapezoid slices get smaller again.

ASSESS

Use the lesson practice to assess your students' understanding of using nets, sketching solids and sketching cross sections of solids. Pay particular attention to Item 17, which has students sketch cross sections that are perpendicular to the base of a solid.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

TEACHER to TEACHER

In Item 15, sketching the different views of a prism may be difficult for some students. Encourage these students to use models of the prism to help them see the views.

In Item 19, have students explore cross sections that are perpendicular to the base of a pyramid. The cross section that goes through the vertex of the pyramid, perpendicular to the base, will be a triangle. Every other cross section that is perpendicular to the base will be a trapezoid.

For Items 16–19, consider having students work in pairs or groups using clear models of the solids along with slicing planes. These will help students visualize the cross sections.

LESSON 18-1 PRACTICE

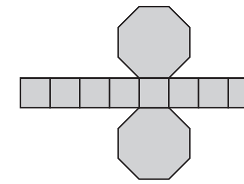
11. octagonal prism
12. pentagonal prism
13. pentagons
14. rectangles
15. Sketches will vary but all views will be rectangles. The top should be larger than the sides; accept a square for the top.

ADAPT

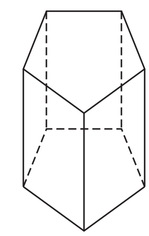
Check students' answers to the Lesson Practice to be sure they understand how to find the perpendicular cross section of a solid. Provide students with additional opportunities to us nets to create solid figures to visualize cross sections.

LESSON 18-1 PRACTICE

11. What is the name of the solid formed by the net?



Use the solid to the right for Items 12–14.

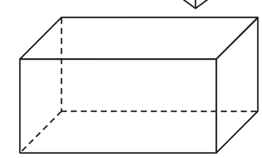


12. What is the name of the solid?

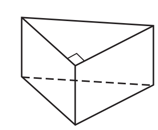
13. Reason abstractly. Imagine making slices through the solid parallel to the bases. What two-dimensional shapes are formed?

14. Reason abstractly. Imagine making slices through the solid perpendicular to the bases. What two-dimensional shapes are formed?

15. Model with mathematics. Sketch and label the bottom, top, and side views of the rectangular prism.



Use the solid to the right for Items 16 and 17.



16. Sketch the cross section that is parallel to the bases.

17. Sketch three different cross sections that are perpendicular to the bases.

For items 18 and 19, consider an octagonal pyramid.

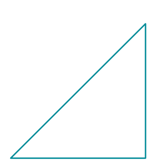
18. Sketch two different cross sections that are parallel to the base of the pyramid.

19. Sketch three different cross sections that are perpendicular to the base of the pyramid.

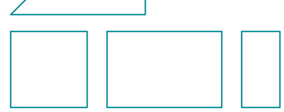
20. Construct viable arguments. Can the cross section of a solid ever be a point? Explain your thinking.

21. Reason abstractly. How can the name of a prism or a pyramid help you visualize the cross sections of the solid?

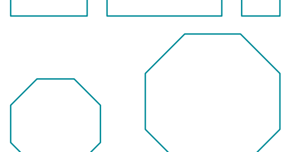
16.



17.



18.



19.

20. Sample answer: Yes, a cross section parallel to the base of a pyramid at the top of the pyramid where the triangular faces meet is a point.

21. Answers may vary. Both solids are named by the shape of their base. Cross sections parallel to the base are always the shape of the base. Cross sections perpendicular to the bases will include rectangles for prisms and triangles and trapezoids for pyramids.

My Notes

Learning Targets:

- Calculate the lateral and total surface area of prisms.

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Create Representations, Summarizing, Paraphrasing, Think-Pair-Share, Visualization, Discussion Groups

The students in the service club will paint the structures in the golf course. They first investigate how to find the surface area of prisms.

1. Work with your group. Look at Net 1, Net 2, and Net 3 on pages 203 and 204. What solids do the nets form?
Net 1: triangular prism; Net 2: pentagonal prism; Net 3: rectangular prism

A ***lateral face*** of a solid is a face that is not a base. A ***right prism*** is a prism on which the bases are directly above each other, making the lateral faces perpendicular to the bases. As a result, all the lateral faces are rectangles. The ***lateral area*** of a solid is the sum of the areas of the lateral faces.

GROUP DISCUSSION TIP

With your group, read the text carefully. Reread definitions of terms as needed to help you comprehend the meanings or words, or ask your teacher to clarify vocabulary terms.

2. For Nets 1–3, what are the shapes of the lateral faces of the figures? Explain.
Rectangles; the lateral faces of any right prism are rectangles.

3. Attend to precision. Find the area of each lateral face and the lateral area of each prism.

	Face 1	Face 2	Face 3	Face 4	Face 5	Lateral Area
Net 1	140	168	140			448 units^2
Net 2	200	140	320	140	200	1,000 in.^{2}
Net 3	35	14	35	14		98 cm^2

MATH TIP

Net 1: Use the square units on the figure to find the lateral areas.

Net 2: Use the given measurements to find the lateral areas.

Net 3: Measure the side lengths to the nearest whole centimeter. Then find the lateral areas.

Differentiating Instruction

Help students verify the formula for the lateral area of a prism, $L = P \times h$, where L represents the lateral area, P represents the perimeter of the base, and h represents the height of the prism. For Net 1, the lateral area is $14 \times 10 + 14 \times 10 + 14 \times 12$. Using the distributive property, the lateral area is $14(10 + 10 + 12)$, or the height of the prism, 14, times the perimeter of the triangular base of the prism, $10 + 10 + 12$. Encourage students to also verify the formula for Net 3.

Lesson 18-2

PLAN

Materials

- model prisms (pages 203–204)
- metric ruler

Pacing: 2 class periods

Chunking the Lesson

#1–2 #3 #4 #5–7
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to make a list of three-dimensional figures they have studied and make connections of these figures to structures they see in everyday life. Have students share their lists with a partner and then in groups. Ask each group to share one example from their discussion.

1–2 Create Representations, Marking the Text, Visualization, Word Wall Visual and kinesthetic learners may benefit from cutting out and folding the nets to form the solids. Have students identify bases and faces of the figures before discussing the term lateral After the students read the introduction to this question, have them add the terms *lateral face* and *lateral area* to the Interactive Word Wall.

3 Create Representations Students will use the nets on pages 203 and 204. As students complete the table, it is not important how the faces are numbered. Students should recognize that lateral area does not include the bases. This may be a new concept for them as they may be accustomed to finding surface area.

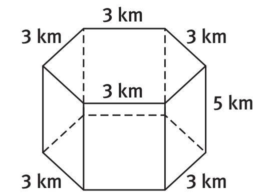
Developing Math Language

Have students add *lateral face* to their list of terms related to solids. A lateral face of a solid is a face that is not a base. The lateral faces of a prism are rectangles if the prism is a *right prism*. That is, the faces are perpendicular to the base. The lateral faces of a pyramid are always triangles. The *lateral area* of a solid is the sum of the area of all the lateral faces. The *surface area* is the lateral area plus the area of the bases. Students can use a formula, if possible, to find the area of each face. Have students add new words and formulas to their math notebooks. Add the words to your Word Wall, and encourage students to use the words correctly as they discuss the lesson and practice problems.

4 Sharing and Responding, Debriefing Student work on this item should be debriefed so that students recognize that all face on this prism are congruent. The final part of this item needs to be discussed thoroughly so students realize that parts b and d present two approaches to answering the same question by using different methods.

My Notes

4. Examine the right hexagonal prism.



a. Find the area of each lateral face. Show your work.
15 km²; The dimensions of each lateral face are 3 km by 5 km, so the area of each lateral face is: 3 • 5 = 15.

b. Find the lateral area of the solid. Show all your work.
90 km^2; 15 km^2 • 6 lateral faces = 90 km^2

c. Determine the perimeter of the base, *P*.
18 km; 3 km per side • 6 sides = 18

d. Multiply the perimeter of the base, *P*, times the height of the prism, *h*.
18 km × 5 km = 90 km^2

e. Compare your responses to Parts b and d. What do you notice?
They are both 90 km^2.

Lesson 18-2
Lateral and Total Surface Area of Prisms

My Notes

The following formula can be used to find the lateral area of a prism: $L = P \times h$, where L represents the lateral area, P represents the perimeter of the base, and h represents the height of the prism.

5. Use the formula to find the lateral area of the prisms in Item 24 for Nets 1 and 2. Are the lateral areas the same as the ones you recorded in the table?
   Net 1:
   32 units $\times$ 14 units = 448 units^2

   Net 2:
   50 in. $\times$ 20 in. = 1,000 in.^{2}
   Yes, they are the same.

The ***surface area*** of a prism is the sum of the areas of the lateral faces and the areas of the bases.

6. **Attend to precision.** Describe the relationship between the lateral area and the surface area of a prism.
   Sample answer: The lateral surface area of a prism is the sum of the areas of its rectangular sides. The surface area of a prism includes the lateral surface area *and* the area of the two bases of the prism.

7. **Reason quantitatively.** Find the surface area of each of the prisms in Item 24. Explain your thinking.
   I added the total lateral surface area I found in Item 24 to the area of the two bases.

   Net 1:
   $448 + 2\left(\frac{1}{2} \cdot 12 \cdot 8\right) = 2(48) = 544 \text{ units}^2$

   Net 2:
   $1{,}000 + \text{area of bases} = 1{,}000 + 2\left[(16 \cdot 7) + \left(\frac{1}{2} \cdot 16 \cdot 6\right)\right] = 2(160) = 1{,}320 \text{ in.}^2$

   Net 3:
   $98 + 2(2 \bullet 5) = 98 + 20 = 118 \text{ cm}^2$

Check Your Understanding

8. **Make use of structure.** Why do you think that the lateral area of a prism is equal to the product of the perimeter and the height of the prism?
9. **Construct viable arguments.** Explain how to use a net to find the lateral and total surface area of a prism.

5–7 Think-Pair-Share, Discussion Groups, Self Revision/Peer Revision In this items students use formulas and are asked to formalize the relationship between lateral area, and surface area. Have students respond to Items 6 and 7 individually, discuss their response in groups, and revise their work. Students should be encouraged to use precise mathematical language in their writing.

Check Your Understanding

Debrief students' answers to these items to gauge their understanding of how to find the surface area of a prism, both from a net and from a formula for a prism.

Answers

8. Sample answer: The lateral area is the total area of the rectangular faces. The formula is similar to drawing a large rectangle composed of all the lateral faces with the perimeter equal to the length and the height equal to the width of the rectangle.
9. Sample answer: To find the lateral area, find the total area of the rectangular faces (except for rectangular bases). To find the total surface area, add the area of the bases to the lateral area.

ASSESS

Use the lesson practice to assess your students' understanding of how to find the surface area of a prism. Pay particular attention to Item 14, which has students draw a net of the prism.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Teacher to Teacher

For Item 14, consider having students work in pairs so they can compare their nets for the problem. Have the pair discuss how to find the lateral areas they found for their nets.

In Item 18, students should notice that while they can find the lateral area and the surface area of a right prism, a real-life area problem may include some restrictions. The display case in this example does not need one base to be covered with film. Point out that that means the lateral area formula for the sides of the display case can be used, but the area of only one base should be included to find the total amount of film.

LESSON 18-2 PRACTICE

10. 100 $in.^2$
11. 81 $in.^2$
12. $(6 + 6 + 6) \cdot 8 = 144$ ft^2
13. $2\left(\frac{1}{2} \cdot 6 \cdot 5.2\right) + 144 = 175.2$ ft^2
14. 40 m^2
15. 88 m^2
16. 280 $in.^2$
17. 400 $in.^2$
18. 340 $in.^2$
19. The larger block has a surface area that is 4 times as great.; Smaller block: 54 $in.^2$; Larger block: 216 $in.^2$; $216 \div 4$

ADAPT

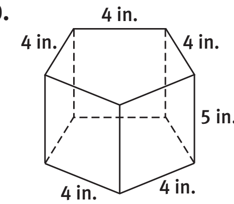
Check students' answers to the Lesson Practice to be sure they understand how to find the lateral area of a prism shown as a solid or as a net. Provide students with a net from which to create a three-dimensional solid. Have students used their model to identify bases and faces and what parts of the model would be included when computing lateral and surface area.

My Notes

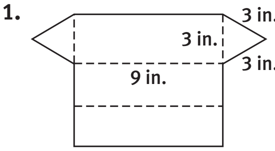
LESSON 18-2 PRACTICE

Find the lateral area of the prisms in Items 10 and 11.

10.



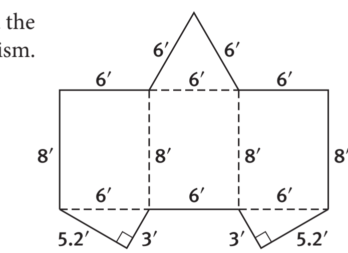
11.



Use the net to the right for Items 12 and 13.

12. Reason quantitatively. Find the lateral area of the triangular prism.

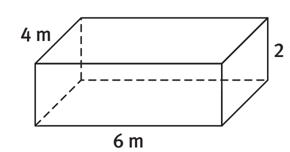
13. Find the surface area of the triangular prism.



Use the prism to the right for Items 14 and 15.

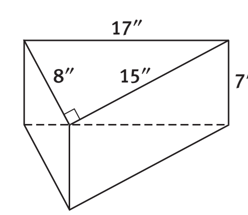
14. Model with mathematics. Draw a net to find the lateral area of the prism.

15. Find the surface area of the prism.



Use the prism for Items 16–18.

16. A display case is shaped like the prism shown. The case needs to be covered with a plastic film. How much film is needed to cover the lateral area?



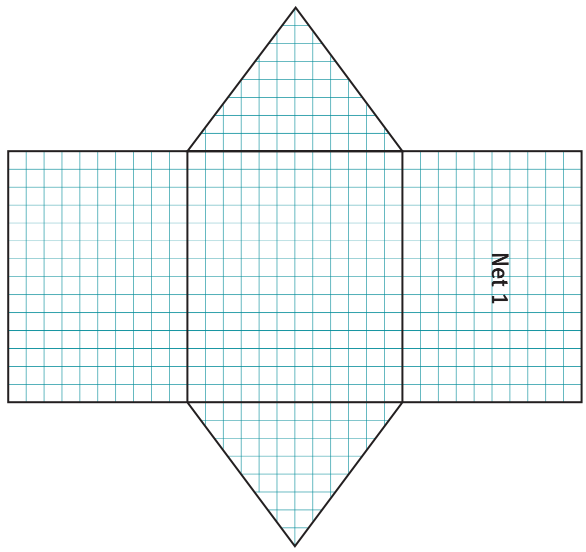
17. How much film is needed to cover the surface area of the display case?

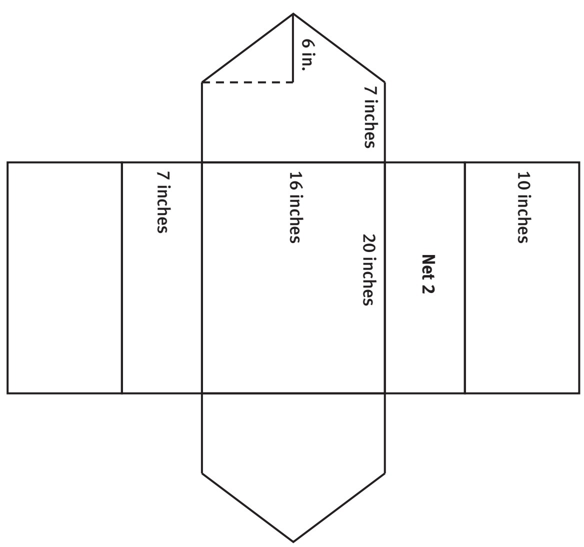
18. Make sense of problems. How much film is needed to cover all but the face the case rests on?

19. Attend to precision. A cube-shaped block has edges that are 3 inches long. A larger block has edges that are twice as long. Compare the surface area of the smaller block to the surface area of the larger block. Support your answer.

Lesson 18-2
Lateral and Total Surface Area of Prisms

My Notes

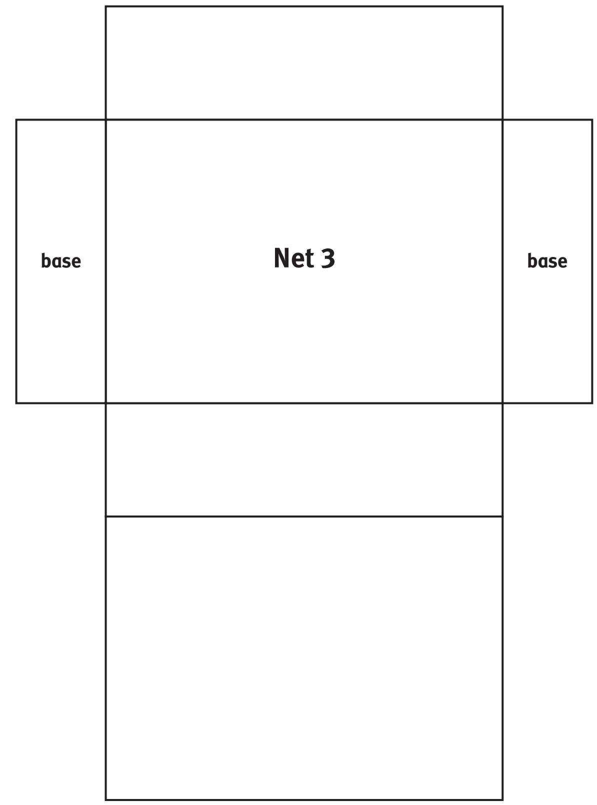




TEACHER to TEACHER

Students will use the nets on this and the next page to answer items in this lesson.

My Notes



Lesson 18-3
Lateral and Total Surface Area of Pyramids

ACTIVITY 18 *continued*

Learning Targets:

- Calculate the lateral and total surface area of pyramids.

SUGGESTED LEARNING STRATEGIES: Create Representations, Group Presentation, Marking the Text, Use Manipulatives, Visualization, Vocabulary Organizer

The students in the service club also investigate how to find the surface area of pyramids.

Two nets of pyramids the students use are on page 210.

The lateral area of a pyramid is the combined area of the faces. The height of a triangular face is the ***slant height*** of the pyramid.

1. Use Net 4, the net of the square pyramid.

a. Draw the slant height on the net.

Check students' work.

b. Why do you think it is called the slant height?

Sample answer: It is slanted instead of vertical on the three-dimensional pyramid.

2. Use Net 4 to find the lateral area of the square pyramid. Explain your thinking.

240 square units; There are four lateral faces, so I found the area of one and multiplied by 4. $\left(\frac{1}{2}\right) = (10)(12)(4) = 240$ **square units**

The surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base.

3. Use Net 4 to find the surface area of the square pyramid. Explain your thinking.

340 square units

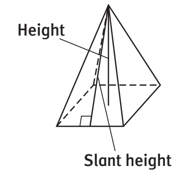
Area of square base: $10 \cdot 10 = 100$ **square units**

Surface area: lateral area + area of square base $= 240 + 100 = 340$ **square units**

My Notes

MATH TIP

Be sure you do not confuse the *slant height* of a pyramid with its *height*. Slant height is a measure along a triangular face. Height is an internal measure from the vertex to the base.



Lesson 18-3

PLAN

Materials

- 6–8 straws per pair of students
- tape

Pacing: 3 class periods

Chunking the Lesson

#1–2 #3–7

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Show students a model of a pyramid. Ask them to write down the two-dimensional figures which compose a pyramid and the formulas which are use to find the area of those figures.

Developing Math Language

Have students add *slant height* to their list of terms related to solids. The slant height of a pyramid is the height of a triangular face of the pyramid. You need the slant height to find the area of the triangle, with the triangle's base being a side of the base. The lateral area of a pyramid is the sum of the area of all the lateral triangles. The *surface area* is the lateral area plus the area of the base, which can be any polygon. Students can use a formula, if possible, to find the area of each face. Have students add new words to their math notebooks. Add the words to your Word Wall, and encourage students to use the words correctly as they discuss the lesson and practice problems.

1–2 Visualization, Manipulatives As students use nets to develop an understanding of slant height and lateral area of pyramids guide them to recognize that a right square pyramid will have congruent isosceles triangles for the lateral faces. The slant height of each triangle will have the same measure, so the lateral area can be represented as 4 times the area of one triangle, or $4\left(\frac{1}{2} \cdot b \cdot l\right)$, where l is the slant height and b is the length of one side of the square base.

ELL Support

To support students' language acquisition, monitor their listening skills and understanding as they participate in group discussions. Carefully group students to ensure that all group members participate in and learn from collaboration and discussion.

3–5 Create representations, Marking the Text, Use Manipulatives, Visualization Students probably will not have any difficulty understanding that the base of a square pyramid is a square. The triangular pyramid may cause some confusion, as the base is identical to each of the other faces. Labeling one surface as the base is arbitrary for the purpose of finding the surface area.

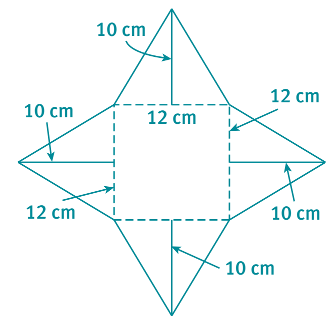
My Notes

CONNECT TO History

The triangular pyramid that can be created using Net 5 has four identical faces. It is an example of a Platonic solid. The Platonic solids are sometimes also called "cosmic figures." There are only five Platonic solids: cube, tetrahedron, dodecahedron, octahedron, and icosahedron.

4. **Attend to precision.** Use a centimeter ruler to measure Net 5. Then find the surface area of the triangular pyramid.
Answers may vary but should be close to 29.9 cm^2.

5. **Model with mathematics.** Consider a square pyramid with base edges 12 cm and slant height 10 cm.
   a. Draw a net to represent the pyramid and label the dimensions of the base edges and the slant height.



   b. Students found the lateral area of the square pyramid using the method shown below. Explain the steps of the method.

Step 1: $\frac{1}{2}\times12\times10+\frac{1}{2}\times12\times10+\frac{1}{2}\times12\times10+\frac{1}{2}\times12\times10$

Step 2: $=\frac{1}{2}\times(12+12+12+12)\times10$

Step 3: $=\frac{1}{2}\times(4\times12)\times10$

Step 4: $=\frac{1}{2}\times48\times10$

Step 5: $=240$ cm2

Step 1: Add the area of the 4 triangular faces.
Step 2: Use the distributive property to factor out 12.
Step 3: Rewrite the repeated addition as multiplication.
Step 4: Multiply 4 by 12.
Step 5: Multiply to find the lateral area.

Lesson 18-3
Lateral and Total Surface Area of Pyramids

My Notes

The following formula can be used to find the lateral area of a regular pyramid:

$L = \frac{1}{2}P \times \ell$, where L represents the lateral area, P represents the perimeter of the base, and ℓ represents the slant height of the pyramid.

6. **Construct viable arguments.** A student says that the above formula can be used to find the lateral area of a rectangular pyramid. Is the student correct? Explain your reasoning.
   No; the base of the pyramid must be a regular polygon so that all the lateral faces are congruent and the slant heights are the same. A rectangle, unless it is a square, is not a regular polygon. However, a pyramid is named by the most specific name of its base, so only a square pyramid has a base that is a square.

7. **Make sense of problems.** The base of a regular triangular pyramid has sides that are 8 meters long and a height of 6.9 meters. The slant height of the pyramid is 6.9 meters.
   a. Find the lateral area of the pyramid. Explain your thinking.
      82.8 m^2; I applied the formula $L = \frac{1}{2}P \times \ell$ to find the lateral area.
      $L = \frac{1}{2} \times (8 + 8 + 8) \times 6.9 = 82.8$
   b. Find the surface area of the pyramid. Explain your thinking.
      110.4 m^2; I added the area of the base to the lateral surface area from part a.
      $A \text{ base} = \frac{1}{2} \times 8 \times 6.9 = 27.6;\ 27.6 + 82.8 = 110.4$

MATH TERMS

A **regular polygon** is a polygon with congruent sides and congruent angles.

A **regular pyramid**, also called a right regular pyramid, is a pyramid with a base that is a regular polygon, and all the lateral faces are congruent.

Check Your Understanding

Write your answers on a separate piece of paper.

8. Why do you need to know the slant height, rather than the height, of a regular pyramid to find the surface area of the pyramid?
9. **Attend to precision.** Explain how to use a net to find the lateral and the total surface area of a pyramid.

6–7 Sharing and Responding Have groups verify that the formula $L = \frac{1}{2} \times P \times l$ for the lateral area L of a pyramid, with P the perimeter of the base and l the slant height, is equivalent to the formula $L = 4\left(\frac{1}{2} \cdot b \cdot l\right)$. Elicit from students that since the perimeter of a square is $4b$, with b the length of a side, $\frac{1}{2} \times P = \frac{1}{2} \times (4b)$. Substituting this into the formula $L = \frac{1}{2} \times P \times l$ gives the equivalent formula $L = 4\left(\frac{1}{2} \cdot b \cdot l\right)$.

Check Your Understanding

Debrief the lesson by having students explain the difference in the slant height of a pyramid and the height of the pyramid, which is the perpendicular distance from the vertex of the pyramid to the base. Make sure they can explain how to find the lateral area and the total surface area of a net for a pyramid.

Answers

8. Sample answer: The slant height is the height of each triangular face. It is needed to find the area of each triangular face. The height of the pyramid does not refer to the height of the triangular faces.
9. Sample answer: Find the lateral area, the total area of the triangular faces. Then find the area of the pyramid's base. To find the total surface area, add the lateral area and the area of the base.

ASSESS

Use the lesson practice to assess your students' understanding of how to find the lateral area and total surface area of a pyramid.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Teacher to Teacher

In Items 10 and 12, have student groups compare the nets they created to find the lateral area of the pyramids. Have them discuss whether there can be more than one net using the giving information (no).

For Item 17, consider having students work in pairs so they can discuss how to find the lateral areas they found for the pyramid.

LESSON 18-3 PRACTICE

10. See students' nets; 42 ft^2
11. 54.25 ft^2
12. See students' nets; 27.6 cm^2
13. 36.8 cm^2
14. 1948.8 m^2
15. 3173.8 m^2

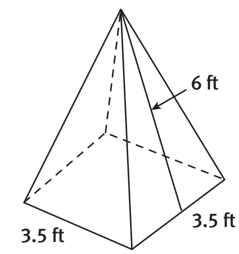
My Notes

LESSON 18-3 PRACTICE

Use the square pyramid for Items 10 and 11.

10. Model with mathematics. Use a net to find the lateral area of the pyramid.

11. Find the surface area of the pyramid.



Use the following information for Items 12 and 13.

A regular triangular pyramid has a base length of 4.6 centimeters and a slant height of 4 centimeters.

12. Use a net to find the lateral area of the pyramid.

13. If the height of the triangular base is approximately 4 cm, what is the surface area of the pyramid?

Use the following information for Items 14 and 15.

The Louvre Pyramid has a square base with sides that are 35 meters long. The slant height of each triangular face of the pyramid is 27.84 meters.

14. What is the lateral area of the Louvre Pyramid?

15. What is the surface area of the Louvre Pyramid?

Lesson 18-3
Lateral and Total Surface Area of Pyramids

My Notes

16. A square pyramid has a slant height of 5 meters. The perimeter of the base is 32 meters. Find the surface area of the pyramid.

17. Construct viable arguments. A regular triangular pyramid has a base length of 3.5 meters, a height of 1.3 meters, and a surface area of 21 square meters. What is the approximate slant height of the pyramid? Justify your answer by showing how you found it.

18. The pyramid of Khufu in Giza, Egypt, is a square pyramid with a base length of 756 feet. The slant height of this great pyramid is 612 feet. What is the lateral area of the pyramid of Khufu?

19. Make use of structure. A party favor is made from two square pyramids joined at their bases. Each edge of the square base is 3 centimeters. The slant height of the triangular faces is 4 centimeters. What is the surface area of the party favor?

20. Make sense of problems. A model of a Mayan pyramid has a square base with sides that are 1.3 meters long. The slant height of the pyramid is 0.8 meter. It costs $4.59 per square meter to paint the pyramid. How much will it cost to paint the lateral area of the model?

ACTIVITY 18 Continued

16. 144 m^2

17. about 3.5 meters; I wrote the formulas I know, substituted the given values into them, and solved for slant height.

$SA = L +$ area of the base

$21 = L + \frac{1}{2} \times 3.5 \times 1.3$

$21 = \left(\frac{1}{2} \times P \times l\right) + 2.275$

$21 = (.05 \times 10.5 \times l) + 2.275$

$21 = 5.25l + 2.275$

$18.725 = 5.25l$

$3.5 \approx l$

18. 925,344 ft^2

19. 48 cm^2

20. $9.55

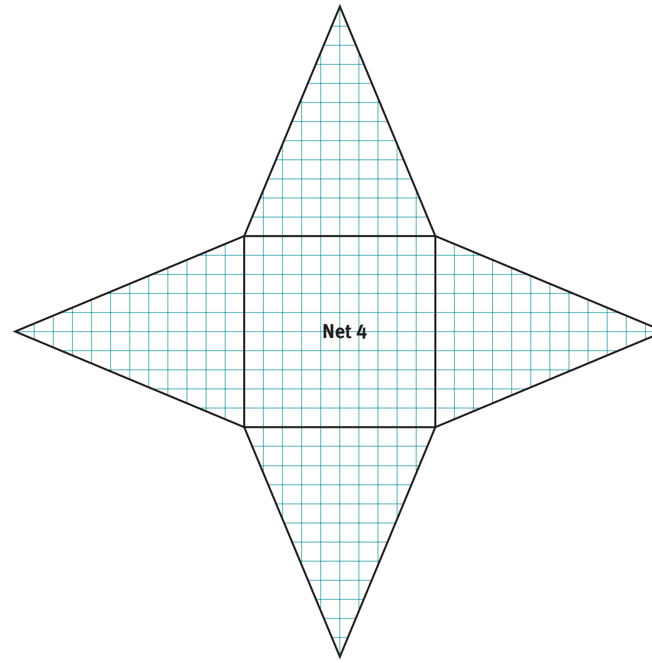
ADAPT

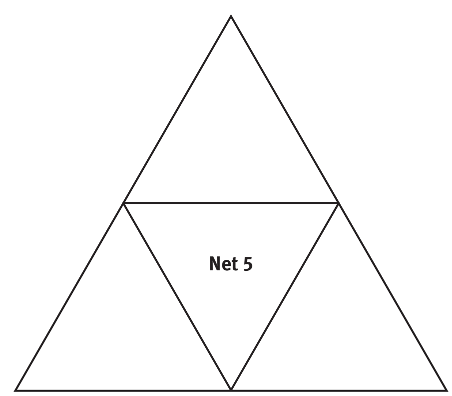
Check students' answers to the Lesson Practice to be sure they understand not only how to find the slant height of a pyramid, but also how to use the slant height to find the lateral area of a pyramid. Guide students to create notes defining slant height, lateral area and outline the steps to find slant height and lateral area.

Teacher to Teacher

Students will use these nets for items in this lesson.

My Notes



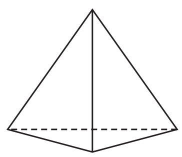


ACTIVITY 18 PRACTICE

Write your answers on a separate piece of paper. Show your work.

Lesson 18-1

Use the solid for Items 1 and 2.

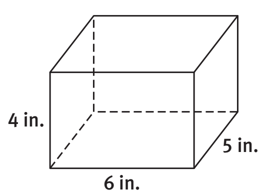


1. Imagine making slices through the solid parallel to the base. What two-dimensional shapes are formed?
2. Imagine making slices through the solid perpendicular to the base. What two-dimensional shapes are formed?

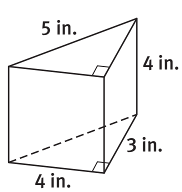
Lesson 18-2

Find the lateral and surface area of the figures in Items 3–5.

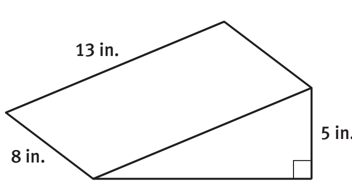
3.



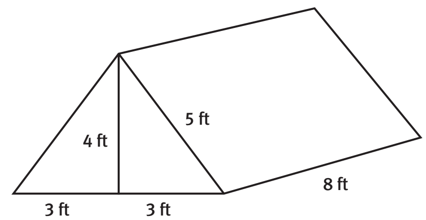
4.



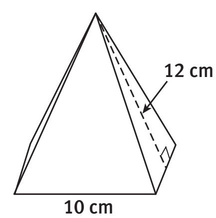
5.



6. A tent with canvas sides and a floor is shown. How much canvas is used to make the sides and floor of the tent?



7. A rectangular prism is 10 meters tall. It has a square base with sides that are 4 meters long. What is the surface area of the prism?
8. Find the lateral and the surface area of the square pyramid.

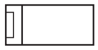


ACTIVITY 18 Continued

ACTIVITY PRACTICE

1. triangles
2. triangles and trapezoids
3. 88 in.^{2}; 148 in.^{2}
4. 48 in.^{2}; 60 in.^{2}
5. 240 in.^{2}; 300 in.^{2}
6. 152 ft^2
7. 192 m^2
8. 240 cm^2; 340 cm^2

9. B

10. Top View: 

Front View: 

Side View:  or 

11. B

12. 1,044 cm²

13. 9 in.

14. 972 in.²

15. 184 ft²

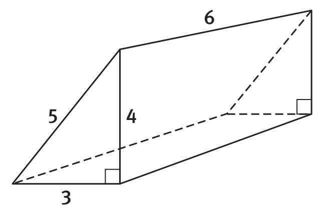
16. Answers may vary. In both the prism and the pyramid, you find the perimeter of the base, *P*. However, in the prism you multiply *P* by the height of the figure, and in a pyramid you multiply $\frac{1}{2} \times P \times$ slant height

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

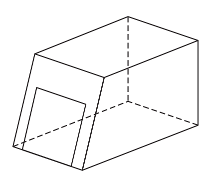
Lesson 18-3

9. The diagram shows the dimensions of a wooden block. The block will be covered with a reflective film. How much of the film is needed to cover the entire block?



A. 72 cm²
B. 84 cm²
C. 96 cm²
D. 108 cm²

Use the information and the drawing for Items 10 and 11. The Pup Company has a new model of dog kennel. The left and right sides are trapezoids and all other faces are shown in the diagram.



10. Sketch the top, front, and side views of the kennel.

11. Which cross sections of the kennel described below will be congruent?
A. all cross sections that are perpendicular to the bottom and parallel to the front and back faces
B. all cross sections that are perpendicular to the bottom and parallel to the left and right faces
C. all cross sections that are parallel to the top and bottom
D. none of the above

12. A cardboard box is 32 centimeters long, 15 centimeters wide, and 6 centimeters tall. The box does not have a top. How much cardboard was used to make the box?

13. The length of a side of the base of a square pyramid is 15 inches. The pyramid has a lateral area of 270 square inches. What is the slant height of the pyramid?

14. Three identical boxes are stacked by placing the bases on top of each other. Each box has a base that is 18 inches by 9 inches and is 4 inches tall. The stack of boxes will be shrink-wrapped with plastic. How much shrink-wrap is needed to cover the boxes?

15. A shed has the shape of a cube with edges that are 6 feet long. The top of the shed is a square pyramid that fits on top of the cube. The slant height of the faces is 5 feet. The shed has a single rectangular door that is 5 feet tall by 4 feet wide. All but the door and the bottom of the shed need to be painted. What is the area of the surface that needs to be painted?

MATHEMATICAL PRACTICES
Attend to Precision

16. Describe the similarities and differences in finding the lateral areas of a prism and a pyramid that have congruent bases.

Volume—Prisms and Pyramids

ACTIVITY 19

Berneen Wick's Candles
Lesson 19-1 Find the Volume of Prisms

Learning Targets:

- Calculate the volume of prisms.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Look for a Pattern, Predict and Confirm, Use Manipulatives

Berneen makes all the candles that she sells in her shop, Wick's Candles. The supplies for each candle cost $0.10 per cubic inch. Berneen wants to find the volume of every type of candle she makes to determine the cost for making the candles.

Volume measures the space occupied by a solid. It is measured in cubic units.

1. Berneen uses unit cubes as models of 1-inch cubes.

a. Use unit cubes to build models of 2-inch cubes and 3-inch cubes. Then complete the table.

Length of Edge (in.)	Area of Face ($in.^2$)	Volume of Cube ($in.^3$)
1	1	1
2	4	8
3	9	27

b. Make use of structure. Describe any relationships you see in the data in the table.

Sample answer: Each area is equal to the length of an edge squared. Each volume is equal to the area of a face times the length of the edge, and to the length of an edge cubed. Area and volume increase at a different rate. Volume increases faster than area.

My Notes

MATH TIP

Cubes are named by the lengths of their edges. A 1-inch cube is a cube with edges that are 1 inch in length. A 2-inch cube is a cube with edges that are 2 inches in length.

Cubes of any size can be used to build larger cubes.

Common Core State Standards for Activity 19

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

ACTIVITY 19
Investigative

Activity Standards Focus
Until now, students have applied volume formulas to simple solids. In Activity 19 they move on to finding the volume of prisms, pyramids, and the complex solids formed when two or more solids are put together.

Lesson 19-1

PLAN

Materials

- unit cubes
- model prisms

Pacing: 2 class period

Chunking the Lesson

#1–2 #3 #4

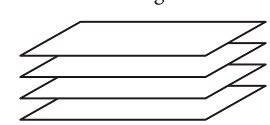
Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Have students use any size deck of playing cards or index cards to visualize how volume can be equal to the area of the base times the height.



Find the area of one card, the base of a rectangular prism, then stack (count) the cards to get the height. The volume is the product of the number of cards (the height) times the area of each card (the area of the base).

Developing Math Language

Help students understand the terms related to volume: *length, width, height,* and *edge*. Students need to know that the length, width, and height of a prism are determined by the lengths of the edges of a prism. Have them identify the terms on a model prism.

1–2 Look for a Pattern, Group Discussion. Students use manipulatives to develop an understanding of volume. After they complete the table for cubes with edges of 1-, 2-, and 3-inches, challenge them to make a conjecture as to the volume of a cube with edge *e*-units.

3 Look for a Pattern, Predict and Confirm Help students see that the volume of a rectangular prism can be found in a similar way to finding the volume of a cube. That is, find the area of the base and then multiply by the height of the prism. Return to the Bell-Ringer activity, if necessary. The volume V of a prism is $V = B \times h$, where B is the area of the base and h is the height of the prism.

Teacher to Teacher

In these items, students are being asked to *look for and make use of structure.* This process of gathering numerical information, making and testing conjectures, and generalizing the results into formulas is one that students will continue to engage in as they progress up to and into AP mathematics.

The formula for the volume, V, of a cube, with edge length e, is $V = e^3$.

2. **Reason quantitatively.** One of Berneen's candles is a cube with sides that are 1.5 inches long.

a. Use the formula to find the volume of this candle. Show your work.
$V = e^3 = 1.5 \times 1.5 \times 1.5 =$ **3.375 in.^{3}**

b. Recall that supplies for each candle cost \$0.10 per cubic inch. How much does it cost to make this candle, to the nearest cent?
\$0.34

Most of the candles Berneen makes are in the shape of rectangular prisms.

3. The formula for the volume of a cube is also equal to the area of the base times the height of the cube.

a. Consider this relationship to help complete the table. Use cubes to build the prisms as needed.

Dimensions of Candle (in.)	Area of Base (in.^{2})	Candle Height (in.)	Candle Volume (in.^{3})
$l = 4$ $w = 2$ $h = 1$	8	1	8
$l = 4$ $w = 2$ $h = 2$	8	2	16
$l = 4$ $w = 2$ $h = 3$	8	3	24
$l = 5$ $w = 3$ $h = 1$	15	1	15
$l = 5$ $w = 3$ $h = 2$	15	2	30

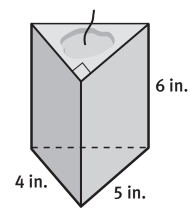
b. Describe any pattern you see for finding volume.
Sample answer: The volume is the product of the area of the base and the height.

Lesson 19-1
Find the Volume of Prisms

My Notes

The volume, V, of a prism is the area of the base, B, times the height, h: $V = B \times h$.

4. Berneen makes a candle in the shape of a triangular prism as shown. The candle is very popular with many customers because of its interesting shape.



a. What is the volume of the candle? Explain your thinking.
60 in.^{3}; The candle is a prism, so apply the formula $V = B \times h$.
$V = \left(\frac{1}{2}\right)(5)(4) \times 6$ in. $= 10 \times 6 = 60$ in.^{3}

b. Make sense of problems. Remember that the cost of the supplies for each candle is \$0.10 per cubic inch. How much profit will Berneen make if she sells this candle for \$8.99? Show your work.
\$2.99; 60 in.^{3} $\times$ \$0.10 per in.^{3} = \$6.00; \$8.99 − 6.00 = \$2.99

Check Your Understanding

5. Construct viable arguments. Can a rectangular prism and a cube have the same volume? Support your opinion with an example or counterexample.

6. Make use of structure. How does the volume of a prism change when one dimension is doubled? When two dimensions are doubled? When three dimensions are doubled? Explain your thinking.

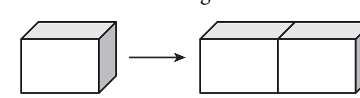
4 Activating Prior Knowledge, Discussion Groups, Group Presentation As students apply the volume formula to a triangular prism, they must recall the method for finding the area of a right triangle. Have students identify the figures which form the bases and faces of the figure and area formulas for those figures. Have groups share their approach to finding the profit . Encourage the use of precise language. Encourage students to critique the work of others.

Teacher to Teacher

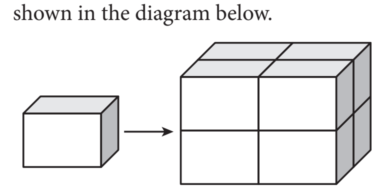
Make sure that students do not just multiply the three dimensions in Item 4 as they correctly did with cubes and rectangular prisms. Since the solid is a triangular prism, they need to find the area of a triangular base, not a rectangular base. Remind them that they cannot find the area of a triangle by multiplying the lengths of two edges.

Check Your Understanding

Debrief students' answers to these items to be sure they understand finding the volume of prisms. For Item 6, help students move from applying a volume formula to understanding how the volume changes when one or more of the dimensions change.



Students can see from the diagram that doubling the length doubles the volume. Doubling all three dimensions increases the volume by a factor of $2^3 = 8$, as shown in the diagram below.



In general, if all three dimensions change with a scale factor of $a : b$, then the volume change has a scale factor $a^3 : b^3$.

Answers

5. Sample explanation: A rectangular prism that measures 2 units by 4 units by 1 units and a cube that measures 2 units by 2 units by 2 units both have a volume of 8 cubic units.

6. When one dimension is doubled, volume is doubled. When two dimensions are doubled, volume increases by a factor of 4. When three dimensions are doubled, volume increases by a factor of 8.

ASSESS

Use the lesson practice to assess your students' understanding of finding the volume of a prism by multiplying the area of the base of the prism by the height.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Teacher to Teacher

In Item 13, students will have to solve the volume formula for h, the height of the refrigerator. Have them check answers with a partner.

Item 15 asks students to find the difference of two volumes, not of two dimensions. Make sure that students find the volume of each pool with different dimensions before they find how much more water is needed to fill the second pool.

For Items 15–16, consider having students use models of prisms. These will help students visualize the dimensions needed to find the volume.

LESSON 19-1 PRACTICE

7. 1,050 cm^3
8. 810 $in.^3$
9. 343 cm^3
10. 125 $in.^3$
11. 80 cubes. Sample explanation: $30 \div 3 = 10$, $12 \div 3 = 4$ and $6 \div 3 = 2$. There will be 2 layers of cubes each with dimensions 10 cubes by 4 cubes.
12. 2,160 $in.^3$
13. 4.5 ft
14. 27 ft^3; 1 yd = 3 ft and $3 \times 3 \times 3 = 27$
15. 360 ft^3
16. The measures are correct but the units are not. The measure of the base should be expressed in square meters and the volume should be expressed in cubic meters.

ADAPT

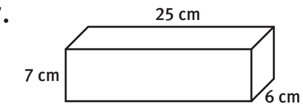
Check students' answers to the Lesson Practice to be sure they understand that the units used for volume are cubic units, and that the reason is that volume is measuring how many cubes fit into the solid. Have students create a graphic organizer on which they sketch various solid figures, label them with a mathematical name (i.e. "cube," not "box"), and list characteristics and formulas of the figures.

My Notes

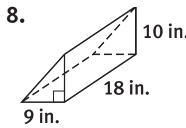
LESSON 19-1 PRACTICE

Find the volume of each figure.

7.



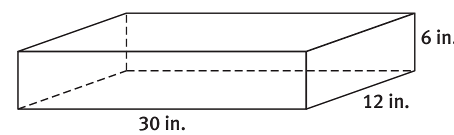
8.



9. A cube with edge length 7 centimeters.

10. A cube that has a face area of 25 square inches.

Use the prism for Items 11 and 12.



11. How many cubes with a side length of 3 inches will fit into the rectangular prism? Explain.

12. Find the volume of the prism.

13. A small refrigerator has a square base with sides that are 3 feet long. The refrigerator has a capacity of 40.5 cubic feet. How tall is the refrigerator?

14. **Reason quantitatively.** How many cubic feet are equivalent to 1 cubic yard? Explain.

15. **Make sense of problems.** The Gray family is putting in a pool in the shape of a rectangular prism. The first plan shows a pool that is 15 feet long, 12 feet wide, and 5 feet deep. The second plan shows a pool with the same length and width, but a depth of 7 feet. How much more water is needed to fill the second pool if both pools are filled to the top?

16. **Attend to precision.** A student says that the volume of a triangular prism with a base area of 12 meters and a height of 5 meters is 60 square meters. Is the student correct? If not, what is wrong with the student's statement?

Lesson 19-2
Find the Volume of Pyramids

My Notes

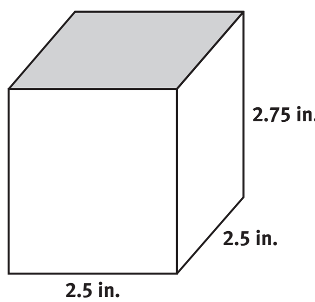
Learning Targets:
- Calculate the volume of pyramids.
- Calculate the volume of complex solids.
- Understand the relationship between the volume of a prism and the volume of a pyramid.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Predict and Confirm, Think-Pair-Share, Use Manipulatives

1. Other candles in Wick's Candles are in the shape of pyramids. To find the volumes, Berneen makes models to look for a relationship between the volume of a prism and the volume of a pyramid.

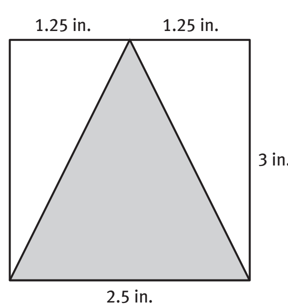
a. Make a model of the prism candle mold.

- Use index cards or card stock to cut out 1 square with side length 2.5 inches and 4 rectangles with length 2.5 inches and width 2.75 inches.
- Tape them together to form a net for a rectangular prism with no top. Then fold the net and tape it together to form a rectangular prism with no top as shown.

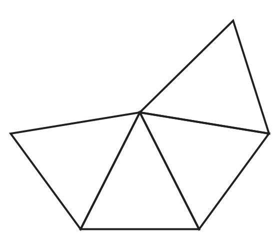


b. Make a model of the pyramid candle mold.

- Use index cards or card stock to cut out 4 isosceles triangles with the dimensions shown in the diagram.



- Tape the triangles together along their congruent sides to form a net for a square pyramid, as shown.
- Tape the net together to form a square pyramid.



Lesson 19-2

PLAN

Materials
- model pyramids and prisms
- index cards

Pacing: 2 class periods

Chunking the Lesson

#1–2 #3–5 #6
Check Your Understanding
Lesson Practice
Activity Practice

TEACH

Bell-Ringer Activity

Have students pair up and find the volume of different objects in the room, such as a box of tissue or their desks. Ask students to share what units of measurement they chose to use and how they calculated the volume of their chosen item.

1–2 Create Representations, Use Manipulatives, Visualization In Items 1 and 2, students create a models. The pyramid will have the same base and height as a prism with a square base that is 2.5 inches on a side and that has a height of 2.75 inches. Have students explain how they know that the pyramid has a square base.

3–5 Create Representations, Use Manipulatives, Visualization. Predict and Confirm Students often look at diagrams or models of a prism and pyramid and think that the volume of a pyramid is half the volume of a prism with the same base and the same height. Visually, it may seem so. Student should make a prediction before beginning their investigation. It is important for students to experience the factor is one-third. Use salt, sand, or water to demonstrate how 3 volumes of a pyramid fit into a prism with the same base and height. Ask students to make the extension from square based prisms and pyramids to prisms and pyramids that have the same heights and have bases with the same areas, and apply their understanding of formulas to the business scenario.

ELL Support

Help students understand that the slant height of a pyramid is not used to find volume, but rather to find surface area. Encourage students to write out the volume formula, including the height *h*, before using it in volume calculations. Students may remember from Activity 18 that the variable used for slant height is *l*, not *h*.

My Notes

Lesson 19-2
Find the Volume of Pyramids

2. Compare the dimensions of the prism and the square pyramid you built in Item 1. What relationships do you notice?
Sample answer: The bases are congruent and the heights are the same.

3. Using the material your teacher distributes, fill your pyramid to the top. Predict the number of times you can fill and empty the pyramid into the rectangular prism to fill the prism to the top. Confirm your prediction by filling and emptying the pyramid into the prism.
Predictions may vary. Sample answer: I predicted 2 times, but it took 3 full pyramids to fill the prism.

The volume, *V*, of a pyramid is one-third the area of the base, *B*, times the height, *h*:

$$V = \frac{1}{3} \times B \times h$$

4. Reason quantitatively. Two of Berneen's candles have congruent rectangular bases. One candle is shaped like a rectangular prism, while the other is shaped like a rectangular pyramid. Both candles are 5 inches tall. What is the relationship between the volumes of the two candles? Explain.
Sample answer: Comparing the volume formulas for a pyramid and a prism shows that the volume of the pyramid is one-third the volume of a prism if the bases and heights are congruent.

5. Make sense of problems. A candle in the shape of a square pyramid has a base edge of 6 inches and a height of 6 inches.

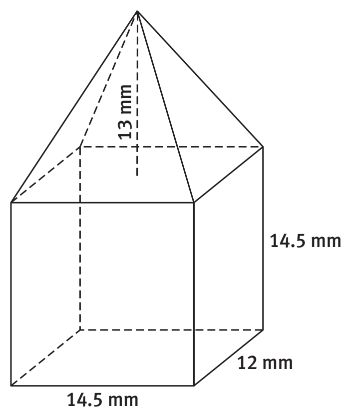
a. What is the volume of the candle?
$V = \frac{1}{3} \times B \times h = \frac{1}{3} \times (6 \times 6) \times 6 = \frac{1}{3} \times 216 = 72 \text{ in.}^3$

b. How much does it cost Berneen to make the candle? Show your work.
$7.20; 72 in.^{3} × $0.10 per in.^{3} = $7.20

Lesson 19-2
Find the Volume of Pyramids

A **complex solid** is formed when two or more solids are put together. The volume of the complex solid is the sum of the volumes of the smaller solids.

6. Consider the complex solid shown.



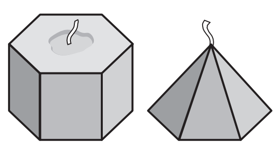
a. Find the volume of the rectangular prism.
$2{,}523 \text{ mm}^3$

b. Find the volume of the rectangular pyramid.
754 mm^3

c. Find the volume of the complex solid.
$3{,}277 \text{ mm}^3$

Check Your Understanding

Use the candles for Items 7 and 8.



7. **Express regularity in repeated reasoning.** The candles shown have congruent bases and heights. What is true about the relationship between the volumes of the candles?

8. **Model with mathematics.** Suppose Berneen makes a candle by setting the pyramid on top of the prism. Write a formula for the volume of this candle.

My Notes

ACTIVITY 19 Continued

6 Use Manipulatives Students make connections between their prior learning of composite two-dimensional figures and complex solids, Use a model of a prism or cube with a pyramid that has the same base area to help students understand how a complex solid is made. Ask students if the pyramid has volume one-third the volume of the prism, and to explain their answer. (No; the height of the prism, 14.5 mm, is not the same as the height of the pyramid, 13 mm.)

Check Your Understanding

Debrief students' answers to these items to be sure they understand how to find the volume of a pyramid.

Answers

7. Answers may vary. Sample answer: The volume of the pyramid-shaped candle is one-third the volume of the prism-shaped candle.

8. $V = \frac{1}{3} \times B \times h + B \times h$, or $\frac{4}{3} \times B \times h$

ASSESS

Use the lesson practice to assess your students' understanding of how to find the volume of a pyramid. Pay particular attention to Item, which has students find the volume of real-life pyramids having large volumes.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Teacher to Teacher

In Items 11 and 12, have students draw triangular pyramids to make sure they do not confuse the height of the triangular base with the height of the pyramid.

For Item 13, consider having students work in pairs so they can compare how to solve the volume formula for the height of the pyramid.

LESSON 19-2 PRACTICE

9. 33.3 cm^3
10. 5060 cm^3
11. 90 $in.^3$
12. The volume of the pyramid (90 $in.^3$) is one-third the volume of the prism (270 $in.^3$); this is indicated by their volume formulas: $\frac{1}{3}Bh$ is one-third of Bh.
13. 9 cm
14. 13 ft
15. 8,400 mm^3
16. 147 $in.^3$ greater
17. about 4,241.25 m^3 greater

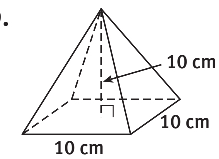
ADAPT

Check students' answers to the Lesson Practice to be sure they understand not only how to find the height of a pyramid, given the volume, but also how to find the volume of a composite pyramid. Have students use nets from previous lessons to create complex solids This will allow them to visualize and label as they apply formulas.

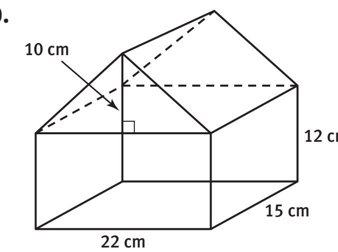
LESSON 19-2 PRACTICE

Find the volume of each figure in Items 9–11.

9.



10.



11. A triangular pyramid with a base area of 18 square inches and a height of 15 inches.

12. How does the volume of the triangular pyramid in item 11 compare with the volume of a triangular prism with a base area of 18 square inches and a height of 15 inches? Use words and symbols to explain.

13. The area of the base of a pyramid is 85 square centimeters. If its volume is 255 cubic centimeters, find the height of the pyramid.

14. A square pyramid 9 feet tall has a volume of 507 cubic feet. How long is each side of the base of the pyramid?

15. Two square pyramids are joined at their bases. Each base is 30 millimeters long. The distance between the vertices of the combined pyramids is 28 millimeters. What is the volume of the solid formed?

16. **Reason quantitatively.** A square pyramid with base lengths of 6 inches is 14 inches tall. The top part of the pyramid is cut off to form a smaller pyramid with base lengths that are 3 inches long and a height of 7 inches. How many square inches greater was the volume of the larger pyramid than that of the new smaller pyramid?

17. **Make sense of problems.** The Pyramid of Cestius in Rome today stands about 27 meters tall with a square base whose sides are about 22 meters long. The pyramid was based on ancient Nubian pyramids. These pyramids average a base area of 25.5 square meters and a height of 13.5 meters. How does the volume of the Pyramid of Cestius compare to the volume of an average Nubian pyramid?

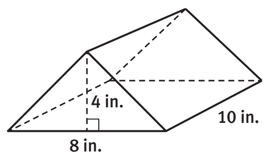
ACTIVITY 19 PRACTICE

Write your answers on a separate piece of paper. Show your work.

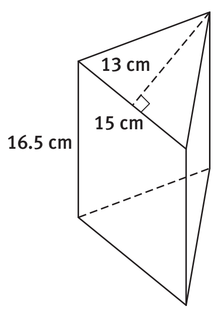
Lesson 19-1

For Items 1–4, find the volume of each figure.

1.



2.



3. A cube with edge length 8 inches.
4. A rectangular prism with sides that are 1.2, 1.8, and 2.5 meters long.
5. A rectangular prism with a square base is 6.4 meters tall. The prism has a volume of 409.6 cubic meters. What are the dimensions of the base of the prism?
6. Mariah is filling a terrarium in the shape of a rectangular prism with sand for her tarantula. The sand will be one-quarter of the way to the top. If the length of the terrarium is 17 inches, the width 12 inches, and the height 12 inches, what is the volume of the sand she uses?

Lesson 19-2

7. A container in the shape of a rectangular prism has a base that measures 20 centimeters by 30 centimeters and a height of 15 centimeters. The container is partially filled with water. A student adds more water to the container and notes that the water level rises 2.5 centimeters. What is the volume of the added water?
   A. 1,500 cm^3
   B. 3,600 cm^3
   C. 4,500 cm^3
   D. 9,000 cm^3

For Items 8–11, find the volume of the figure described.

8. A triangular pyramid with a base area of 43.3 meters and a height of 12 meters.
9. A square pyramid with base edge 10 centimeters and height 12 centimeters.
10. A triangular pyramid with a base length of 9 inches, a base height of 10 inches, and a height of 32 inches.
11. A square pyramid with a base length of 4 centimeters and a height of 6 centimeters resting on top of a 4-centimeter cube.
12. The area of the base of a triangular pyramid is 42 square feet. The volume is 1,197 cubic feet. Find the height of the pyramid.
13. The square pyramid at the entrance to the Louvre Museum in Paris, France, is 35.42 meters wide and 21.64 meters tall. Find the volume of the Louvre Pyramid.

ACTIVITY 19 Continued

ACTIVITY PRACTICE

1. 160 $in.^3$
2. 1608.75 cm^3
3. 512 $in.^3$
4. 5.4 m^3
5. 8 m by 8 m
6. 612 $in.^3$
7. A
8. 173.2 m^3
9. 400 cm^3
10. 480 $in.^3$
11. 96 cm^3
12. 85.5 ft
13. 9,049.7 m^3

14. Answers may vary. To find the volume of both prisms and pyramids, first multiply the area of the base by the height. This shows that the area of the base is related to volume by the height of the solid. The shape of a solid is related to its volume because a pyramid has only one-third the volume of a prism with the same base and height. This is reflected in the formulas for the two figures: V prism $= B \times h$, while V pyramid $= \frac{1}{3} \times B \times h$

15. Answers may vary. By comparing the formulas for the volumes, the volume of the pyramid is one-third the volume of the prism since the bases and heights are congruent.

16. A

17. Answers may vary.
$V_1 = 42$ in.^{3}, \$4.20;
$V_2 = 39$ in.^{3}, \$3.90;
$V_3 = 24$ in.^{3}, \$2.40

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

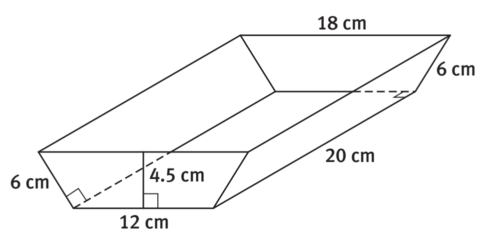
14. For prisms and pyramids, how are the area of the base of the solid *and* the shape of the solid related to the volume?

15. a. A triangular prism and a triangular pyramid have congruent bases and heights. What is the relationship between the volumes of the two figures? Explain in words using an example.

b. Explain the relationship between the volumes using their formulas.

16. A plastic tray is shown, with the dimensions labeled. The bottom and two of the sides are rectangles. The other two sides are congruent isosceles trapezoids. What is the volume of the tray?

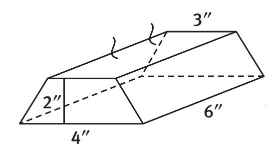
A. 1,350 cm^3
B. 1,080 cm^3
C. 1,440 cm^3
D. 1,620 cm^3



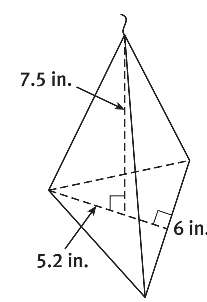
MATHEMATICAL PRACTICES
Attend to Precision

17. Berneen Wick wants to offer a gift set containing the three candles shown. Remember: The cost per cubic inch of a candle is \$0.10. Prepare a report for Berneen in which you provide her with:

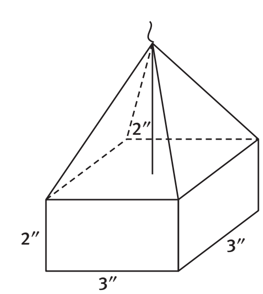
- a name and a cost for each candle and the method of calculating each cost
- your recommendation for the price of the gift set
- your reasons for the recommendation



Candle 1



Candle 2



Candle 3

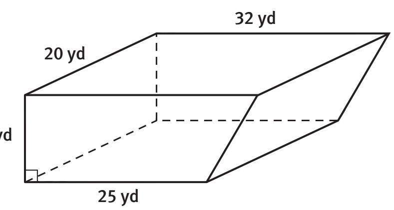
Surface Area and Volume
UNDER THE SEA

Embedded Assessment 3
Use after Activity 19

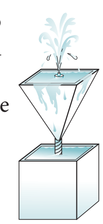
Mackeral "Mack" Finney is designing a new aquarium called Under the Sea. He plans to include several different types of saltwater tanks to house the aquatic life.

1. Mack begins by designing the smallest fish tank. This tank is a rectangular prism with dimensions 4 feet by 2 feet by 3 feet.
   a. Draw and label a net to represent the aquarium.
   b. The tank will have a glass covering on all six sides. Find the surface area of the tank. Explain your reasoning.
   c. Find the volume of the tank. Show your work.

2. Near the main entrance to the aquarium, Mack has decided to put a larger pool for four dolphins. Its shape is the trapezoidal prism shown.
   a. Sketch and label the dimensions of a cross section parallel to the bases of the prism.
   b. Find the amount of water needed to fill the pool. Explain your thinking.

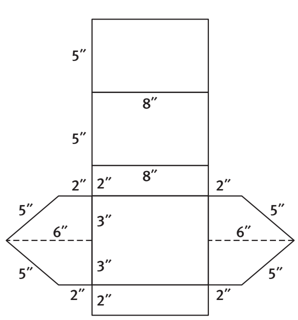


3. Mack designed a water fountain with a square pyramid flowing into a cube, as shown at right. The edges of the bases of the pyramid and the cube have the same length and the heights of the pyramid and the cube are the same. Describe the relationship between the volume of the cube and the volume of the pyramid.

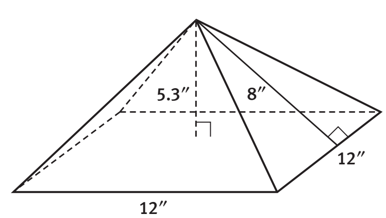


In addition to tanks for the aquatic life, Mack designs some hanging birdhouses for the trees around the aquarium.

4. The net for one birdhouse is shown below. What is the total surface area of the solid? Show your work.



5. Another birdhouse design is in the shape of a square pyramid, as shown below. Find the surface area and volume of the birdhouse.



Common Core State Standards for Embedded Assessment 3

7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

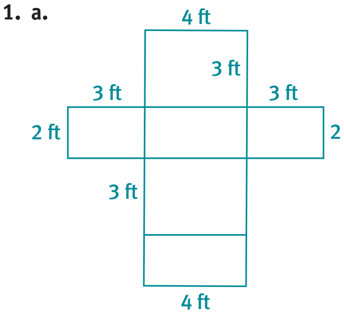
Embedded Assessment 3

Assessment Focus

- Nets for a prism
- Surface area of a prism
- Cross section of a solid

Answer Key

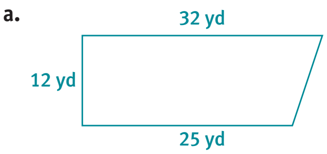
1. a.



b. 52 ft^2; I found the lateral surface area (4 sides) and added it to the area of the bottom and top of the tank.

c. 24 ft^3; $V = lwh = 4 \times 3 \times 2$

2. a.



b. $B = \frac{1}{2}(12)(25 + 32) = 342$ yd^2, $V = 342 \times 20 = 6{,}840$ yd^3.

3. Answers may vary. Sample answer: the volume of the pyramid is $\frac{1}{3}$ the volume of the cube.

4. 208 in.^{2}

5. surface area = 336 in.^{2}, volume = 254.4 in.^{2}

Teacher to Teacher

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

Embedded Assessment 3
Use after Activity 19

Surface Area and Volume
UNDER THE SEA

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-c, 2b, 3, 4, 5)	• Accurately and efficiently finding the surface area and volume of prisms and pyramids.	• Finding the surface area and volume of prisms and pyramids.	• Difficulty finding the surface area and volume of prisms and pyramids.	• No understanding of finding the surface area and volume of prisms and pyramids.
Problem Solving (Items 1b-c, 2b, 4, 5)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-b, 2a, 4, 5)	• Clear and accurate understanding of how a net represents a three-dimensional figure.	• Relating a net to the surfaces of a three-dimensional figure.	• Difficulty recognizing how a net represents a three-dimensional figure.	• No understanding of how a net represents a three-dimensional figure.
Reasoning and Communication (Items 1b, 2b, 3)	• Precise use of appropriate terms to explain finding surface area and volume of solids. • A precise and accurate description of the relationship between the volume of a pyramid and a cube.	• An adequate explanation of finding surface area and volume of solids. • A basically correct description of the relationship between the volume of a pyramid and the cube.	• A partially correct explanation of finding surface area and volume of solids. • A partial description of the relationship between the volume of a pyramid and a cube.	• An incomplete or inaccurate explanation of finding surface area and volume of solids. • A partial description of the relationship between the volume of a pyramid and a cube.