

Geometry

3

Unit Overview

In this unit you will continue your study of angles and triangles and explore the Pythagorean Theorem. You will investigate 2- and 3-dimensional figures and apply formulas to determine the area and volume of those figures. You will explore rigid transformations of figures, including translations, rotations, and reflections of two-dimensional figures.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- alternate
- transform

Math Terms

- angle
- ray
- complementary angles
- supplementary angles
- congruent
- transversal
- alternate exterior angles
- alternate interior angles
- corresponding angles
- vertical angles
- exterior angle of a triangle
- remote interior angle
- diagonal
- transformation
- preimage
- image
- translation
- reflection
- line of reflection
- equidistant
- rotation
- center of rotation
- composition of transformations
- similar figures
- similarity statement
- proportion
- scale factor
- dilation
- center of dilation
- scale factor of dilation
- hypotenuse
- legs
- Pythagorean Theorem
- surface area
- lateral area

ESSENTIAL QUESTIONS



What are transformations and how are they useful in solving real-world problems?



How are two- and three-dimensional figures related?

EMBEDDED ASSESSMENTS

These assessments, following activities 17, 19, 21, 24, and 26, will give you an opportunity to demonstrate how you can use your understanding of angles, triangles, transformation, and geometric formulas to solve problems.

Embedded Assessment 1:

Angle Measures p. 229

Embedded Assessment 2:

Rigid Transformations p. 263

Embedded Assessment 3:

Similarity and Dilations p. 293

Embedded Assessment 4:

The Pythagorean Theorem p. 325

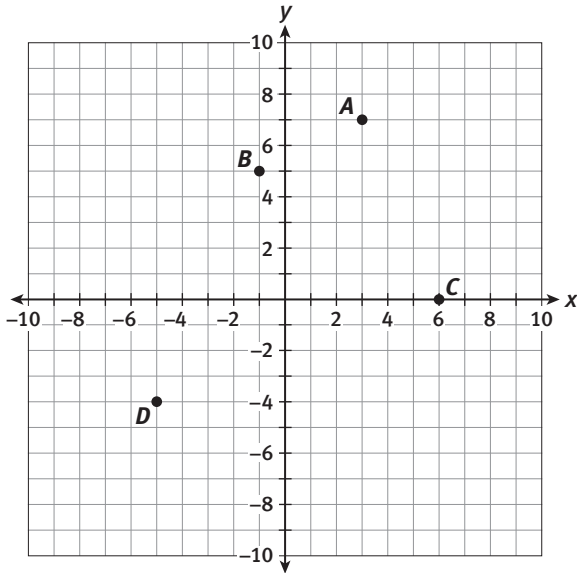
Embedded Assessment 5:

Surface Area and Volume p. 353

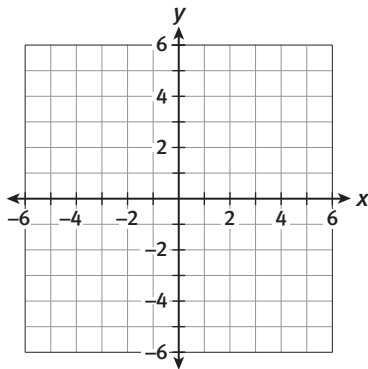
Getting Ready

Write your answers on notebook paper.
Show your work.

1. Give the coordinates of points A, B, C, and D on the graph below.



2. On the grid below, draw a square that has $(-2, 4)$ and $(3, -1)$ as two of its vertices. Label the other two vertices.



3. Define the following terms:

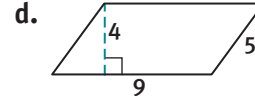
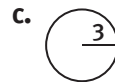
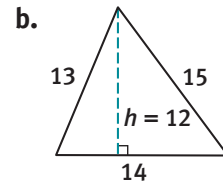
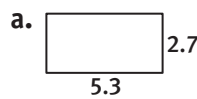
- a. acute triangle
- b. right triangle
- c. obtuse triangle

4. Write three ratios equivalent to $\frac{2}{5}$.

5. Find the value of x in each of the following.

a. $\frac{2}{3} = \frac{6}{x}$ b. $\frac{4}{7} = \frac{x}{21}$

6. Find the perimeter or circumference of each of the figures below.



7. Find the area of each figure in Item 6.

8. Explain using specific formulas how you could find the area of the shaded area of the figure below.



Angle-Pair Relationships

The Winning Angle

Lesson 16-1 Complementary and Supplementary Angles

Learning Targets:

- Identify and determine the measure of complementary angles.
- Identify and determine the measure of supplementary angles.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Graphic Organizer, Identify a Subtask

Bob Toose, football coach and geometry teacher at Johnny Unitas High School, names his football plays after different geometric terms. He knows that the players from other schools won't know what is coming at them with these names.

Coach Toose gives playbook quizzes to make sure his players know their plays. A portion of one of his quizzes is below.

1. Match each play shown at the right with the mathematical term that best describes it.
Draw a line to connect the plays and the terms.

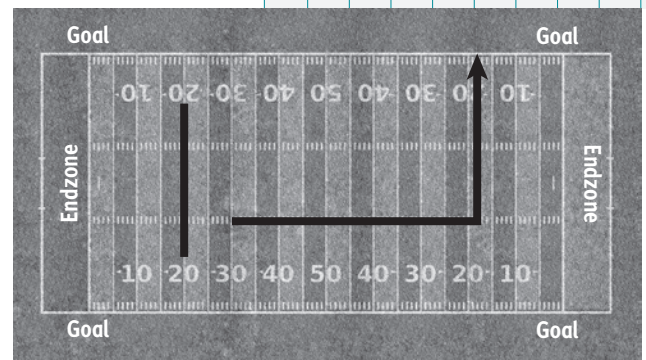
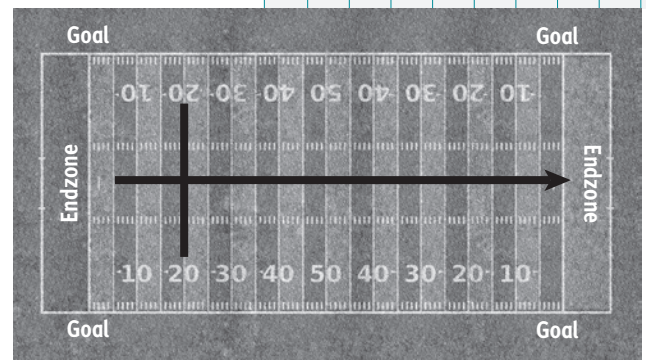
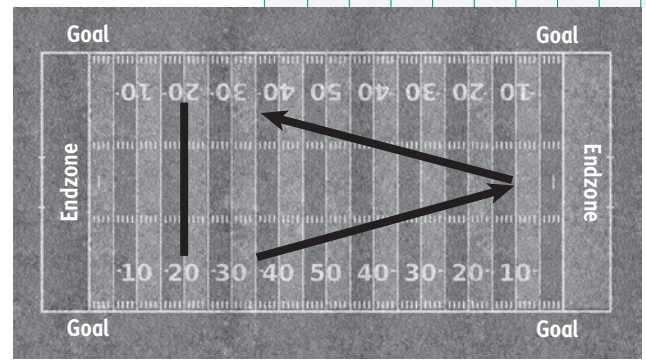
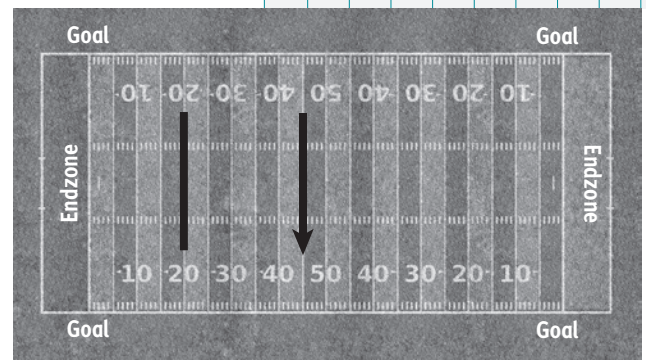
Angle

Perpendicular Lines

Parallel Lines

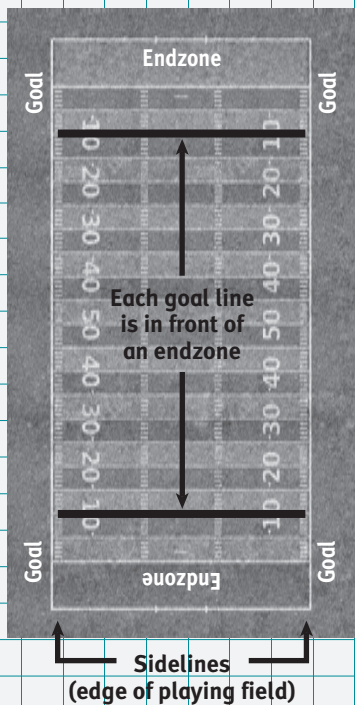
Right Angle

My Notes									

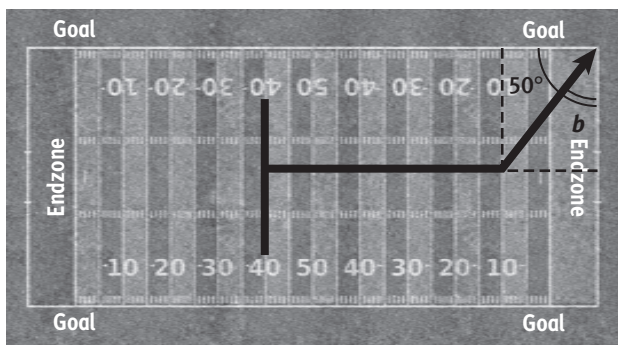


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My Notes



Coach Toose is very particular about the routes that his players run. He told his receiver that this “corner” route needed to be run at a 50° angle to the sideline of the end zone.

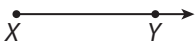


2. What is the measure of angle b in the diagram above?
3. Two angles are **complementary** if the sum of their measures is 90° . Explain why these two angles can be classified as complementary.
4. Coach Toose wanted his players to run other corner routes as well. Identify the **angle** complementary to the one listed. Draw a diagram to illustrate the angle and its complement.
 - a. 20°
 - b. 73°

MATH TERMS

An **angle** is the union of two rays with a common endpoint called the vertex.

A ray is part of a line with one endpoint. It extends indefinitely in one direction.

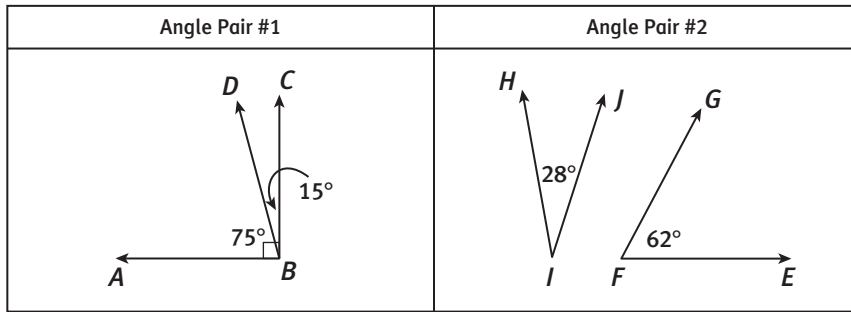


Ray XY can be written \overrightarrow{XY} . Its endpoint is X and it extends forever through point Y .

Lesson 16-1

Complementary and Supplementary Angles

5. Make use of structure. Shown below is an example of two pairs of angles. Compare and contrast the angle pairs.



Make notes about the math terms and academic vocabulary used in Example A below and other examples that follow. Review your notes and use new vocabulary when you write and discuss your responses to items.

Example A

The measure of angle A is $(3x)^\circ$ and the measure of its complement, angle B, is $(x + 6)^\circ$. Determine the measures of the angles.

Step 1: Write an equation that shows the relationship between $\angle A$ and $\angle B$.
 The sum of the angle measures is 90° . $m\angle A + m\angle B = 90^\circ$.
 Substitute the expressions for the angle measures. $3x + x + 6 = 90$

Step 2: Solve the equation.
 Original equation $3x + x + 6 = 90$
 Combine like terms. $4x + 6 = 90$
 Subtract 6 from both sides. $4x + 6 - 6 = 90 - 6$
 Simplify. $4x = 84$
 Divide both sides by 4. $\frac{4x}{4} = \frac{84}{4}$
 Simplify. $x = 21$

Step 3: Determine the measure of the two angles.

$$m\angle A = (3x)^\circ = (3 \cdot 21)^\circ = 63^\circ$$

$$m\angle B = (x + 6)^\circ = (21 + 6)^\circ = 27^\circ$$

Solution: $m\angle A = 63^\circ$, and $m\angle B = 27^\circ$.

Try These A

Angle P and angle Q are complementary. Determine the measures of the angles.

- a. $m\angle P = (2x - 5)^\circ$ and $m\angle Q = (x + 20)^\circ$
- b. $m\angle P = (x + 4)^\circ$ and $m\angle Q = (5x - 4)^\circ$

My Notes

READING MATH

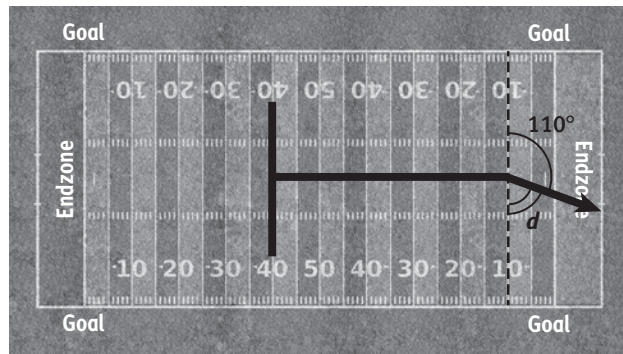
$m\angle A$ is read "the measure of angle A."

My Notes

Another route that Coach Toose has his players run is a “post” route. The route can be used to show **supplementary** angles.

6. Tell what it means for angles to be supplementary and sketch an example below.

7. This “post” route is seen below as it passes over the goal line. Give the measure of angle d .



8. Coach Toose’s team runs a variety of “post” routes. Identify the angle supplementary to the one listed. Draw a diagram to illustrate the angle and its supplement.

a. 20°

b. 153.1°

9. One of Coach Toose’s players claims that two angles do not need to be adjacent to be supplementary. Draw a pair of nonadjacent supplementary angles and explain why they are supplementary.

Lesson 16-1

Complementary and Supplementary Angles

ACTIVITY 16

continued

10. The measures of two supplementary angles are $(x + 1)^\circ$ and $(2x - 1)^\circ$.
- Write an equation that shows the relationship between the two angle measures and determine the value of x .
 - Determine the measure of the two angles.

Check Your Understanding

- Determine the complement and/or supplement of each angle. If it is not possible, explain.
 - 57.2°
 - 93°
- Determine whether angles with measures 47° and 53° are complementary. Explain why or why not.
- Determine whether angles with measures 37° and 143° are supplementary. Explain why or why not.

LESSON 16-1 PRACTICE

- Determine the measure of two **congruent**, complementary angles.
- Draw a pair of adjacent, complementary angles.
- Angle C and angle D are complementary. The measure of angle C is $(2x)^\circ$ and the measure of angle D is $(3x)^\circ$. Determine the measure of the two angles. Show the work that leads to your answer.
- Angle E and angle F are supplementary. The measure of angle E is $(x + 10)^\circ$ and the measure of angle F is $(x + 40)^\circ$. Determine the measure of the two angles. Show the work that leads to your answer.
- Construct viable arguments.** Determine whether the following statement is true or false. Justify your reasoning. "Two right angles are always supplementary."

My Notes

MATH TERMS

Angles that have the same measure are called **congruent**.

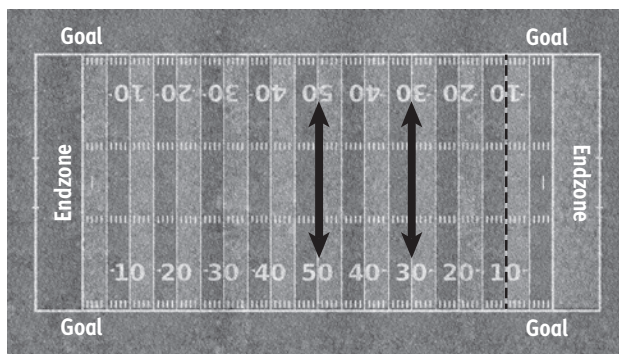
My Notes

Learning Targets:

- Determine the measure of angles formed by parallel lines and transversals.
- Identify angle pairs formed by parallel lines and transversals.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think-Pair-Share, Interactive Word Wall, Graphic Organizer

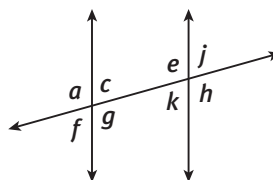
The coach uses a diagram like the one below to show plays to his team. Your teacher will give you tape to recreate these same play lines on your desk or on a piece of paper.



MATH TERMS

A **transversal** is a line that intersects two or more other lines to form eight or more angles.

Now using the tape, add a “slant” route to your diagram and label the angles as seen below.



Coach Toose calls this route the “**transversal**.”

1. On the above diagram, mark each of the eight angles formed by the parallel lines and the transversal as acute or obtuse.
2. Measure angle j on your diagram.
3. Without measuring, predict which other angles are the same size as angle j and list them below.
4. Now measure these angles. Were your predictions correct?
5. What is true about the measures of the remaining angles?

Lesson 16-2

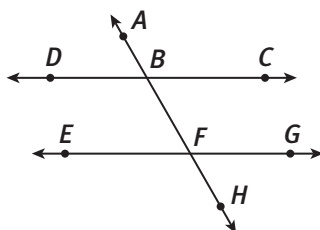
Angles Formed by Parallel Lines

ACTIVITY 16

continued

6. Using the diagram that you made on your desk and your observations in the previous items, what can you say about the measures of the angles formed by two parallel lines cut by a transversal?

In the diagram, $\overline{CD} \parallel \overline{EG}$.



7. Determine whether each pair of angles is congruent or supplementary.
- $\angle ABD$ and $\angle CBH$
 - $\angle ABD$ and $\angle EFH$
 - $\angle DBH$ and $\angle CBF$
 - $\angle ABC$ and $\angle BFG$
 - $\angle CBF$ and $\angle EFB$
8. **Critique the reasoning of others.** In the diagram, $m\angle CBF = (x + 10)^\circ$ and $m\angle EFB = (3x - 54)^\circ$. Students were asked to determine the value of x . One student's solution is shown below. Determine whether or not the solution is correct. If it is correct, explain the reasoning used by the student. If it is incorrect, identify the student's error and explain to the student how to correctly solve the problem.

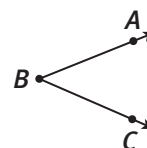
$$\begin{aligned} x + 10 + 3x - 54 &= 180 \\ 4x - 44 &= 180 \\ 4x &= 224 \\ x &= 56 \end{aligned}$$

My Notes

READING MATH

The symbol \parallel is used to indicate parallel lines. $\overline{CD} \parallel \overline{EG}$ is read "line CD is parallel to line EG ."

READING MATH



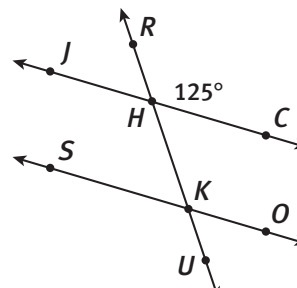
To read this angle, say "angle ABC ," "angle CBA ," or "angle B ."

My Notes

Check Your Understanding

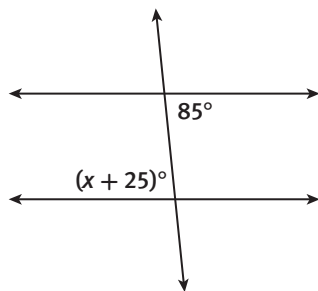
9. In the diagram, $\overline{JC} \parallel \overline{SO}$. Copy and complete the table to find the missing angle measures.

Angle	Measure
$\angle RHC$	125°
$\angle JHK$	
$\angle RHJ$	
$\angle CHK$	
$\angle HKS$	
$\angle SKU$	
$\angle OKU$	
$\angle HKO$	

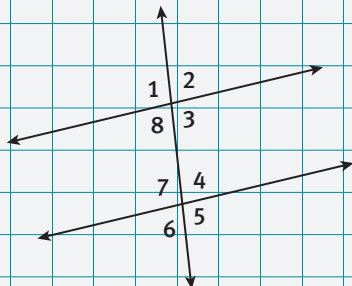
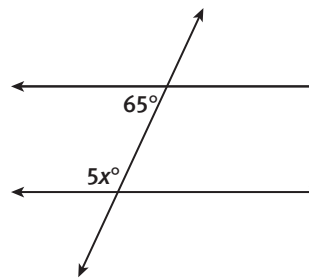


Each diagram shows parallel lines cut by a transversal. Solve for the value of x .

10.



11.



12. Refer to the diagram in the My Notes section.

- What does the term *exterior* mean in everyday language? Give at least two examples.
- Which angles in the figure do you think are exterior angles? Explain.
- What does the term *interior* mean in everyday language? Give at least two examples.
- Which angles in the figure do you think are interior angles? Explain.

Lesson 16-2

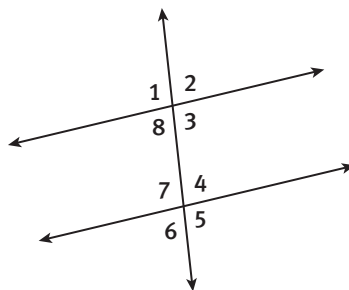
Angles Formed by Parallel Lines

ACTIVITY 16

continued

13. Alternate angles are on opposite sides of the transversal and have a different vertex. There are two pairs of angles in the diagram that are referred to as **alternate exterior angles** and two pairs of angles that are referred to as **alternate interior angles**.

a. Explain what it means for angles to be *alternate exterior angles*.



b. Name the two pairs of alternate exterior angles in the diagram.

c. Explain what it means for angles to be *alternate interior angles*.

d. Name the two pairs of alternate interior angles in the diagram.

14. The above diagram shows a pair of parallel lines cut by a transversal.

a. If the measure of $\angle 2$ is 70° , determine the measures of the other angles.

b. What relationship do you notice about the measures of the alternate exterior angles?

c. What relationship do you notice about the measures of the alternate interior angles?

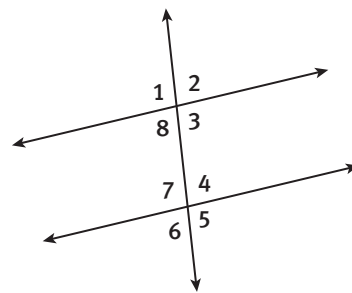
My Notes

ACADEMIC VOCABULARY

As a verb, the word **alternate** means to shift back and forth between one state and another.

My Notes

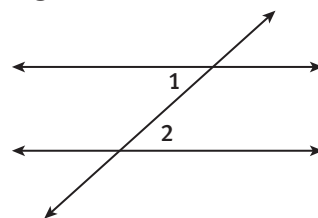
15. Another classification for angle pairs that exist when two lines are cut by a transversal is **corresponding angles**. In the diagram, $\angle 2$ and $\angle 4$ are corresponding.



- a. What do you think is meant by the term *corresponding*?
- b. Name the three other pairs of corresponding angles in the diagram and tell what you notice about the measures of these angles.

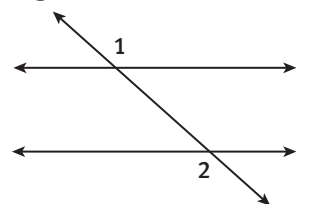
16. In Figure A, two parallel lines are cut by a transversal. The measure of $\angle 1 = 42^\circ$. Find $m\angle 2$ and describe the relationship that helped you determine the measure.

Figure A



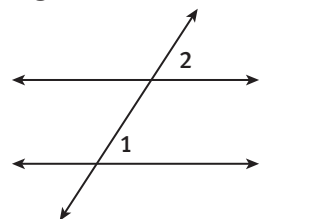
17. In Figure B, two parallel lines are cut by a transversal. The measure of $\angle 1 = 138^\circ$. Find $m\angle 2$ and describe the relationship that helped you determine the measure.

Figure B



18. In Figure C, two parallel lines are cut by a transversal. The measure of $\angle 1 = 57^\circ$. Find $m\angle 2$ and describe the relationship that helped you determine the measure.

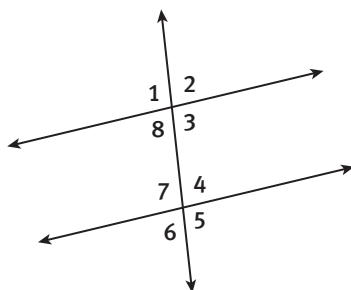
Figure C



Lesson 16-2

Angles Formed by Parallel Lines

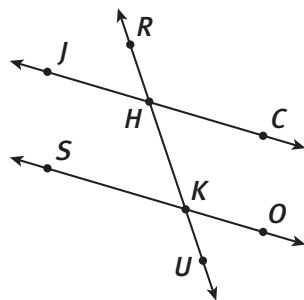
19. Two pairs of **vertical angles** are formed when two lines intersect. They share a vertex but have no common rays. List the pairs of vertical angles in the diagram and tell what you notice about the measures of these angles.



Check Your Understanding

20. Identify each pair of angles as alternate interior, alternate exterior, corresponding, or vertical.

- $\angle RHJ$ and $\angle RKS$
- $\angle CHK$ and $\angle SKH$
- $\angle OKU$ and $\angle HKS$
- $\angle CHK$ and $\angle UKO$
- $\angle RHJ$ and $\angle OKU$



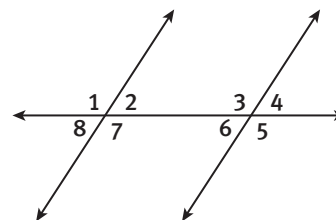
21. Angles ABC and ADF are alternate interior angles. The measure of $\angle ABC = (8x + 4)^\circ$ and the measure of $\angle ADF = (10x - 20)^\circ$. Determine the measure of each of the angles.

My Notes																							

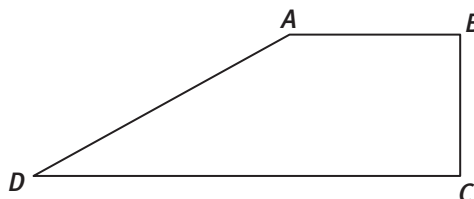
My Notes

LESSON 16-2 PRACTICE

The figure shows a pair of parallel lines that are intersected by a transversal. Use the figure for Items 22–26.



22. Name all pairs of vertical angles in the figure.
23. Name all of the angles in the figure that are supplementary to $\angle 8$.
24. If $m\angle 2 = 57^\circ$, find $m\angle 3$ and $m\angle 4$.
25. If $m\angle 6 = (5x + 1)^\circ$ and $m\angle 8 = (7x - 23)^\circ$, find $m\angle 6$ and $m\angle 8$.
26. Suppose $\angle 9$, which is not shown in the figure, is complementary to $\angle 4$. Given that $m\angle 1 = 153^\circ$, what is $m\angle 9$?
27. Two parallel lines are intersected by a transversal. The transversal forms four right angles with one of the parallel lines. Can you conclude that the transversal forms four right angles with the other parallel line? Justify your answer.
28. **Model with mathematics.** DeMarco is designing a skateboard ramp as shown in the figure. He wants the sides \overline{AB} and \overline{CD} to be parallel to each other. He also wants the measure of $\angle A$ to be five times the measure of $\angle D$. Explain how he can find the correct measures of these two angles.

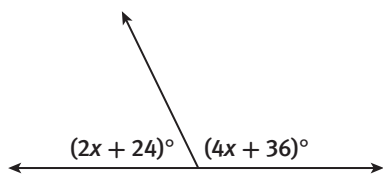


ACTIVITY 16 PRACTICE

Write your answers on notebook paper.
Show your work.

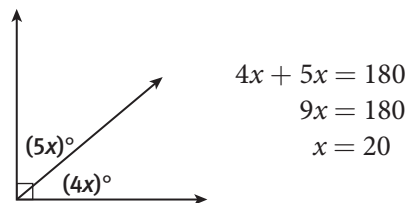
Lesson 16-1

- Are angles with measures of 11° and 89° complementary? Why or why not?
- Can two obtuse angles be supplementary? Explain why or why not.
- What is the measure of an angle that is supplementary to an angle that measures 101° ?
- What is the measure of an angle that is complementary to an angle that measures 75° ?
- The measures of two complementary angles are $(3y - 1)^\circ$ and $(4y + 7)^\circ$.
 - Determine the value of y .
 - Calculate the measure of each of the angles.
- The measures of two supplementary angles are $(\frac{1}{2}x)^\circ$ and $(x + 30)^\circ$.
 - Determine the value of x .
 - Calculate the measure of each of the angles.
- In the figure below, determine the value of x .



- Suppose $\angle A$ is complementary to $\angle B$ and $\angle B$ is supplementary to $\angle C$. If $m\angle A$ is 21° , find $m\angle C$.

- A student determined the value of x as shown. Explain the student's error.

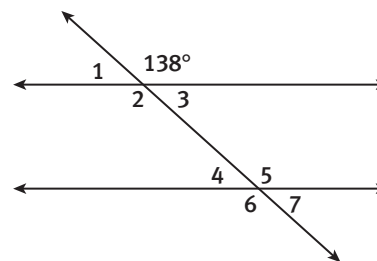


- $\angle 1$ and $\angle 2$ are supplementary. Which of the following statements cannot be true?
 - $\angle 1$ is obtuse and $\angle 2$ is acute.
 - $\angle 1$ and $\angle 2$ are adjacent angles.
 - $\angle 1$ and $\angle 2$ are congruent angles.
 - $\angle 1$ and $\angle 2$ are complementary.
- $\angle A$ and $\angle B$ are complementary angles. The measure of $\angle A$ is 4 times the measure of $\angle B$. Which of these is the measure of $\angle B$?

A. 18°	B. 22.5°
C. 36°	D. 72°

Lesson 16-2

- The diagram below shows parallel lines cut by a transversal. Determine the measures of $\angle 1$ through $\angle 7$.



- Name a pair of alternate interior angles in the above figure.

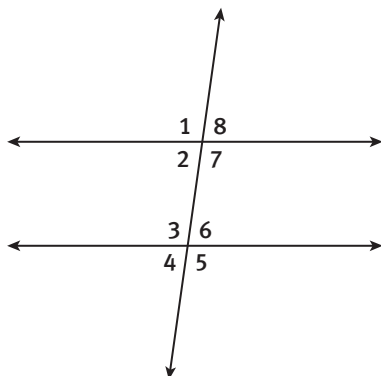
ACTIVITY 16

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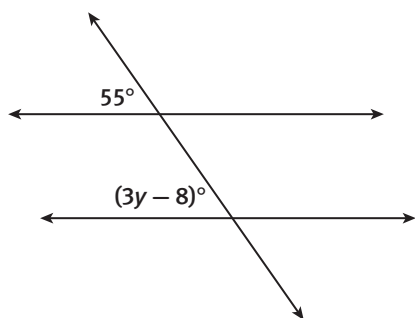
Angle-Pair Relationships

The Winning Angle

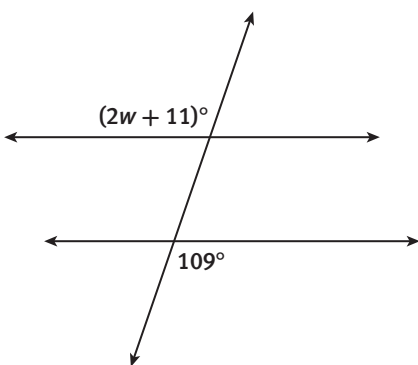
14. Two parallel lines are cut by a transversal as shown below. Find each of the following measures if $m\angle 2 = 82^\circ$. Explain your answers.



- a. $m\angle 6$ b. $m\angle 7$
 c. $m\angle 4$ d. $m\angle 8$
15. The figure shows parallel lines cut by a transversal. Find the value of y .



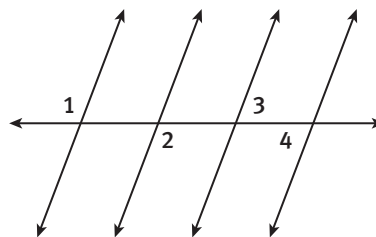
16. The figure shows parallel lines cut by a transversal. Find the value of w .



Draw and label a figure to match each statement.

17. Parallel lines are cut by a transversal and a pair of alternate interior angles are right angles.
 18. Parallel lines are cut by a transversal and a pair of corresponding angles are complementary.

19. The figure shows several parking spaces at a mall. The parking spaces were created by drawing four parallel lines and a transversal.



- a. If $m\angle 1 = 111^\circ$, find $m\angle 4$.
 b. If $m\angle 2 = (3x - 15)^\circ$ and $m\angle 3 = (2x - 30)^\circ$, find the value of x .

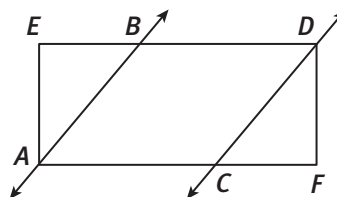
Determine whether each statement is always, sometimes, or never true.

20. When two parallel lines are intersected by a transversal, all of the corresponding angles are right angles.
 21. When two parallel lines are intersected by a transversal, every pair of angles are either congruent or supplementary.
 22. When two parallel lines are intersected by a transversal, there is one pair of alternate exterior angles that are not congruent.
 23. If $\angle Q$ and $\angle R$ are vertical angles, then $\angle Q$ is congruent to $\angle R$.

MATHEMATICAL PRACTICES

Make Sense of Problems

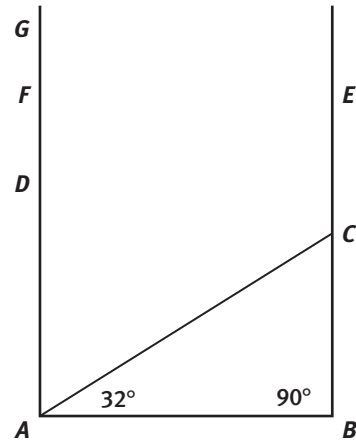
24. The figure shows rectangle $EDFA$. The opposite sides of the rectangle are parallel. Also, $\overline{AB} \parallel \overline{CD}$. If $m\angle EBA = 50^\circ$, is it possible to determine $m\angle DCF$? If so, explain how. If not, explain why not.



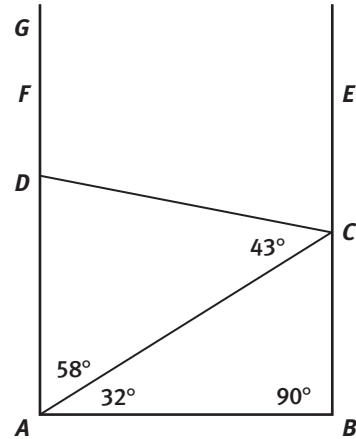
My Notes

During the game, before each triangle appears, the player must select the measure (in degrees) of one angle in the triangle.

2. In one game, Chip's first triangle with a 90° angle came to rest and displayed the measure of $\angle CAB$ to be 32° .
 - a. Determine the measure of $\angle CAD$.
 - b. Explain why the measure of $\angle CAD$ must equal the measure of $\angle ACB$.



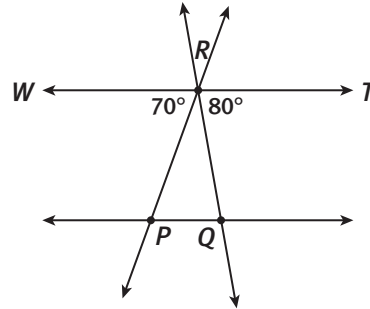
3. When $\triangle ACD$ came down the chute, Chip selected the 58° angle and the computer selected the 43° angle. Determine the measure of each of the following angles.
 - a. $\angle ECD$
 - b. $\angle CDA$
 - c. $\angle FDC$



4. List the measures of the three angles in $\triangle ACD$. Then list the measures of the three nonoverlapping angles whose vertex is at C. How do the two lists compare?

My Notes

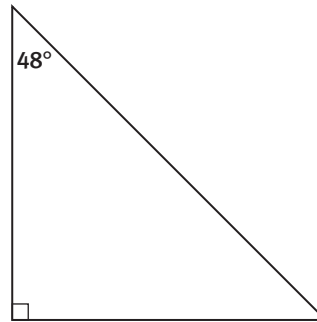
8. In the diagram, $\overline{WT} \parallel \overline{PQ}$.



a. Use what you know about parallel lines and transversals to determine the measures of $\angle RPQ$ and $\angle RQP$.

b. Explain how this diagram supports your conjecture in Item 7.

9. Determine the measure of the unknown angle in the triangle below.



My Notes

LESSON 17-1 PRACTICE

In Items 14 and 15, the measures of two angles of a triangle are given. Find the measure of the third angle of the triangle.

14. $23^\circ, 78^\circ$

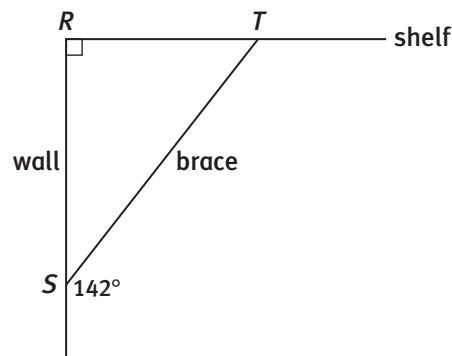
15. $105^\circ, 40^\circ$

16. The measures of the three angles in a triangle are $(4x)^\circ$, $(3x - 3)^\circ$, and $(5x + 3)^\circ$. Write an equation, solve for x , and determine the measure of each angle.

17. In $\triangle ABC$, $\angle A$ and $\angle B$ have the same measure. The measure of $\angle C$ is twice the measure of $\angle A$. Find the measures of the angles in the triangle.

18. Eliana claimed that she drew a triangle with two right angles. Draw a sketch of such a triangle or explain why it is not possible.

19. **Model with mathematics.** Brian is building a brace for a shelf. The figure shows the plans for the brace.



- Based on the information given in the figure, is it possible to determine the three angles of $\triangle RST$? If so, find the measures. If not, explain why not.
- Brian wants to know the measure of the obtuse angle formed by the brace and the shelf. Explain how he can determine this.

My Notes

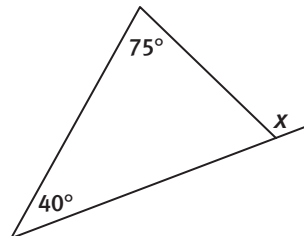
- b. For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles**. Name the two remote interior angles for each exterior angle of $\triangle SBM$.

Exterior Angle	Two Remote Interior Angles
$\angle SMT$	
$\angle RSB$	
$\angle QBM$	

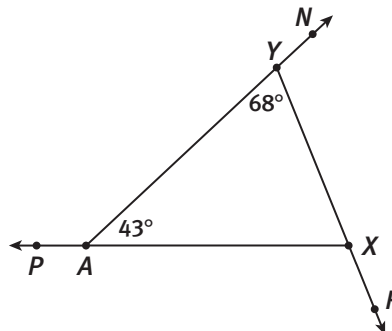
- c. Chip claims that there is a relationship between the measure of an exterior angle and its remote interior angles. Examine the measures of the exterior angles and the measures of their corresponding remote interior angles to write a conjecture about their relationship.

Check Your Understanding

3. Determine the value of x .



4. Determine the measure of each of the exterior angles of $\triangle YAX$.



5. The measures of two interior angles of a triangle are 75° and 65° . Determine the measure of the largest exterior angle.

Lesson 17-2

Exterior Angles and Angles in Quadrilaterals

ACTIVITY 17

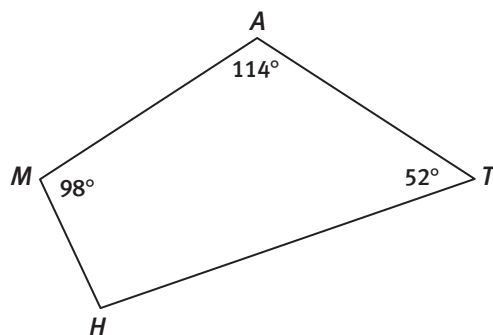
continued

6. Now consider quadrilaterals.
- Draw a **diagonal** from *one* vertex in each quadrilateral.

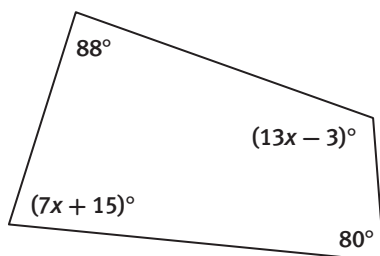


- Construct viable arguments.** What is the sum of the measures of the interior angles in any quadrilateral? Explain your reasoning.

7. Find the unknown angle measure in quadrilateral *MATH*.



8. Determine the value of x in the quadrilateral.



My Notes

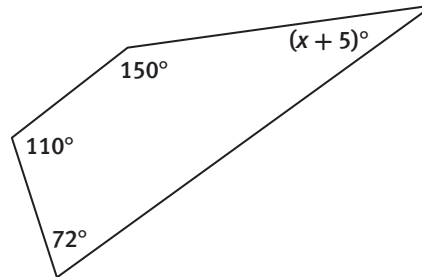
MATH TERMS

A **diagonal** of a polygon is a line segment connecting two nonconsecutive vertices.

My Notes

Check Your Understanding

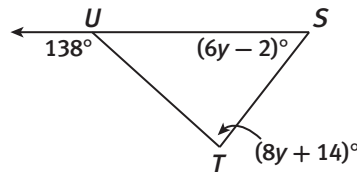
9. Determine the value of x .



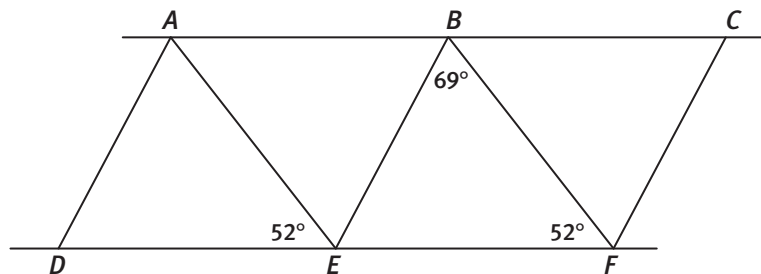
10. In quadrilateral $RSTU$, all of the angles of the quadrilateral are congruent. What can you conclude about the angles? What can you conclude about the quadrilateral?

LESSON 17-2 PRACTICE

11. Determine the value of y . Then find $m\angle S$ and $m\angle T$.



12. A portion of a truss bridge is shown in the figure. Explain how to determine the measure of $\angle AEB$.



13. A quadrilateral contains angles that measure 47° , 102° , and 174° . What is the measure of the fourth angle of the quadrilateral?
14. In quadrilateral $DEFG$, $\angle D$ is a right angle. The measure of $\angle E$ is half the measure of $\angle D$. The measure of $\angle F$ is three times the measure of $\angle E$. Sketch the quadrilateral and label the measure of each angle.
15. **Make use of structure.** Can an exterior angle of a triangle ever be congruent to one of its remote interior angles? Justify your answer.

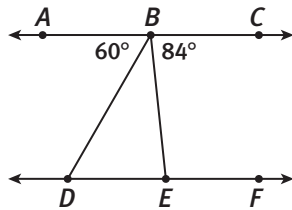
ACTIVITY 17 PRACTICE

Write your answers on notebook paper.

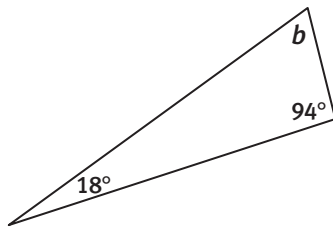
Show your work.

Lesson 17-1

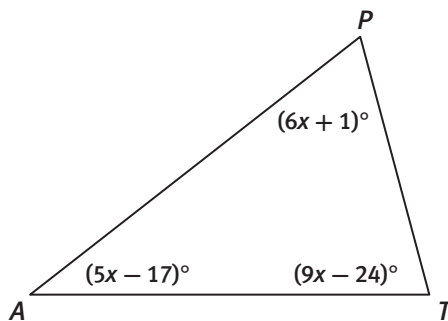
- Two angles of a triangle measure 32° and 70° . Find the measure of the third angle.
- In the diagram below, $\overleftrightarrow{AC} \parallel \overleftrightarrow{DF}$. Determine the measure of each of the angles in $\triangle BDE$.



- Determine the value of b .

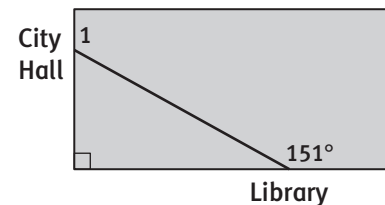


- Write an equation, solve for x , and determine the measure of each angle in $\triangle PAT$.



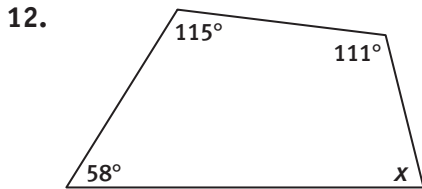
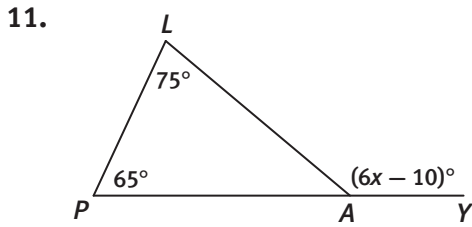
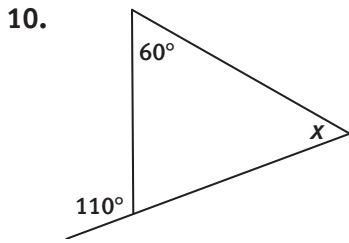
- The measures of the three interior angles of a triangle are 85° , 20° , and 75° . Determine the measures of the three exterior angles.

- The measures of two angles of a triangle are 38° and 47° . What is the measure of the third angle?
 - 85°
 - 95°
 - 133°
 - 142°
- In $\triangle DEF$, the measure of $\angle D = (3x - 6)^\circ$, the measure of $\angle E = (3x - 6)^\circ$, and the measure of $\angle F = (2x)^\circ$. Which of the following is the measure of $\angle F$?
 - 24°
 - 46°
 - 48°
 - 66°
- In $\triangle PQR$, $\angle P$ is an obtuse angle. Which of the following statements about the triangle must be true?
 - The other two angles must be congruent.
 - The other two angles must be acute angles.
 - One of the other two angles could be a right angle.
 - One of the other two angles could also be an obtuse angle.
- The figure shows a rectangular lawn at a civic center. Over time, people have cut across the lawn to walk from the library to city hall and made a straight path in the lawn, as shown. What is the measure of $\angle 1$ in the figure?

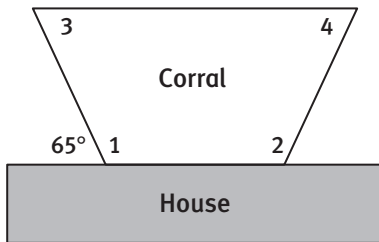


Lesson 17-2

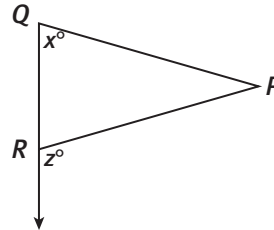
In Items 10–12, determine the value of x .



13. Determine the measure of each angle in quadrilateral $DEFG$ with $m\angle D = (12x - 4)^\circ$, $m\angle E = (18x + 4)^\circ$, $m\angle F = (15x + 10)^\circ$, and $m\angle G = (5x)^\circ$.
14. The figure shows a plan for a corral in the shape of a trapezoid. One side of the corral is formed by a house and the other three sides are formed by a fence. Given that $\angle 1$ and $\angle 2$ are congruent, and that $\angle 3$ and $\angle 4$ are congruent, find the measures of the four angles.



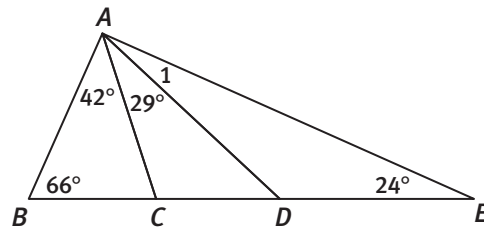
15. In quadrilateral $ABCD$, $m\angle A = (5x - 5)^\circ$, $m\angle B = (9x)^\circ$, $m\angle C = (12x + 15)^\circ$, and $m\angle D = (15x - 60)^\circ$. Which angle has the greatest measure?
- A. $\angle A$ B. $\angle B$
 C. $\angle C$ D. $\angle D$
16. Which expression represents the measure of $\angle P$?



- A. $(z + x)^\circ$ B. $(z - x)^\circ$
 C. $(x - z)^\circ$ D. z°
17. Sketch a quadrilateral that contains a 50° angle and a 170° angle. Give possible measures for the other two angles.

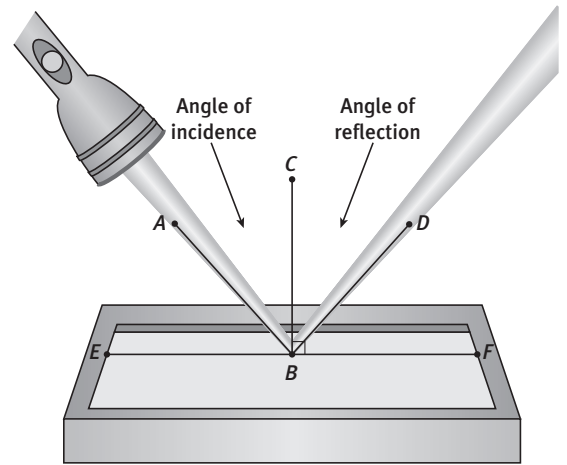
MATHEMATICAL PRACTICES
Critique the Reasoning of Others

18. Nick and LaToya are painting a backdrop of a mountain for a stage set. A sketch for the backdrop is shown below. Nick says there is not enough information to determine the measure of $\angle 1$. LaToya says there is enough information to determine this angle measure. Who is correct? Explain.

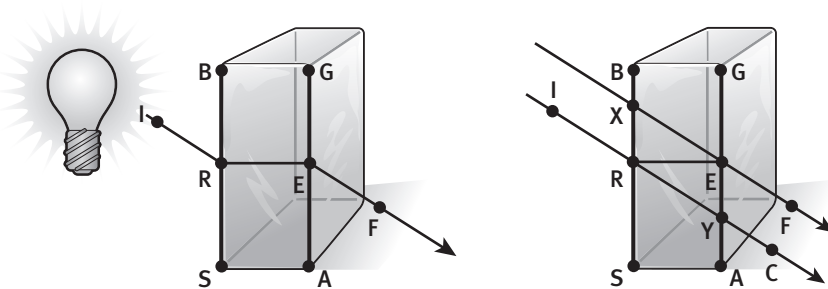


A beam of light and a mirror can be used to study the behavior of light. When light hits the mirror it is reflected so that the angle of incidence and the angle of reflection are congruent.

1. Name a pair of nonadjacent complementary angles in the diagram.
2. Name a pair of adjacent supplementary angles in the diagram.
3. In the diagram, $m\angle CBD = (4x)^\circ$ and $m\angle FBD = (3x - 1)^\circ$.
 - a. Solve for the value of x .
 - b. Determine $m\angle CBD$, $m\angle FBD$, and $m\angle DBE$.



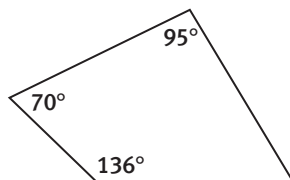
Light rays are bent as they pass through glass. Since a block of glass is a rectangular prism, the opposite sides are parallel and a ray is bent the same amount entering the piece of glass as exiting the glass.



This causes \overline{XF} to be parallel to \overline{RY} , as shown.

4. If the measure of $\angle YEX$ is 130° , determine the measure of each of the following angles. Explain how you arrived at your answer.
 - a. $\angle BXE$
 - b. $\angle GEF$
 - c. $\angle SRY$
5. If $m\angle CYA = (5x)^\circ$ and $m\angle SRY = (6x - 10)^\circ$, then the value of x is _____.
6. If $m\angle XRE = 90^\circ$ and $m\angle REX = 30^\circ$, then $m\angle RXE =$ _____. Explain how you arrived at your answer.

- The measures of the angles of a triangle are $(2x)^\circ$, $(x + 14)^\circ$, and $(x - 38)^\circ$. Determine the value of x and the measures of each of the three angles.
- One of the quadrilaterals in a mural design is shown below. Determine the measure of the missing angle.



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3a-b, 4a-c, 5, 6, 7, 8)	<ul style="list-style-type: none"> Clear and accurate understanding of angle relationships, and finding angle measures in a triangle and quadrilateral. 	<ul style="list-style-type: none"> An understanding of angle relationships and finding angle measures in a triangle and quadrilateral. 	<ul style="list-style-type: none"> Partial understanding of angle relationships and finding angle measures in a triangle and quadrilateral. 	<ul style="list-style-type: none"> Little or no understanding of angle relationships and finding angle measures in a triangle and quadrilateral.
Problem Solving (Items 3a-b, 4a-c, 5, 6, 7, 8)	<ul style="list-style-type: none"> Interpreting a problem accurately in order to find missing angle measures. 	<ul style="list-style-type: none"> Interpreting a problem to find missing angle measures. 	<ul style="list-style-type: none"> Difficulty interpreting a problem to find missing angle measures. 	<ul style="list-style-type: none"> Incorrect or incomplete interpretation of a problem.
Mathematical Modeling / Representations (Items 1, 2, 3a-b, 4a-c, 5, 6, 7, 8)	<ul style="list-style-type: none"> Accurately interpreting figures in order to characterize angle pairs and find angle measures. 	<ul style="list-style-type: none"> Interpreting figures in order to find angle pairs and find missing angle measures. 	<ul style="list-style-type: none"> Difficulty interpreting figures in order to find angle pairs and find missing angle measures. 	<ul style="list-style-type: none"> Incorrectly interpreting figures in order to find angle pairs and find missing angle measures.
Reasoning and Communication (Items 4a-c, 6)	<ul style="list-style-type: none"> Precise use of appropriate terms to describe finding angle measures. 	<ul style="list-style-type: none"> An adequate description of finding of missing angle measures. 	<ul style="list-style-type: none"> A confusing description of finding missing angle measures. 	<ul style="list-style-type: none"> An inaccurate description of finding missing angle measures.

Move It!

Lesson 18-1 What Is a Transformation?

Learning Targets:

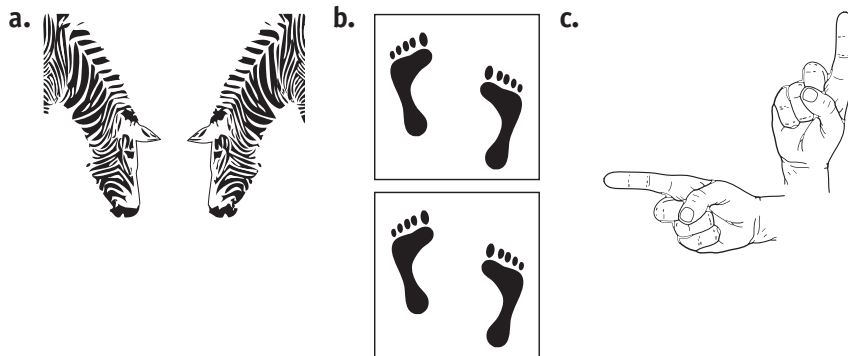
- Recognize rotations, reflections, and translations in physical models.
- Explore rigid transformations of figures.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Vocabulary Organizer, Paraphrasing

A **transformation**, such as a flip, slide, or turn, changes the position of a figure. Many graphic artists rely on graphic design software to transform images to create logos or promotional materials.

A **preimage** is a figure before it has been transformed and the **image** is its position after the transformation. You can tell whether a figure has been transformed if the preimage can be moved to coincide with its image.

1. Each set of pictures below shows the preimage and image of some familiar objects. Use the terms *flip*, *slide*, and *turn* to describe what transformation will make the two objects coincide.



2. Make a conjecture about the preimage and image of a transformed object based on your observations of the pictures.

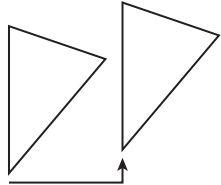
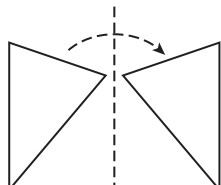
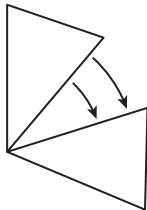
My Notes

ACADEMIC VOCABULARY




The word **transform** means “to change.”

My Notes

3. Make use of structure. The table below shows the proper name for transformations and the corresponding definition. Match each transformation with the words *flip*, *slide*, and *turn*.

Transformation	Definition	Example
Translation	Each point of a figure is moved the same distance in the same direction.	
Reflection	Each point of a figure is reflected over a line, creating a mirror image.	 line of reflection
Rotation	Each point of a figure is rotated through a given angle in a given direction around a fixed point.	

4. For each capital letter shown below, visualize the movement the letter takes while being transformed. Identify the transformation by its proper name.

a.  b.  c. 

My Notes

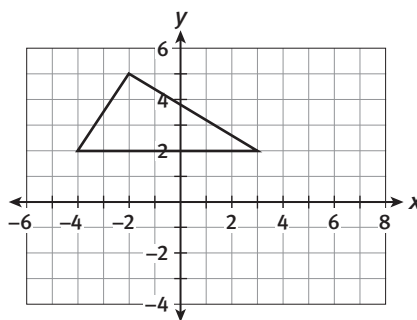
Learning Targets:

- Determine the effect of translations on two-dimensional figures using coordinates.
- Represent and interpret translations involving words, coordinates, and symbols.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Create Representations, Identify a Subtask, Interactive Word Wall

A **translation** changes only a figure's position. A verbal description of a translation includes words such as *right*, *left*, *up*, and *down*.

1. Consider the triangle shown on the coordinate plane.



Coordinates of Triangle	Coordinates of Image

- a. Record the coordinates of the vertices of the triangle in the table.
- b. Translate the triangle down 2 units and right 5 units. Graph the translation.
- c. Record the coordinates of the vertices of the **image** in the table.

A **symbolic representation** of a transformation is an algebraic way to show the changes to the x - and y -coordinates of the vertices of the original figure, or **preimage**.

- d. **Make use of structure.** Use the information in the table to help you complete the symbolic representation for the translated triangle:
 $(x, y) \rightarrow (x + 5, y - 2)$

MATH TERMS

A **symbolic representation** of a transformation is an algebraic way to show the changes to the x - and y -coordinates of the vertices of the original figure, or preimage.

A **preimage** is a figure before it has been transformed and the **image** is its position after the transformation.

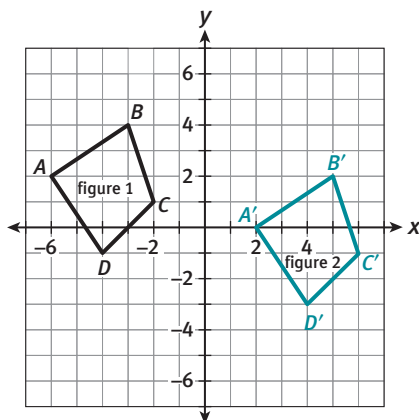
Lesson 18-2

Translations and Coordinates

ACTIVITY 18

continued

2. Figure 2 is the image of Figure 1 after a translation, as shown in the coordinate plane.



- a. Record the coordinates of the vertices of the preimage and image.

Figure 1: Preimage		Figure 2: Image	
A		A'	
B		B'	
C		C'	
D		D'	

- b. **Make sense of problems.** Refer to the table and graph.
 Was the figure translated up or down? _____ By how much?

 Was the figure translated to the left or right? _____ By how
 much? _____
- c. Write a verbal description to describe the translation.
- d. Describe the translation using a symbolic representation.
 symbolic representation: $(x, y) \rightarrow (x + 8, y - 2)$

My Notes

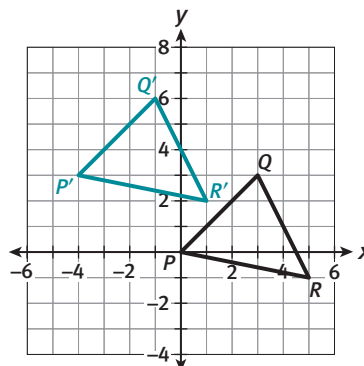
READING MATH

A prime symbol (') is placed after the letter for the original point to show that the new point is its image.

Example: Point A' is the image of point A.

My Notes

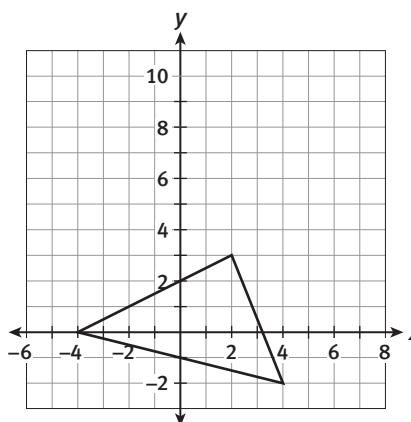
3. The coordinate plane shows $\triangle P'Q'R'$ after $\triangle PQR$ undergoes a translation.



- Write a verbal description to describe the translation.
- Describe the translation using a symbolic representation.
symbolic representation: $(x, y) \rightarrow (x - 4, y + 3)$

Check Your Understanding

4. The triangle shown on the coordinate plane is translated according to the following symbolic representation: $(x, y) \rightarrow (x + 1, y + 6)$.



- Describe how the symbolic representation can be used to determine if the triangle is translated left or right, and up or down.
 - Write a verbal description of the translation.
 - Attend to precision.** Sketch the image of the triangle according to the symbolic representation.
5. **Construct viable arguments.** Explain how the change in the coordinates of a translated point is related to the symbolic representation.

My Notes

MATH TERMS

Equidistant means to be the same distance from a given point or line.

CONNECT TO AP

Translations and reflections of figures in the coordinate plane are preparing you to successfully translate and reflect graphs of functions. This is a helpful tool for visualizing and setting up the graphs for many problems you will solve in calculus.

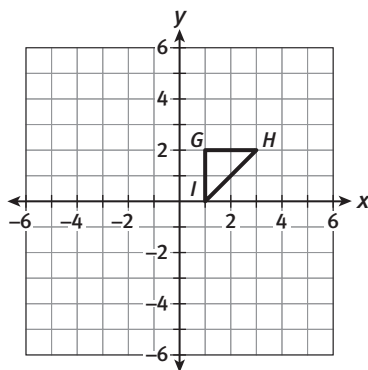
Learning Targets:

- Determine the effect of reflections on two-dimensional figures using coordinates.
- Represent and interpret reflections involving words, coordinates, and symbols.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Interactive Word Wall, Construct an Argument, Summarizing

To perform a **reflection**, each point of a preimage is copied on the opposite side of the line of reflection and remains **equidistant** from the line.

1. $\triangle GHI$ is shown on the coordinate plane below.



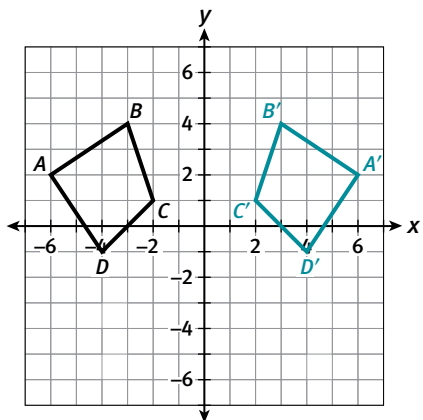
Coordinates of Triangle	Coordinates of Image

- a. Record the coordinates of the vertices of $\triangle GHI$ in the table.
- b. Sketch the reflection of $\triangle GHI$ over the x -axis.
- c. Record the coordinates of the vertices of the image $\triangle G'H'I'$ in the table.

The symbolic representation for this transformation is $(x, y) \rightarrow (x, -y)$.

- d. Explain how the change in the coordinates of the vertices is related to the symbolic representation for this transformation.

2. Figure 2 is the image of figure 1 after a reflection, as shown in the coordinate plane.

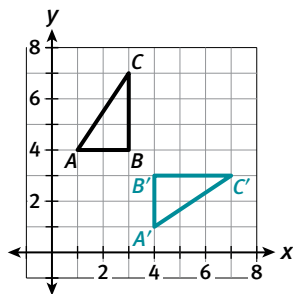


Preimage: Figure 1		Image: Figure 2	
A		A'	
B		B'	
C		C'	
D		D'	

- Record the coordinates of the vertices of the preimage and image.
- The line across which an object is reflected is called the **line of reflection**. Identify the line of reflection in the transformation of Figure 1.
- A verbal description of a reflection includes the line of reflection. Write a verbal description of the reflection.
- Describe the reflection using a symbolic representation.
Symbolic Representation: $(x, y) \rightarrow$

Check Your Understanding

3. Triangle ABC and its reflected image are shown on the coordinate plane.



Coordinates of $\triangle ABC$		Coordinates of $\triangle A'B'C'$	
A		A'	
B		B'	
C		C'	

- Complete the table.
- Identify the line of reflection. Write a verbal description of the transformation.
- Describe the reflection using symbolic representation.

My Notes

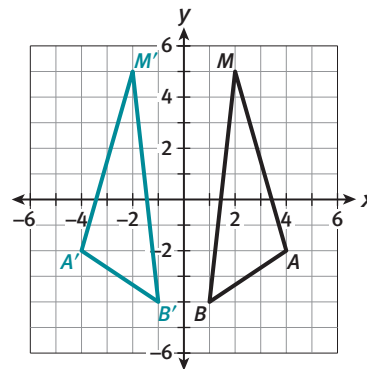
A large grid area for taking notes, with the title 'My Notes' at the top left of the grid.

My Notes

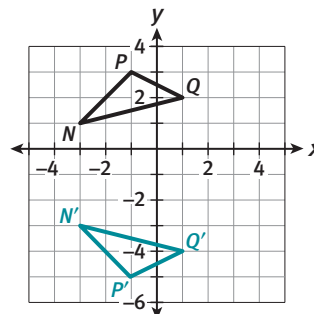
4. **Express regularity in repeated reasoning.** Write a summary statement describing which coordinate stays the same when a figure is reflected over the y -axis.
5. Modify your statement in Item 4 describing which coordinate stays the same when a figure is reflected over the x -axis.

LESSON 18-3 PRACTICE

6. Triangle BAM is shown along with its image $\triangle B'A'M'$ on the coordinate plane below.



- a. Write a verbal description of the reflection.
 - b. Describe the reflection using symbolic representation.
7. Suppose $\triangle CDF$, whose vertices have coordinates $C(-2, 1)$, $D(4, 5)$, and $F(5, 3)$, is reflected over the x -axis.
 - a. Explain a way to determine the coordinates of the vertices of $\triangle C'D'F'$.
 - b. Find the coordinates of $\triangle C'D'F'$.
 8. **Critique the reasoning of others.** Filip claims $\triangle N'P'Q'$ is a reflection of $\triangle NPQ$ over the x -axis. Is Filip correct? Justify your answer.



Learning Targets:

- Determine the effect of rotations on two-dimensional figures using coordinates.
- Represent and interpret rotations involving words, coordinates, and symbols.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Look for a Pattern, Interactive Word Wall

A **rotation** is a transformation that describes the motion of a figure about a fixed point. To perform a rotation, each point of the preimage travels along a circle the same number of degrees.

1. The point $(3, 1)$ is rotated in a counterclockwise direction about the origin 90° , 180° , and 270° .

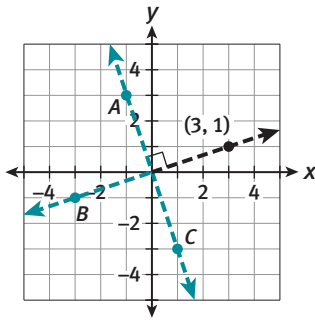


Image Point	Coordinates	Measure of Angle of Rotation
A		
B		
C		

- a. Write the coordinates of each image point A, B, and C in the table.
- b. Complete the table by giving the angle of rotation for each image point.
- c. **Reason abstractly.** Describe in your own words why the origin is the **center of rotation** in this rotation transformation.
- d. **Construct viable arguments.** Make a conjecture about the changes of the x - and y -coordinates when a point is rotated counterclockwise 90° , 180° , and 270° about the origin.

- e. What are the coordinates of the point $(3, 1)$ after a 360° rotation about the origin? Explain your answer.

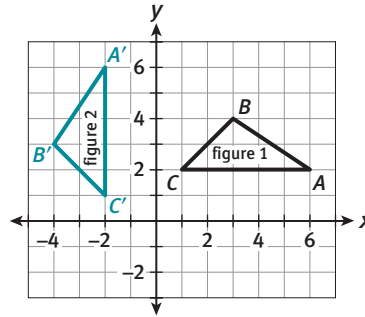
My Notes

MATH TIP

If the direction of a rotation is counterclockwise, the measure of the angle of rotation is given as a positive value. If the direction of a rotation is clockwise, the measure of the angle of rotation is given as a negative value.

My Notes

2. Figure 2 is a 90° counterclockwise rotation about the origin of figure 1.



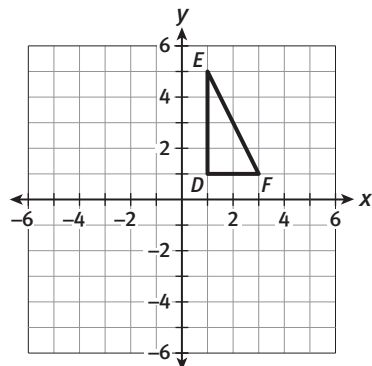
Determine the coordinates of the vertices for each figure.

Preimage: Figure 1		Image: Figure 2	
<i>A</i>		<i>A'</i>	
<i>B</i>		<i>B'</i>	
<i>C</i>		<i>C'</i>	

3. **Make sense of problems.** Complete the summary statement:

When a figure in Quadrant I of the coordinate plane is rotated 90° counterclockwise about the origin, its image is located in Quadrant _____.

4. Use appropriate tools strategically. Consider $\triangle DEF$ shown on the coordinate plane.



- Trace $\triangle DEF$ and the positive x -axis on a piece of tracing paper. Label the vertices and the axis.
- Rotate the triangle 90° counterclockwise by aligning the origin and rotating the tracing paper until the positive x -axis coincides with the positive y -axis.
- Record the coordinates of the vertices of the image in the table.

Preimage	$D(1, 1)$	$E(1, 5)$	$F(3, 1)$
Image	$D'(\quad)$	$E'(\quad)$	$F'(\quad)$

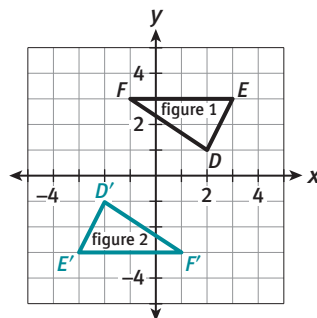
- Sketch $\triangle D'E'F'$ on the coordinate plane above.

Check Your Understanding

5. **Make use of structure.** Use your results from Items 1, 2, and 3 to write a symbolic representation for a 90° counterclockwise rotation.
 $(x, y) \rightarrow (\quad, \quad)$

6. **Critique the reasoning of others.** Sven recognized the 180° rotation of $\triangle DEF$ about the origin in the coordinate plane and determined the symbolic representation to be $(x, y) \rightarrow (-x, -y)$.

Determine whether the symbolic representation is correct. Justify your answer.



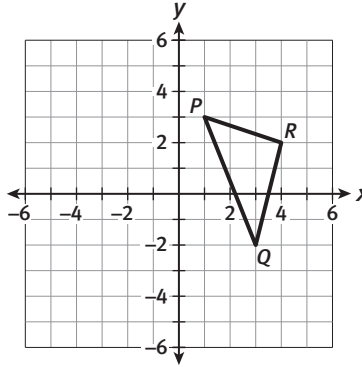
7. A point with coordinates (x, y) is rotated 360° in a counterclockwise direction about the origin. Write the symbolic representation for this transformation:
 $(x, y) \rightarrow (\quad, \quad)$.

What does the symbolic representation indicate?

My Notes

LESSON 18-4 PRACTICE

8. Triangle PQR with vertices $P(1, 3)$, $Q(3, -2)$, and $R(4, 2)$ is shown on the coordinate plane. Graph each given rotation about the origin.
 - a. 90° counterclockwise
 - b. 180° counterclockwise



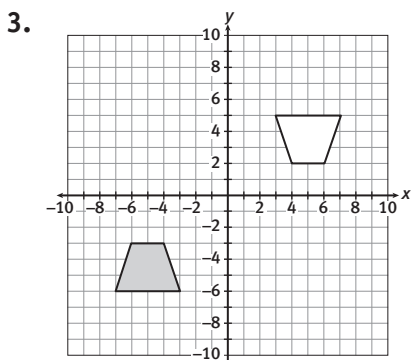
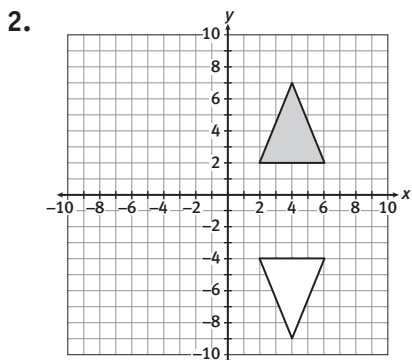
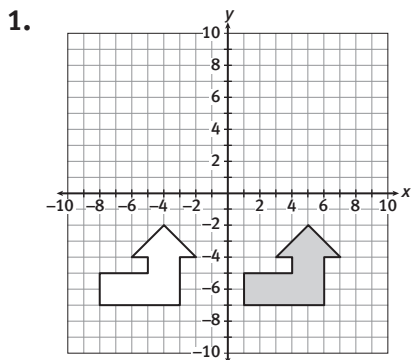
9. The preimage of point A is located at $(-1, 5)$. What are the coordinates of the image, A' , after a 270° counterclockwise rotation?
10. Complete the summary statements:
 - a. When a figure in Quadrant I of the coordinate plane is rotated 180° counterclockwise about the origin, its image is located in Quadrant _____.
 - b. When a figure in Quadrant I of the coordinate plane is rotated 270° counterclockwise about the origin, its image is located in Quadrant _____.
11. **Reason quantitatively.** Use your answer from Item 9 to write a conjecture about the symbolic representation for a 270° counterclockwise rotation.
12. Draw a figure on a coordinate plane. Rotate the figure counterclockwise 270° about the origin. How does your drawing confirm your conjecture in Item 11?

ACTIVITY 18 PRACTICE

Write your answers on notebook paper.
Show your work.

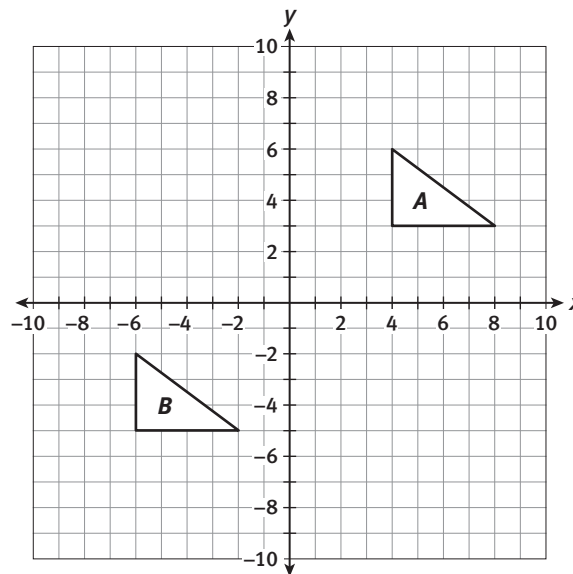
Lesson 18-1

For Items 1–3, the shaded figure is the preimage and the unshaded figure is the image. Identify the single transformation that will make the figures coincide.

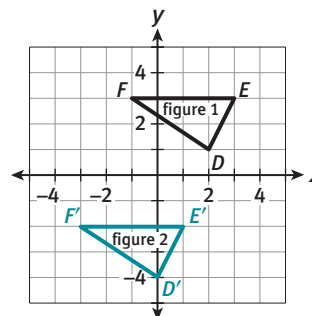


Lesson 18-2

4. Figure B is the image of figure A after a transformation, as shown in the coordinate plane.



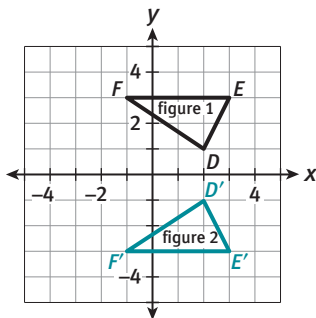
- a. Write a verbal description of the transformation.
 - b. Write a symbolic representation of the transformation.
5. The vertices of $\triangle MOV$ are located at $M(-2, -2)$, $O(4, -2)$, and $V(4, 3)$. Determine the coordinates of the vertices of the image after $\triangle MOV$ is translated 3 units up and 2 units to the right.
6. Which symbolic representation describes the transformation shown on the coordinate plane?



- A. $(x, y) \rightarrow (x + 2, y - 5)$
- B. $(x, y) \rightarrow (x - 2, y - 5)$
- C. $(x, y) \rightarrow (x - 2, y + 5)$
- D. $(x, y) \rightarrow (x + 2, y + 5)$

Lesson 18-3

7. The vertices of $\triangle QRS$ are located at $Q(2, 2)$, $R(-4, 2)$, and $S(-4, -4)$. Determine the coordinates of the vertices of each image of $\triangle QRS$ after the following transformations are performed:
 - a. $\triangle QRS$ is reflected over the x -axis.
 - b. $\triangle QRS$ is reflected over the y -axis.
8. Triangle FED and its transformed image is shown on the coordinate plane.



- a. Identify the line of reflection.
- b. Write a verbal description of the transformation.
- c. Write a symbolic representation of the transformation.

Lesson 18-4

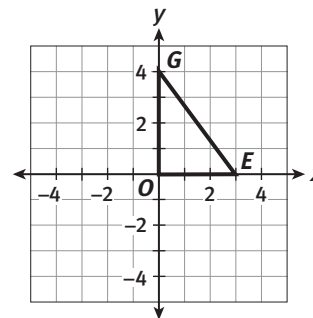
9. The vertices of $\triangle XYZ$ are located at $X(-2, -2)$, $Y(4, -2)$, and $Z(4, 3)$. Determine the coordinates of the vertices of each image of $\triangle XYZ$ after the following transformations are performed:
 - a. $\triangle XYZ$ is rotated 90° counterclockwise about the origin.
 - b. $\triangle XYZ$ is rotated 180° about the origin.
10. The preimage of point B is located at $(-1, 4)$. Determine the coordinates of the image, B' , for each counterclockwise rotation.
 - a. 90°
 - b. 180°
 - c. 270°

11. Triangle ABC has vertices $A(2, 4)$, $B(5, 7)$, and $C(-1, 5)$. If $\triangle ABC$ is rotated 270° counterclockwise about the origin, in what quadrant(s) would you find the image of $\triangle ABC$?
 - A. Quadrant I
 - B. Quadrant III
 - C. Quadrants II and III
 - D. Quadrants I and IV

MATHEMATICAL PRACTICES

Make Use of Structure

12. Determine the coordinates of the vertices for each image of $\triangle GEO$ after each of the following transformations is performed.



- a. Translate $\triangle GEO$ 2 units to the left and reflect over the x -axis.
 - b. Reflect $\triangle GEO$ over the x -axis and translate 2 units to the left.
13. Does the order in which multiple transformations, such as rotations, reflections, and translations, are performed on a preimage have an effect on the image?

All the Right Moves

Lesson 19-1 Properties of Transformations

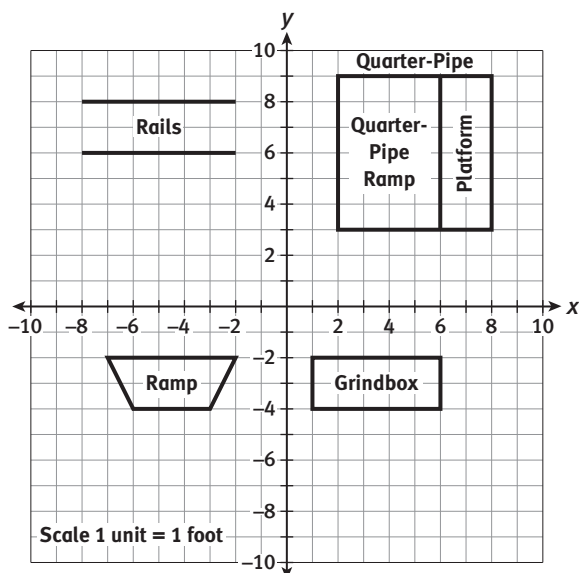
Learning Targets:

- Explore properties of translations, rotations, and reflections on two-dimensional figures.
- Explore congruency of transformed figures.

SUGGESTED LEARNING STRATEGIES: Visualization, Identify a Subtask, Create Representations, Critique Reasoning, Predict and Confirm

Skip and Kate are designing a skateboard park for their neighborhood. They want to include rails, a grindbox, a quarter-pipe, and a ramp. They are deciding where to place the equipment. Kate sketches her plan for the layout on a coordinate plane.

Using the origin as the center of their park, Kate sketched figures to represent the equipment on the coordinate plane, as shown below.



Kate uses the layout on the coordinate plane to determine the dimensions and the area of each figure.

- 1. Model with mathematics.** Use the scale on Kate's layout to complete the table of dimensions for each piece of equipment.

Equipment	Base (ft)	Height (ft)	Area (ft ²)
Quarter-Pipe (ramp and platform)			
Ramp	base 1: base 2:		
Grindbox			

My Notes

MATH TIP

The coordinates of the origin on a coordinate plane are (0, 0).

MATH TIP

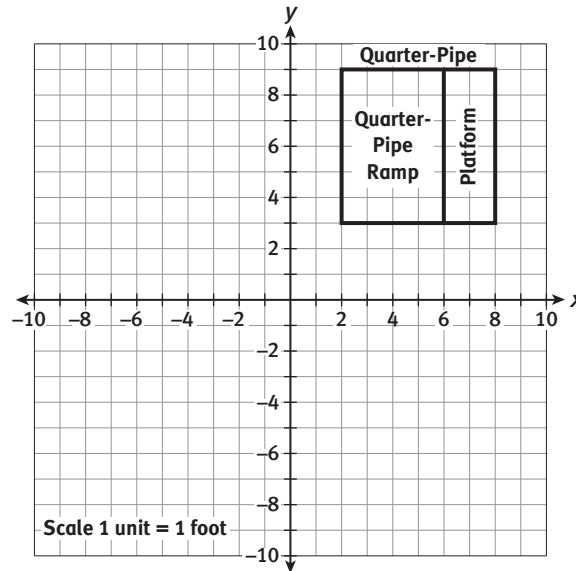
The area of a trapezoid can be found using the formula

$\text{Area} = \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the bases.

My Notes

Skip reviewed Kate's plan for the skateboarding park. To improve the layout, Skip suggested transformations for each piece of equipment as described.

2. The original placement of the quarter-pipe is shown on the coordinate plane.



- a. Reflect the figure representing the quarter-pipe ramp and platform over the y -axis. Label each vertex of the image with an ordered pair.
- b. Determine the dimensions of the image, in feet.
- c. Compare the areas of the original figure and the image.
- d. Explain why the image is congruent to the original figure.

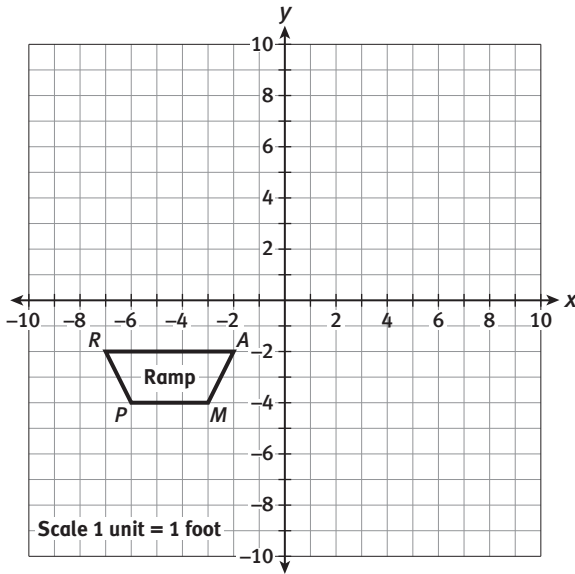
Lesson 19-1

Properties of Transformations

ACTIVITY 19

continued

3. The original placement of the ramp is shown on the coordinate plane.

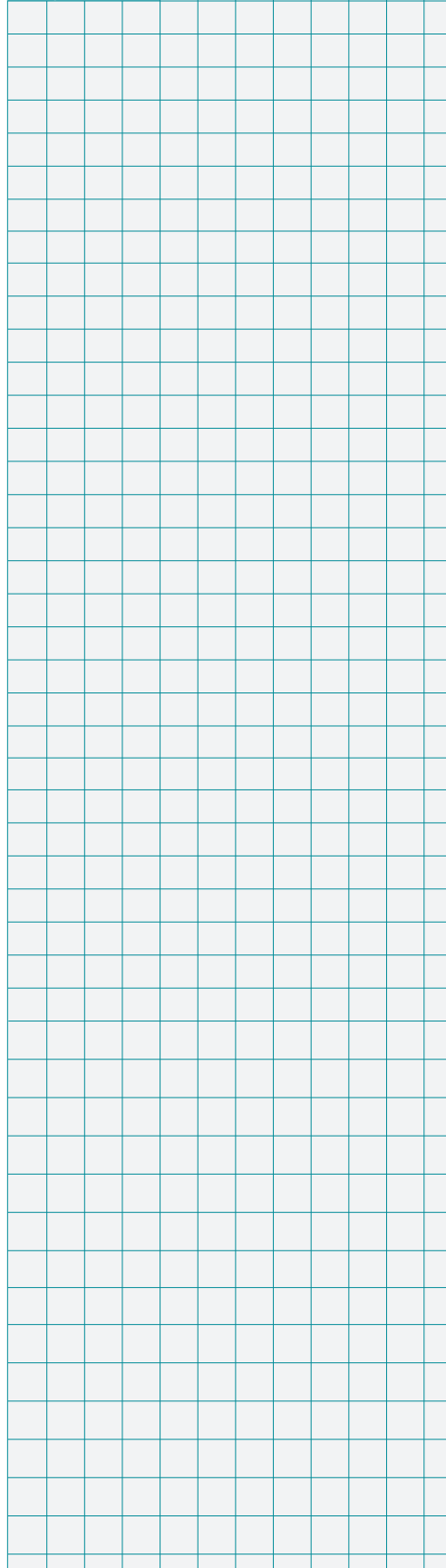


- Rotate the figure representing the ramp 90° counterclockwise about the origin. Label the vertices of the image R' , A' , M' , and P' .
- Critique the reasoning of others.** Kate states that this rotation will change the shape and size of the figure. Skip reassures her that the image is congruent to the original figure. With whom do you agree? Justify your reasoning.

Congruent figures have corresponding angles as well as corresponding sides.

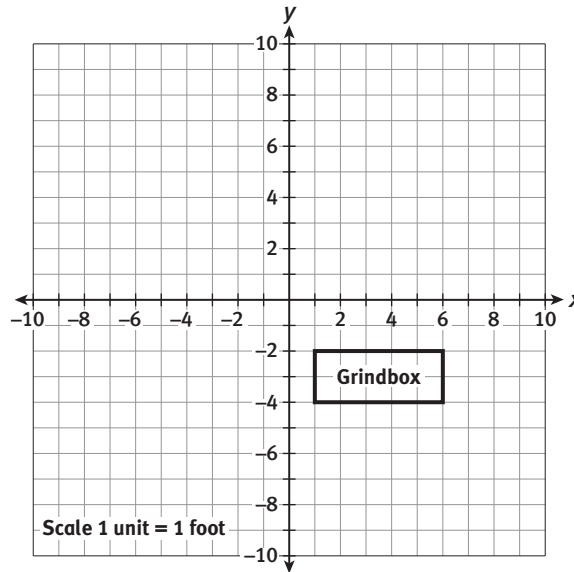
- List the pairs of corresponding angles in trapezoids $RAMP$ and $R'A'M'P'$.
- Construct viable arguments.** Make a conjecture about the corresponding angles of congruent figures.

My Notes



My Notes

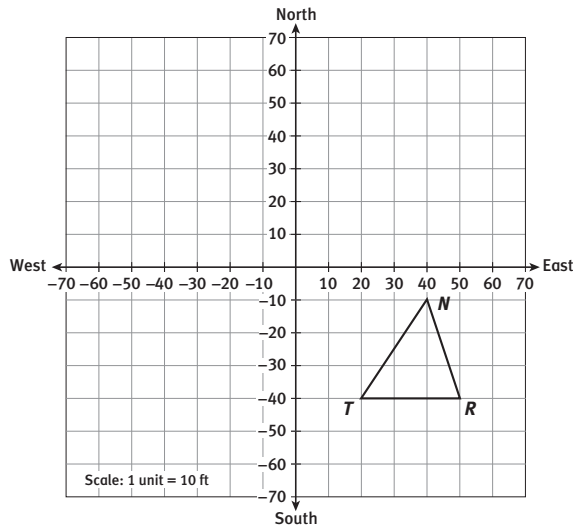
4. The original placement of the grindbox is shown on the coordinate plane.



- a. Plot the image of this figure using the transformation whose symbolic representation is $(x, y) \rightarrow (x + 2, y + 9)$.
 - b. Write a verbal description of the transformation.
- c. Is the image of the grindbox congruent to the preimage of the grindbox? Justify your answer.
5. **Reason abstractly.** After using reflections, rotations, and translations to create images of figures, what can you infer about the preimage and its image under all of these transformations?

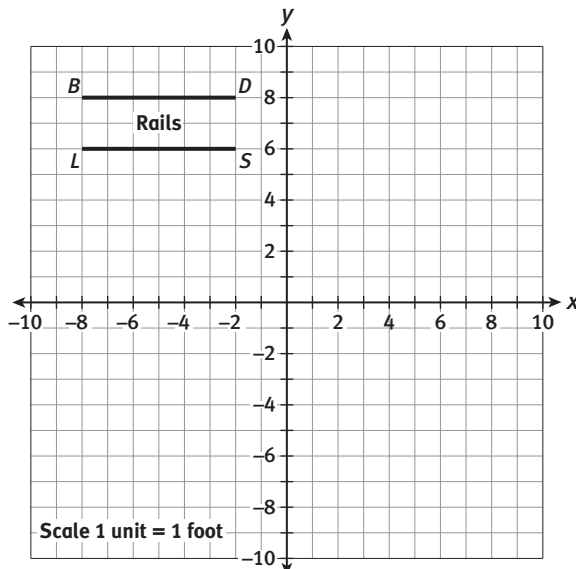
Check Your Understanding

Consider $\triangle NTR$ shown on the coordinate grid.



6. Rotate $\triangle NTR$ 180° about the origin. Label the vertices T' , R' , and N' .
7. Find the area, in square units, of $\triangle NTR$ and $\triangle N'T'R'$. Show the calculations that led to your answer.
8. Write a supporting statement justifying how you know that $\triangle NTR$ and $\triangle N'T'R'$ are congruent triangles.
9. **Express regularity in repeated reasoning.** Could your statement in Item 8 be used to support other types of transformations of $\triangle NTR$? Explain.

Finally, Skip decides to move the location of the rails. The original placement of the rails is shown on the coordinate plane.



My Notes

MATH TIP

The area of a triangle can be found using the formula

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}.$$

My Notes

MATH TERMS

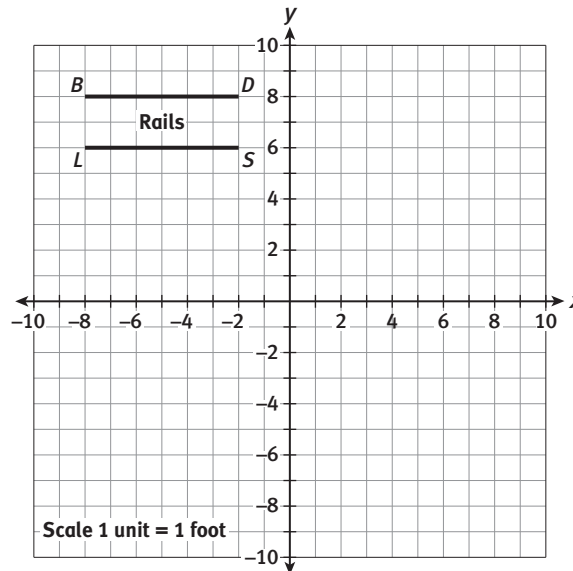
Performing two or more transformations on a figure is called a **composition of transformations**.

READING MATH

A prime symbol (') is placed after the letter of the original point to show that the new point is its image. Two prime symbols (") are placed after the letter of the original point to show that the new point has been transformed twice.

10. Construct viable arguments. Skip claims that the rails are parallel and that moving them, using a reflection, rotation, or translation, will not affect this relationship. Confirm or contradict Skip's claim. Use examples to justify your answer.

11. Skip decides to move the rails using a **composition of transformations**.



- a. Reflect the graph of each rail, \overline{BD} and \overline{LS} , over the x -axis. Label the image points B' , D' , L' , and S' .
- b. Then, translate the reflected image 3 feet up and 1 foot left. Label the image points B'' , D'' , L'' , and S'' .

My Notes

Learning Targets:

- Explore composition of transformations.
- Describe the effect of composition of translations, rotations, and reflections on two-dimensional figures using coordinates.

SUGGESTED LEARNING STRATEGIES: Self Revision/Peer Revision, Visualization, Discussion Groups, Create Representations, Close Reading

To explore composition of transformations, you and a partner will play a game called All the Right Moves. Cut out the five All the Right Moves cards on page 259 and two game pieces. You and your partner will use only one set of All the Right Moves game cards to play the game, but you both need a game piece.

DISCUSSION GROUP TIPS

As you read and discuss the rules of All the Right Moves, ask and answer questions to be sure you have a clear understanding of not only all the terminology used, but also how the game is to be played.

All the Right Moves Rules

1. As partners, lay out the 5 All the Right Moves cards face down.
2. Take turns choosing an All the Right Moves card. You will each take 2 cards. The extra card may be used later as a tiebreaker.
3. Working independently, each of you will use your All the Right Moves cards to complete the two game sheets on pages 255 and 256.
4. To complete the first game sheet, follow these steps:
 - a. Record the number of one of your All the Right Moves cards on your game sheet. You may use either one first.
 - b. Plot and label the points for Position 0 on the grid. Then use those points as the vertices to draw a triangle.
 - c. Follow the directions on the All the Right Moves card to find the coordinates of the vertices for Position 1.
 - d. Record the new coordinates on your game sheet, plot the new points on the coordinate plane, and draw a triangle. Use your game piece to identify the transformation you made and record its name on your game sheet.
 - e. Continue until you have moved the figure to all 5 positions on the All the Right Moves card. Then record the coordinates of the composition of transformations, which is Position 5.
5. Repeat the process with your other All the Right Moves card for the second game sheet.
6. When you and your partner have completed your two cards, exchange game sheets and check each other's work.
7. Score your game sheets: You get 2 points for each transformation you correctly identify and 5 points for the correct coordinates of each composition of transformations.
8. The player with the greater number of points wins the game.

My Notes

**All the Right Moves
Game Sheet**

Player: _____

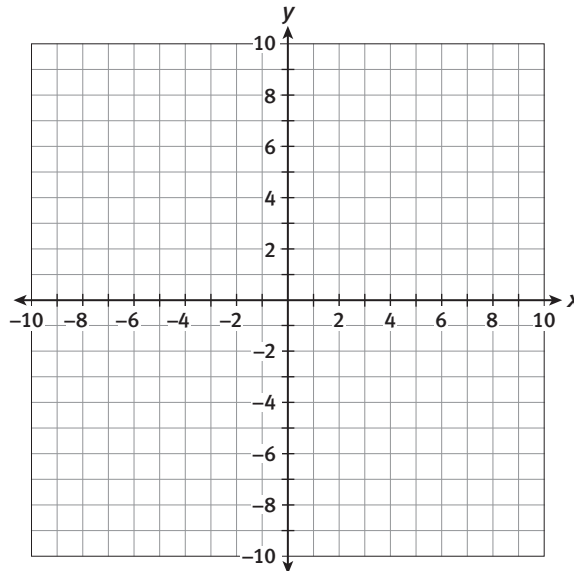
All the Right Moves Card: _____

Position 0: A(), B(), C()	Type of Transformation
Position 1: A(), B(), C()	
Position 2: A(), B(), C()	
Position 3: A(), B(), C()	
Position 4: A(), B(), C()	
Position 5: A(), B(), C()	

Composition of Transformations:

A(), B(), C()

Points Earned for All the Right Moves Card: _____



Total Points Earned: _____

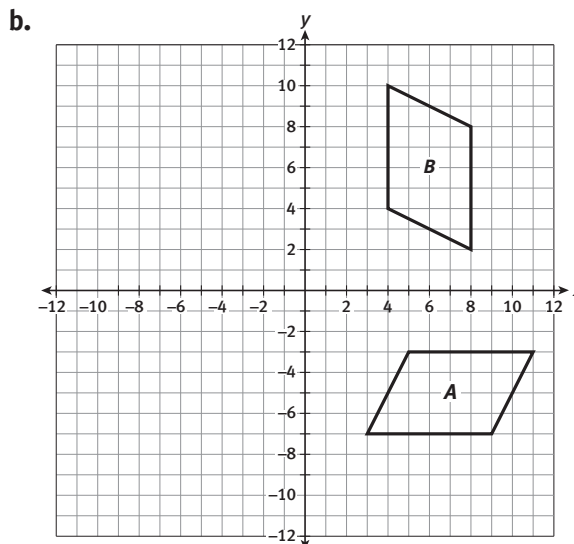
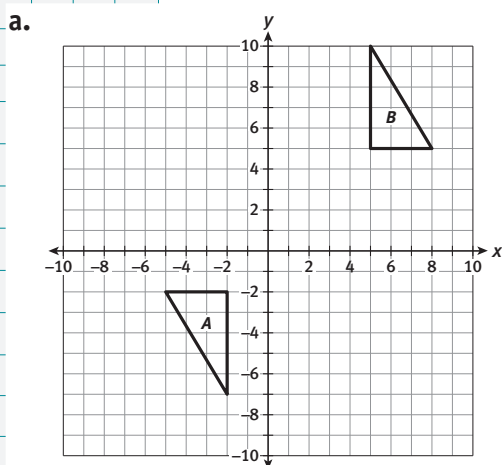
My Notes

Check Your Understanding

10. The point $T(5, -1)$ is reflected across the x -axis, then across the y -axis. What are the coordinates of T' and T'' ?
11. $\triangle ABC$ has vertices $A(-5, 2)$, $B(0, -4)$, and $C(3, 3)$.
 - a. Determine the coordinates of the image of $\triangle ABC$ after a translation 2 units right and 4 units down followed by a reflection over the y -axis.
 - b. What are the coordinates of the image of $\triangle ABC$ after a reflection over the y -axis followed by a translation 2 units right and 4 units down?

LESSON 19-2 PRACTICE

12. The point $(1, 3)$ is rotated 90° about the origin and then reflected across the y -axis. What are the coordinates of the image?
13. **Attend to precision.** Find a single transformation that has the same effect as the composition of translations $(x, y) \rightarrow (x - 2, y + 1)$ followed by $(x, y) \rightarrow (x + 1, y + 3)$. Use at least three ordered pairs to confirm your answer.
14. **Reason abstractly.** Describe a single transformation that has the same effect as the composition of transformations reflecting over the x -axis followed by reflecting over the y -axis. Use at least three ordered pairs to confirm your answer.
15. Write a composition of transformations that moves figure A so that it coincides with figure B .



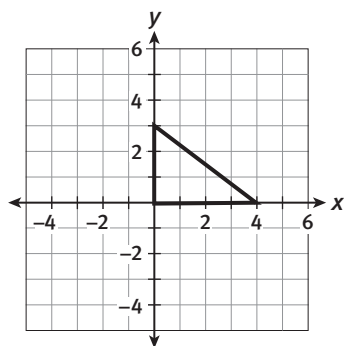
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ACTIVITY 19 PRACTICE

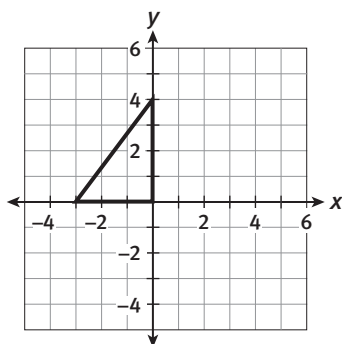
Write your answers on notebook paper.
Show your work.

Lesson 19-1

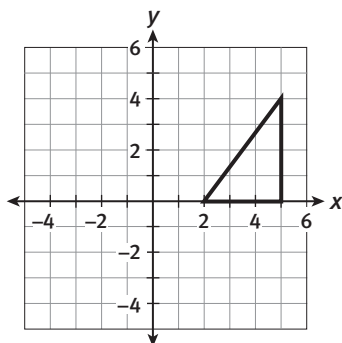
Each figure in Items 1–4 is an image of the figure shown on the coordinate plane below. Describe the transformations that were performed to obtain each image.



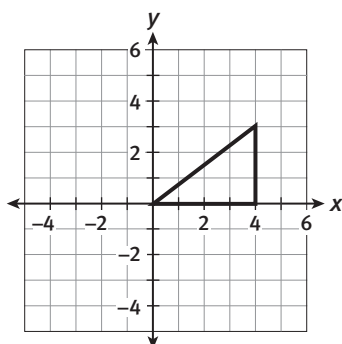
1.



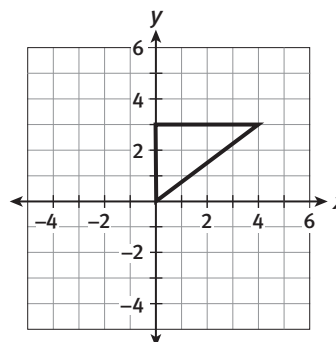
2.



3.



4.

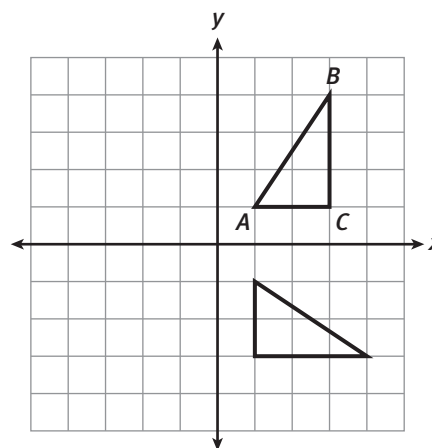


5.

- Compare the figures in Items 1–4.
 - a. How do the areas of each figure compare to the area of the original figure?
 - b. What can you determine about the corresponding sides of each figure?
 - c. What can you determine about the corresponding angles of each figure?
 - d. Can you determine if the images of each figure are congruent to the original figure? Provide reasoning for your answer.

6.

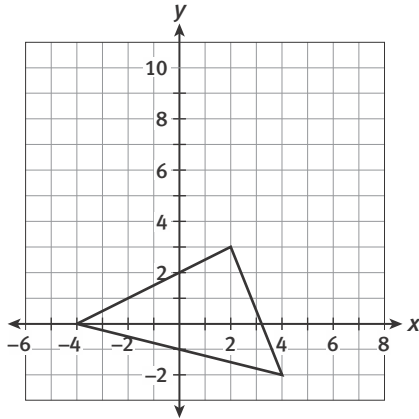
The coordinate plane below shows $\triangle ABC$ and a 90° clockwise rotation of $\triangle ABC$ about the origin.



- a. Sketch the 180° clockwise rotation of $\triangle ABC$.
- b. Sketch the 90° counterclockwise rotation of $\triangle ABC$.
- c. How do the images compare with $\triangle ABC$?

Lesson 19-2

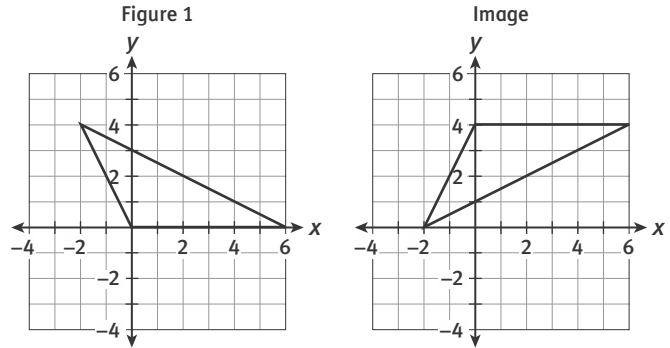
7. The preimage of a triangle is shown on the coordinate plane.



Which of the following types of transformation results in an image where corresponding angles and sides are NOT congruent?

- A. reflection
 - B. rotation
 - C. translation
 - D. none of the above
8. To create a logo, Henry transforms a quadrilateral by reflecting it over the x -axis, translating it 4 units up and then rotating the image 270° counterclockwise about the origin. Does the order in which Henry performs the transformations on the preimage change the size or shape of the image? Explain.

9. Figure 1 shows the preimage of a figure.



Which of the following transformation(s) have been performed on Figure 1 to obtain the image?

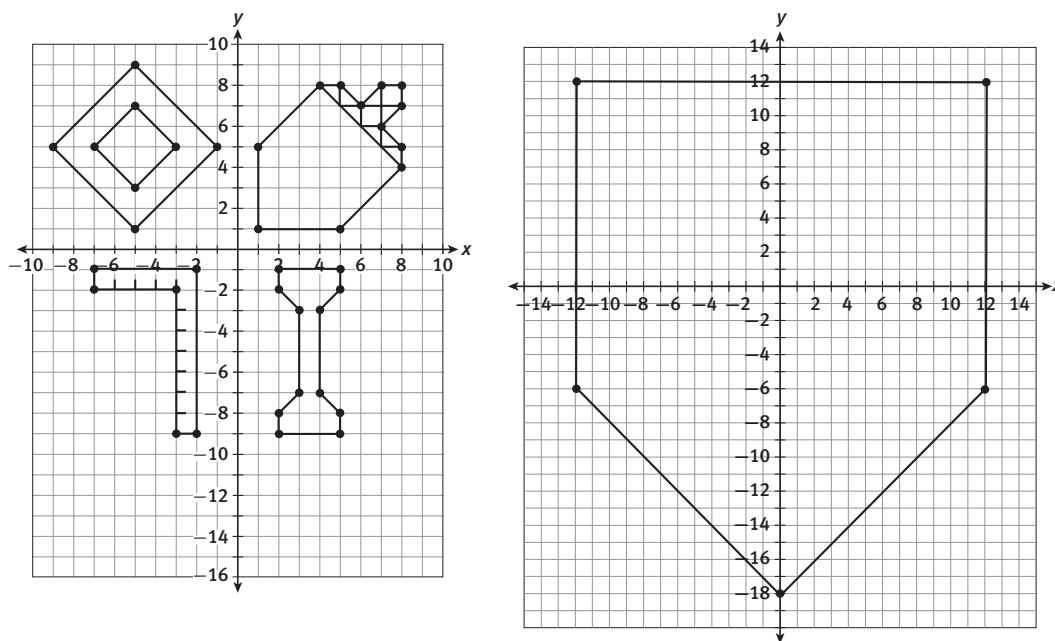
- A. Rotate 180° .
 - B. Shift down 2 units and reflect over the line $y = 2$.
 - C. Reflect over the x -axis and shift up 4 units.
 - D. Reflect over the y -axis and shift up 4 units.
10. List two transformations and then name one transformation that gives the same result as the two transformations.
11. Find a translation that has the same effect as the composition of translations $(x, y) \rightarrow (x + 7, y - 2)$ followed by $(x, y) \rightarrow (x - 3, y + 2)$.

MATHEMATICAL PRACTICES

Reason Abstractly

12. How many and what types of reflections would have to be performed on a preimage to get the same image as a 180° rotation?

In medieval times, a person was rewarded with a coat of arms in recognition of noble acts. In honor of your noble acts so far in this course, you are being rewarded with a coat of arms. Each symbol on the coordinate plane below represents a special meaning in the history of heraldry.



Transform the figures from their original positions to their intended positions on the shield above using the following descriptions.

- The **acorn** in Quadrant I stands for antiquity and strength and is also the icon used in the SpringBoard logo.
 - Reflect the acorn over the x -axis. Sketch the image of the acorn on the shield.
 - Write the symbolic representation of this transformation.
- The **masle** in Quadrant II represents the persuasiveness you have exhibited in justifying your answers.
 - Rotate the masle 270° counterclockwise about the origin. Sketch the image of the masle on the shield.
 - Write the symbolic representation of this transformation.
- The **carpenter's square** in Quadrant III represents your compliance with the laws of right and equity. The location of the carpenter's square is determined by a composition of transformations. Rotate the carpenter's square 90° counterclockwise about the origin followed by a reflection over the y -axis.
 - Copy and complete the table by listing the coordinates of the image after the carpenter's square is rotated 90° counterclockwise about the origin.
 - Sketch the image of the carpenter's square after the composition of transformations described.

Preimage	Image
$(-7, -1)$	
$(-7, -2)$	
$(-2, -1)$	
$(-3, -2)$	
$(-3, -9)$	
$(-2, -9)$	

4. Finally, the **column** in Quadrant IV represents the determination and steadiness you've shown throughout your work in this course.
 - a. Sketch the column using the transformation given by the symbolic representation $(x, y) \rightarrow (x - 9, y + 10)$.
 - b. Write a verbal description of the transformation.
5. Explain why each of the symbols on your coat of arms is congruent to the preimage of the symbol on the original coordinate plane.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-b, 2a-b, 3a-b, 4a-b, 5)	<ul style="list-style-type: none"> Clear and accurate understanding of reflections, rotations, and translations in the coordinate plane. 	<ul style="list-style-type: none"> An understanding of reflections, rotations, and translations in the coordinate plane with few errors. 	<ul style="list-style-type: none"> Partial understanding of reflections, rotations, and translations in the coordinate plane. 	<ul style="list-style-type: none"> Incorrect understanding of reflections, rotations, and translations in the coordinate plane.
Problem Solving (Items 1a-b, 2a-b, 3a-b, 4a-b)	<ul style="list-style-type: none"> Interpreting a problem accurately in order to carry out a transformation. 	<ul style="list-style-type: none"> Interpreting a problem to carry out a transformation. 	<ul style="list-style-type: none"> Difficulty interpreting a problem to carry out a transformation. 	<ul style="list-style-type: none"> Incorrect or incomplete interpretation of a transformation situation.
Mathematical Modeling / Representations (Items 1a, 2a, 3a, 4a)	<ul style="list-style-type: none"> Accurately transforming pre-images and drawing the images. 	<ul style="list-style-type: none"> Transforming pre-images and drawing the images with few, if any, errors. 	<ul style="list-style-type: none"> Difficulty transforming pre-images and drawing the images. 	<ul style="list-style-type: none"> Incorrectly transforming pre-images and drawing the images.
Reasoning and Communication (Items 4b, 5)	<ul style="list-style-type: none"> A precise explanation of congruent transformations. 	<ul style="list-style-type: none"> An understanding of transformations that retain congruence. 	<ul style="list-style-type: none"> A confusing explanation of congruent transformations. 	<ul style="list-style-type: none"> An inaccurate explanation of congruent transformations.

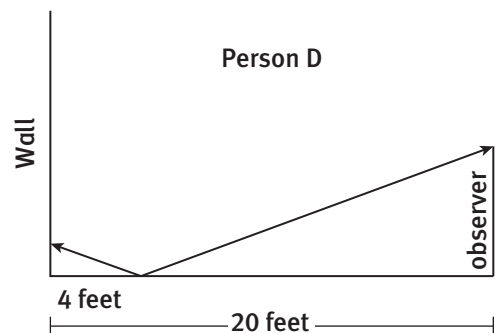
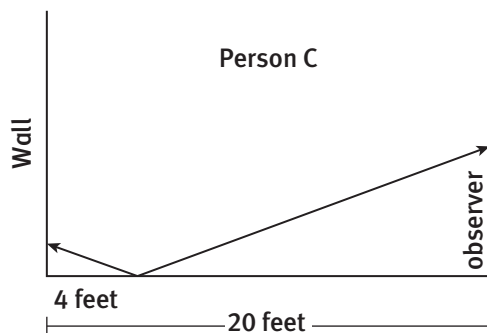
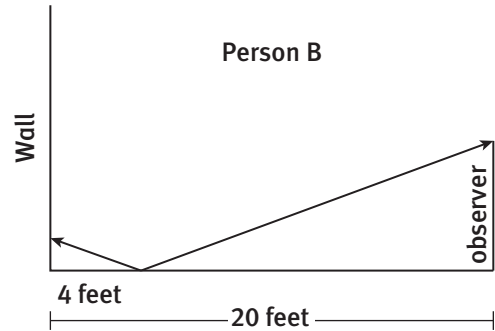
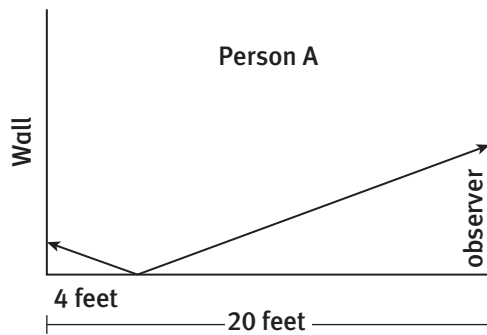
My Notes

Distance from the Wall to the Mirror (in feet)	Height of the Point on the Wall Reflected in the Mirror (in feet)			
	Person A	Person B	Person C	Person D
4				
8				
10				

2. Measure the eye-level height for each member of the group and record it in the table below.

Eye-Level Height for Each Group Member			
Person A	Person B	Person C	Person D

3. Consider the data collected when the mirror was 4 feet from the wall.
- On the diagrams below, label the height of each group member and the height of the point on the wall determined by the group member.



Lesson 20-1

Exploring Similarity

ACTIVITY 20

continued

- b. For each person in the group, determine the ratio of the height of the point on the wall to the eye-level height of the observer.

Ratio of height of the point on the wall to eye level of observer	Person A	Person B	Person C	Person D
	Ratio as a fraction			
Ratio as a decimal				

- c. **Express regularity in repeated reasoning.** What appears to be true about the ratios you found?
4. If the eye-level height of a five-year-old observer is 3.6 feet, what height can you predict for the point on the wall? Explain your reasoning.
5. Consider the data collected when the mirror was 8 feet from the wall. For each group member, determine the ratio of the height of the point on the wall to the eye-level height of the observer. What appears to be true?
6. Consider the data collected when the mirror was 10 feet from the wall. For each group member, determine the ratio of the height of the point on the wall to the eye-level height of the observer. What appears to be true?

My Notes

My Notes

MATH TERMS

Similar polygons are polygons in which the lengths of the corresponding sides are in proportion, and the corresponding angles are congruent.

MATH TERMS

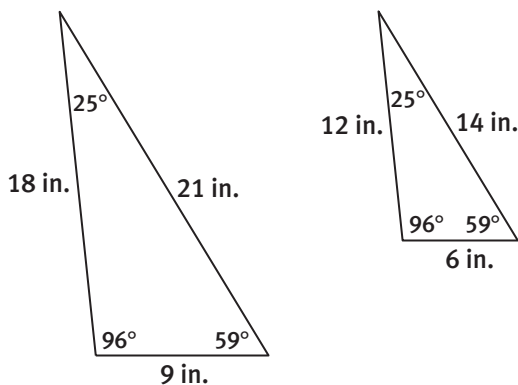
When two ratios are equivalent, then they form a **proportion**. For example, the ratios $\frac{3}{7}$ and $\frac{12}{28}$ are equivalent. Setting these ratios equal generates the proportion $\frac{3}{7} = \frac{12}{28}$.

WRITING MATH

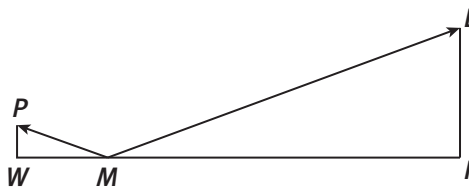
The symbol \sim is used to denote two similar figures.

Similar polygons are polygons in which the lengths of the corresponding sides are in **proportion**, and the corresponding angles are congruent.

For example, in the following triangles, the corresponding angles are congruent, and the corresponding sides are in proportion. Therefore, the triangles are similar.



A similarity statement for the triangles below is $\triangle PWM \sim \triangle EFM$. A **similarity statement** indicates that the corresponding angles are congruent, and the corresponding sides are proportional.

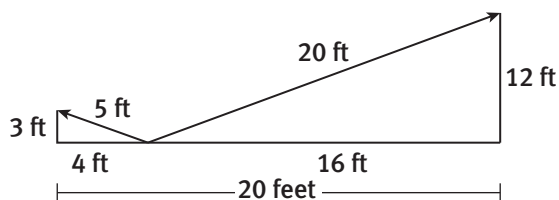


Lesson 20-1

Exploring Similarity

My Notes

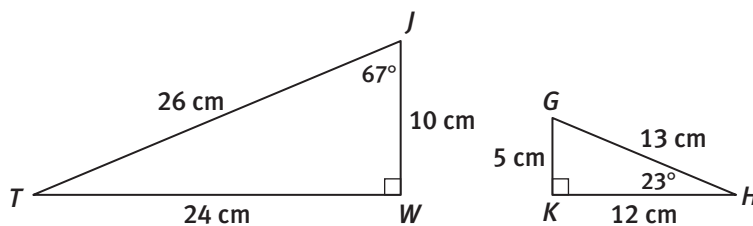
7. The diagram below shows two similar triangles like the triangles you worked with in Item 3.



- a. Use the lengths of the three pairs of corresponding sides to create three ratios in the form $\frac{\text{side length in small triangle}}{\text{corresponding length in large triangle}}$
- b. Compare the ratios written in part a. Then explain how these ratios relate to the ratios you created in Item 3.

Check Your Understanding

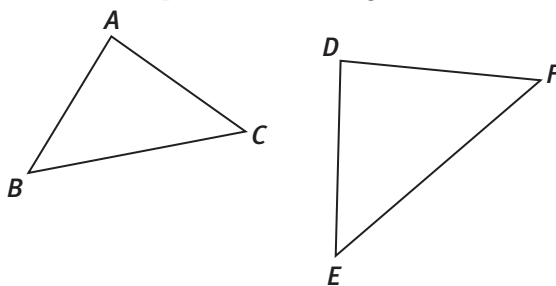
8. Are the triangles shown below similar? If so, explain why and write a similarity statement. If not, explain why not.



9. In the figure, $\triangle ABC \sim \triangle DEF$. Complete the following.

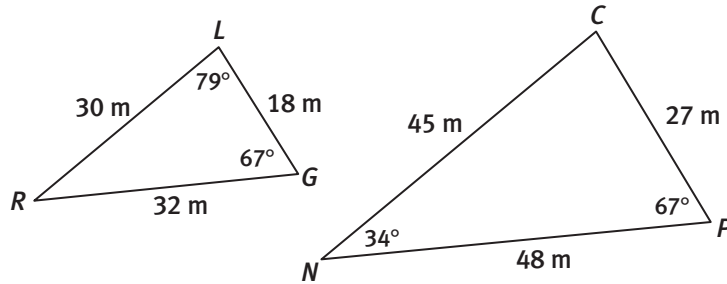
a. $m\angle F =$ _____

b. $\frac{AB}{DE} = \frac{\square}{DF}$

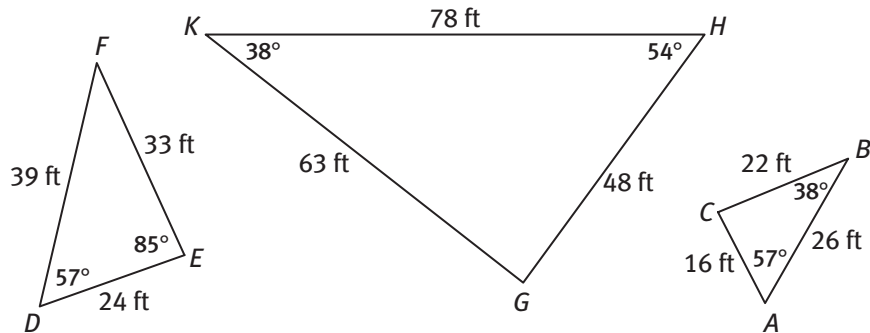


LESSON 20-1 PRACTICE

10. Are the triangles below similar? If so, explain why and write a similarity statement. If not, explain why not.



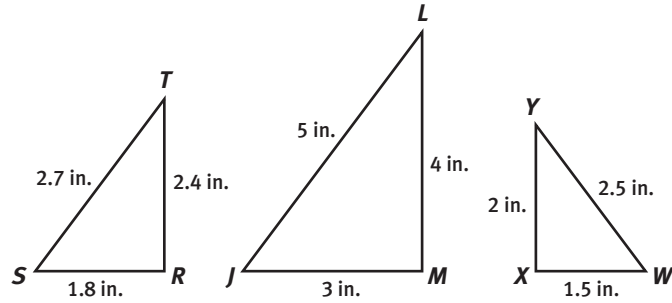
Use the figure below for Items 11–13.



11. Identify the pair of similar triangles in the figure. Explain your answer.
12. Write a similarity statement for the triangles you identified in Item 11. Is there more than one correct way to write the statement?
13. What are the pairs of corresponding sides in the triangles you identified in Item 11?
14. **Construct viable arguments.** Malia is a jewelry designer. She created two silver triangles that she would like to use as earrings, but she is not sure if the two triangles are similar. One triangle has angles that measure 51° and 36° . The other triangle has angles that measure 36° and 95° . Is it possible to determine whether or not the triangles are similar? Justify your answer.

My Notes

3. Consider the three triangles below.



- a. Compare ratios to identify any similar triangles.

- b. Write a similarity statement to identify the similar triangles.

- c. State the scale factor for the similar triangles.

- d. What are the pairs of corresponding angles of the similar triangles?

Lesson 20-2

Properties and Conditions of Similar Triangles

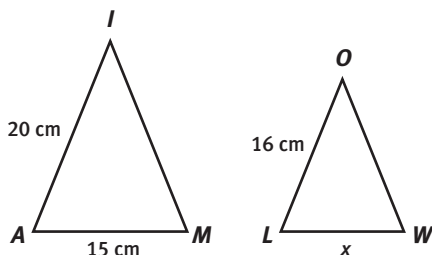
ACTIVITY 20

continued

The scale factor can be used to determine an unknown side length in similar figures.

Example A

Solve for x if $\triangle AIM \sim \triangle LOW$.



Step 1: Find the scale factor using known corresponding lengths.

The scale factor is $\frac{20 \text{ cm}}{16 \text{ cm}}$ or $\frac{5}{4}$.

Step 2: Write a proportion using the scale factor.

$$\frac{5}{4} = \frac{15 \text{ cm}}{x}$$

Step 3: Solve the proportion.

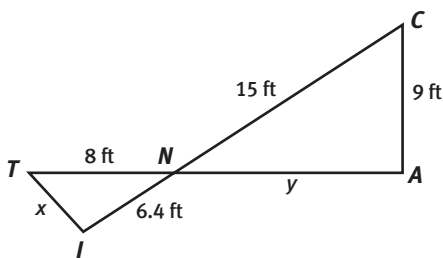
$$5x = 60.$$

$$x = 12$$

Solution: $x = 12 \text{ cm}$

Try These A

Given $\triangle TIN \sim \triangle CAN$.

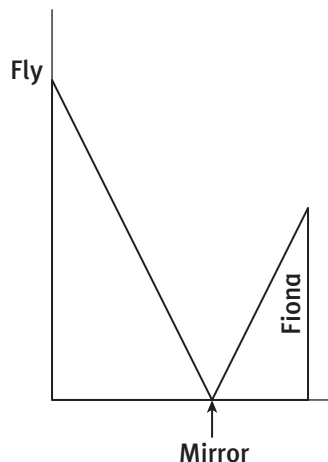


- Determine the scale factor.
- Solve for x and y .

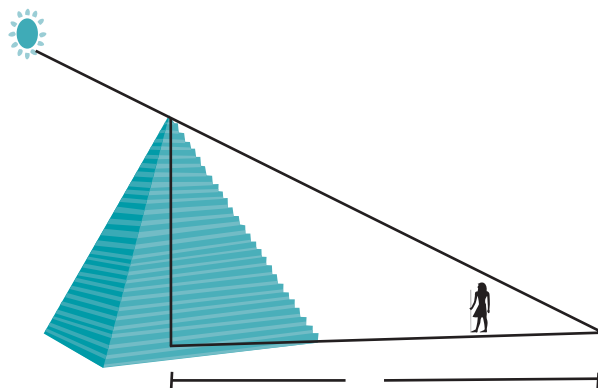
My Notes

My Notes

4. Suppose that a fly has landed on the wall and a mirror is lying on the floor 5 feet from the base of the wall. Fiona, whose eye-level height is 6 feet, is standing 3 feet away from the mirror and 8 feet away from the wall. She can see the fly reflected in the mirror.
- a. Use the information provided to label the distances on the diagram.



- b. Show how to use the properties of similar triangles to calculate the distance from the floor to the observed fly.
5. **Model with mathematics.** In his research, Thales determined that the height of the Great Pyramid could easily be calculated by the using the length of its shadow relative to the length of Thales's own shadow. Assume Thales was 6 feet tall and the shadow of the pyramid was 264 feet at the same time the shadow of Thales was 3.5 feet.
- a. Using these data, label the distances on the diagram.



- b. Determine the height of the Great Pyramid. Round your answer to the nearest tenth.

Lesson 20-2
Properties and Conditions of Similar Triangles

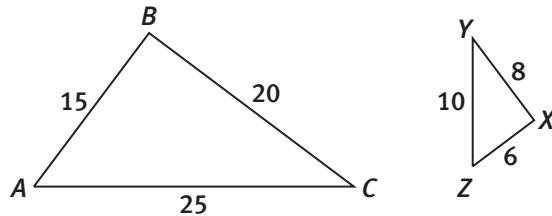
ACTIVITY 20
continued

My Notes

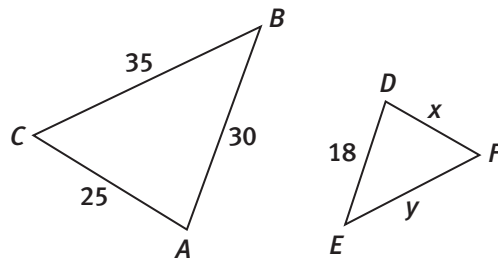
6. In $\triangle JKL$, $m\angle J = 32^\circ$ and $m\angle K = 67^\circ$. In $\triangle PQR$, $m\angle P = 32^\circ$ and $m\angle Q = 67^\circ$. Is $\triangle JKL \sim \triangle PQR$? Explain.

Check Your Understanding

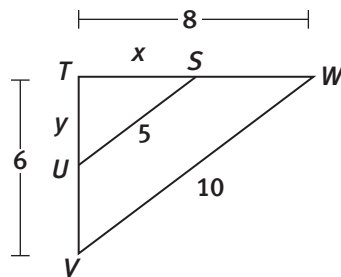
7. Are the two triangles shown below similar? If so, write a similarity statement and determine the scale factor. If not, explain why not.



8. Given $\triangle ABC \sim \triangle DEF$. Determine the value of x and y .

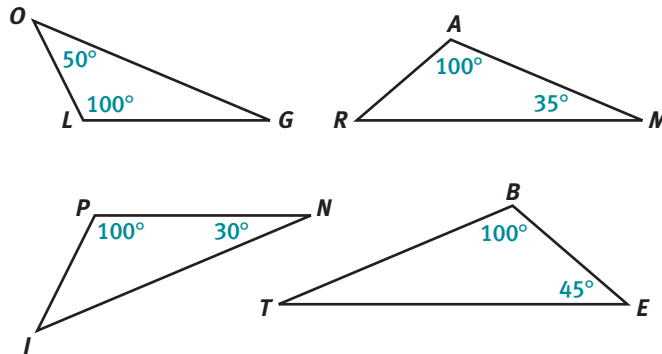


9. Given $\triangle TUS \sim \triangle TVW$. Determine the value of x and y .

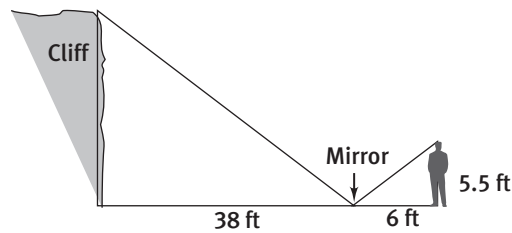


LESSON 20-2 PRACTICE

10. Write similarity statements to show which triangles are similar.



11. Before rock climbing to the top of a cliff, Chen wants to know how high he will climb. He places a mirror on the ground and walks backward until he sees the top of the cliff in the mirror, as shown in the figure. What is the height of the cliff?



12. Given: $\triangle ABC \sim \triangle RST$
 $AB = 44$ in., $BC = 33$ in., and $AC = 22$ in.
 $RS = 20$ in. and $ST = 15$ in.

Find RT .

13. If two triangles are similar, how does the ratio of their perimeters compare to the scale factor? Use an example to justify your answer.

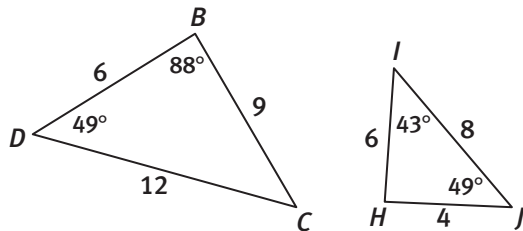
14. **Critique the reasoning of others.** Lucas claims, "If triangles have two pairs of congruent corresponding angles, then the third angles must also be congruent and the triangles must be similar." Is Lucas correct? Justify your answer.

ACTIVITY 20 PRACTICE

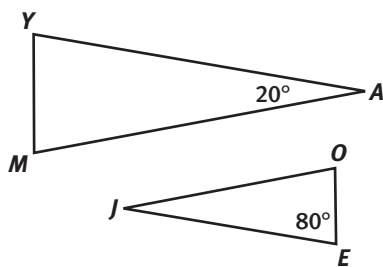
Write your answers on notebook paper.
Show your work.

Lesson 20-1

- Determine whether the triangles are similar. If so, write a similarity statement. If not, explain why not.



- If $\triangle JOE \sim \triangle AMY$, find the measure of each of the following angles.

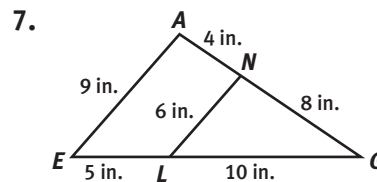
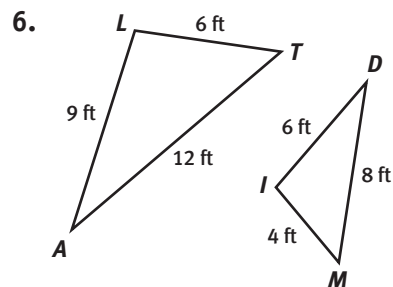


- $m\angle J$
 - $m\angle O$
 - $m\angle Y$
 - $m\angle M$
- $\triangle ABC$ has side lengths 15 cm, 20 cm, and 25 cm. What could be the side lengths of a triangle similar to $\triangle ABC$?
 - 7 m, 8 m, and 9 m
 - 6 m, 8 m, and 10 m
 - 5 cm, 10 cm, and 15 cm
 - 30 mm, 40 mm, and 55 mm
 - In $\triangle PQR$, $m\angle P = 27^\circ$ and $m\angle R = 61^\circ$. In $\triangle XYZ$, $m\angle Y = 92^\circ$.
 - Is it possible for $\triangle PQR$ to be similar to $\triangle XYZ$? Explain your reasoning.
 - Can you conclude that $\triangle PQR$ is similar to $\triangle XYZ$? Why or why not?

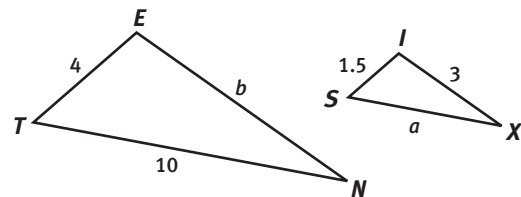
- Given that $\triangle ABC$ is similar to $\triangle GHJ$, which of the following statements must be true?
 - Both triangles have the same side lengths.
 - If $\triangle ABC$ has a right angle, then $\triangle GHJ$ has a right angle.
 - The perimeter of $\triangle ABC$ is greater than the perimeter of $\triangle GHJ$.
 - If $\triangle ABC$ has a side of length 2 cm, then $\triangle GHJ$ has a side of length 2 cm.

Lesson 20-2

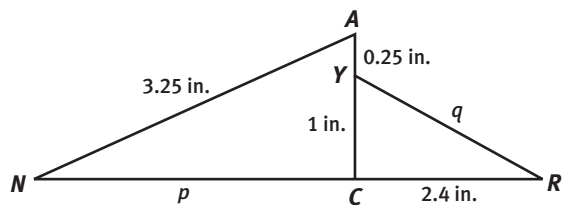
For Items 6 and 7, determine whether the triangles shown are similar. If so, write a similarity statement for the triangles and determine the scale factor. If not, explain why not.



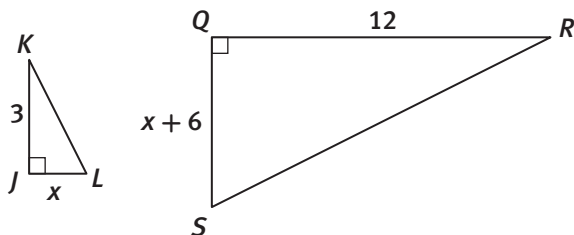
- Given $\triangle SIX \sim \triangle TEN$, find a and b .



9. Given $\triangle CAN \sim \triangle CYR$, find p and q .



10. Given: $\triangle JKL \sim \triangle QRS$. Determine the value of x .



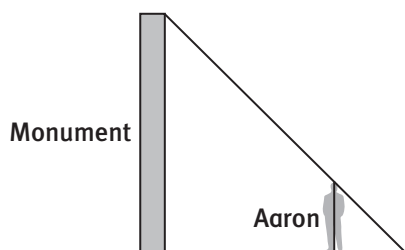
11. $\triangle MON \sim \triangle WED$, $m\angle M = 37^\circ$, and $m\angle E = 82^\circ$. Find the measure of each of the following angles.

- a. $\angle O$ b. $\angle W$
c. $\angle N$ d. $\angle D$

12. Tell the measure of each angle of $\triangle ABC$ and $\triangle PQR$ if $\triangle ABC \sim \triangle PQR$, $m\angle A = 90^\circ$, and $m\angle B = 56^\circ$.

13. Aaron is 6.25 ft tall, and he casts a shadow that is 5 ft long. At the same time, a nearby monument casts a shadow that is 25 ft long.

- a. Copy the figure and label the dimensions on the figure.



- b. Determine the height of the monument.

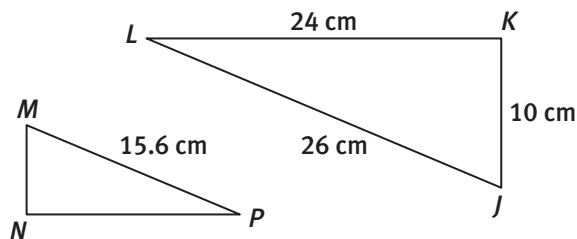
14. $\triangle ABC \sim \triangle DEF$ and the scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{4}{3}$. If $AB = 60$, what is DE ?

15. Sonia is 124 centimeters tall and casts a shadow that is 93 centimeters long. She is standing next to a tree that casts a shadow that is 135 meters long. How tall is the tree?

16. $\triangle STU \sim \triangle XYZ$, $ST = 6$, $SU = 8$, $XZ = 12$, and $YZ = 15$. What is the scale factor of $\triangle STU$ to $\triangle XYZ$?

- A. $\frac{2}{5}$ B. $\frac{1}{2}$
C. $\frac{8}{15}$ D. $\frac{2}{3}$

17. In the figure, $\triangle JKL \sim \triangle MNP$. What is the perimeter of $\triangle MNP$?



18. $\triangle ABC \sim \triangle DEF$. $AB = 12$, $AC = 16$, $DE = 30$, and $DF = x + 5$. What is the value of x ?

- A. 30 B. 35
C. 40 D. 45

MATHEMATICAL PRACTICES

Look For and Make Use of Structure

19. An equiangular triangle is a triangle with three congruent angles. Explain why all equiangular triangles are similar.

Dilations

Alice's Adventures in Shrinking and Growing Lesson 21-1 Stretching and Shrinking Geometric Figures

Learning Targets:

- Investigate the effect of dilations on two-dimensional figures.
- Explore the relationship of dilated figures on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Predict and Confirm, Create Representations, Visualization

In the story *Alice's Adventures in Wonderland* written by Lewis Carroll, Alice spends a lot of time shrinking and growing in height. The height changes occur when she drinks a potion or eats a cake.

1. Complete the table by finding Alice's new height after she eats each bite of cake or drinks each potion.

Starting Height (inches)	Change in Height	New Height (inches)
56	$\frac{1}{8}$ times as tall	
60	$\frac{2}{5}$ times as tall	
60	1.5 times as tall	
24	$\frac{5}{3}$ times as tall	
30	2.2 times as tall	

2. Each change in height resulted in a decrease or increase to Alice's starting height.
 - a. Alice's starting height decreased when it was multiplied by which two factors?

My Notes

CONNECT TO LITERATURE

Alice's Adventures in Wonderland is a novel published in 1865. It is the story of a young girl named Alice who wants to escape being bored by adulthood. In a dream she follows a white rabbit and falls down a deep tunnel and the adventure begins.

My Notes

- b.** Write a conjecture regarding the number you multiply by to decrease Alice’s height.

- c.** Confirm your conjecture by providing two additional examples that show that Alice’s starting height decreases.

- d.** Write a conjecture regarding the number you multiply by to increase Alice’s starting height.

- e.** Confirm your conjecture regarding Alice’s increase in height by providing two additional examples that show that Alice’s starting height increases.

Alice’s height changes—shrinking and growing—are a type of transformation known as a dilation.

Lesson 21-1

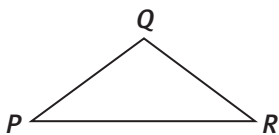
Stretching and Shrinking Geometric Figures

ACTIVITY 21

continued

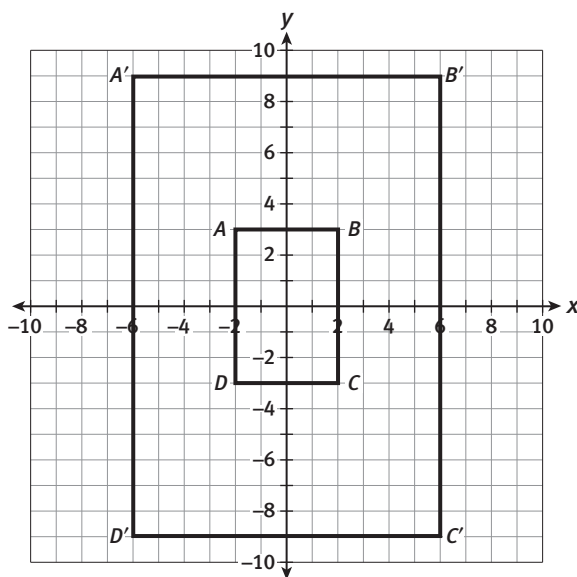
A **dilation** is a transformation where the image is *similar* to the preimage; the size of the image changes but the shape stays the same.

3. Use appropriate tools strategically. Given the preimage of $\triangle PQR$ below, use a ruler to draw the image of $\triangle PQR$ if it is dilated:



- a. by a factor of 2 b. by a factor of $\frac{1}{2}$

4. Rectangles $ABCD$ and $A'B'C'D'$ are shown on the coordinate plane with the **center of dilation** at the origin, O .



My Notes

MATH TERMS

A **dilation** is a transformation that changes the size but not the shape of an object.

MATH TERMS

The **center of dilation** is a fixed point in the plane about which all points are expanded or reduced. It is the only point under a dilation that does not move.

My Notes

- a. Determine the length of each side of rectangles $ABCD$ and $A'B'C'D'$.

Side	Length (in units)	Side	Length (in units)
\overline{AB}		$\overline{A'B'}$	
\overline{BC}		$\overline{B'C'}$	
\overline{CD}		$\overline{C'D'}$	
\overline{AD}		$\overline{A'D'}$	

- b. Describe the relationship between the side lengths of rectangle $ABCD$ and rectangle $A'B'C'D'$.

- c. Determine the coordinates of each of the vertices of both rectangles.

Rectangle $ABCD$		Rectangle $A'B'C'D'$	
A		A'	
B		B'	
C		C'	
D		D'	

- d. Describe the relationship between the coordinates of the vertices of $ABCD$ and the coordinates of the vertices of $A'B'C'D'$.

- e. The point $(\frac{1}{3}, -3)$ is a point on rectangle $ABCD$. What are the coordinates of the image of the point on $A'B'C'D'$? Explain how you determined your answer.

My Notes

Example A

Quadrilateral $SQRE$ is dilated to quadrilateral $S'Q'R'E'$ as shown on the coordinate plane. What is the relationship between the side lengths, perimeter, and area of the two figures?

Step 1: Compare the side lengths of corresponding sides of quadrilateral $S'Q'R'E'$ to quadrilateral $SQRE$.

$$\frac{S'Q'}{SQ} = \frac{10}{2} = \frac{5}{1}; \quad \frac{S'E'}{SE} = \frac{10}{2} = \frac{5}{1}$$

$$\frac{E'R'}{ER} = \frac{10}{2} = \frac{5}{1}; \quad \frac{R'Q'}{RQ} = \frac{10}{2} = \frac{5}{1}$$

The side lengths of quadrilateral $S'Q'R'E'$ are 5 times as great as the side lengths of quadrilateral $SQRE$.

Step 2: Find the perimeter of each quadrilateral. Then write the ratio of the perimeter of quadrilateral $S'Q'R'E'$ to the perimeter of quadrilateral $SQRE$.

Perimeter of quadrilateral $SQRE = 8$ units
 Perimeter of quadrilateral $S'Q'R'E' = 40$ units

$$\text{ratio: } \frac{\text{Perimeter of } S'Q'R'E'}{\text{Perimeter of } SQRE} = \frac{40}{8} = \frac{5}{1}$$

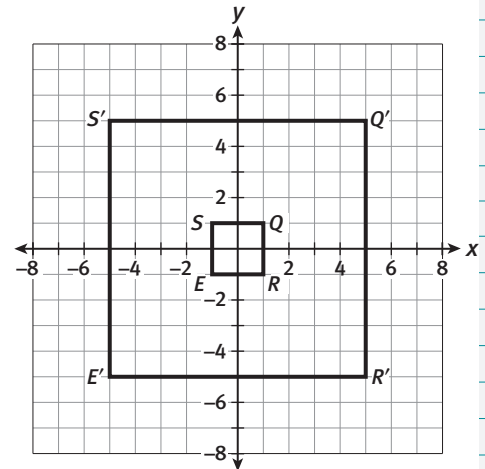
Solution: The perimeter of quadrilateral $S'Q'R'E'$ is 5 times as great as that of quadrilateral $SQRE$.

Step 3: Find the area of each quadrilateral. Then write the ratio of the area of quadrilateral $S'Q'R'E'$ to the area of quadrilateral $SQRE$.

Area of quadrilateral $SQRE = 4$ square units
 Area of quadrilateral $S'Q'R'E' = 100$ square units

$$\text{ratio: } \frac{\text{Area of } S'Q'R'E'}{\text{Area of } SQRE} = \frac{100}{4} = \frac{25}{1}$$

Solution: The area of quadrilateral $S'Q'R'E'$ is 25 times as great as that of quadrilateral $SQRE$.

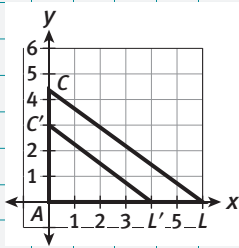


READING MATH

The fraction bar in a ratio is read aloud as "to." For example, the ratio $\frac{4}{1}$ is read as "4 to 1."

As you discuss Example A, make notes about the notation and vocabulary used so you can review them later to aid your understanding of dilating geometric figures.

My Notes



MATH TIP

The area of a triangle can be found using the formula

$$\text{Area} = \frac{1}{2} \text{base} \times \text{height}.$$

In a right triangle, the legs can be used as the base and height.

Try These A

Triangle ALC is dilated to $\triangle AL'C'$ as shown on the coordinate plane. Triangle ALC has vertices $A(0, 0)$, $L(6, 0)$, $C(0, 4\frac{1}{2})$. The length of $\overline{C'L'}$ is 5 units.

- a. Substitute known values into the proportion to find the length of \overline{CL} .

$$\frac{LA}{L'A} = \frac{CL}{C'L'}$$

- b. Determine the ratio of the perimeter of $\triangle AL'C'$ to the perimeter of $\triangle ALC$.

- c. Determine the ratio of the area of $\triangle AL'C'$ to the area of $\triangle ALC$.

Check Your Understanding

- Triangle ABC is dilated to $\triangle A'B'C'$. The ratio of the perimeter of $\triangle A'B'C'$ to the perimeter of $\triangle ABC$ is $\frac{4}{1}$. Explain how you can use this information to determine if the image has a larger or smaller perimeter than the preimage.
- Square $TUVW$ is enlarged to form square $T'U'V'W'$. What must be true about the relationship between corresponding sides for the enlargement to be considered a dilation?
- Reason abstractly.** Bradley states that in theory circles with different diameters are all dilations of each other. Susan states that in theory rectangles with different side lengths are all dilations of each other. Do you agree with either, both, or neither statement? Explain your reasoning.

My Notes

MATH TERMS

The **scale factor of dilation** is the factor by which each linear measure of the figure is multiplied.

Learning Targets:

- Determine the effect of the value of the scale factor on a dilation.
- Explore how scale factor affects two-dimensional figures on a coordinate plane.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Graphic Organizer, Create Representations

In the story *Alice's Adventures in Wonderland*, when Alice drinks a potion or eats a cake, she physically becomes taller or shorter, depending on a given factor. When this height change occurs, Alice changes size, but she does not change shape. Each dimension of her body is proportionally larger or smaller than her original self.

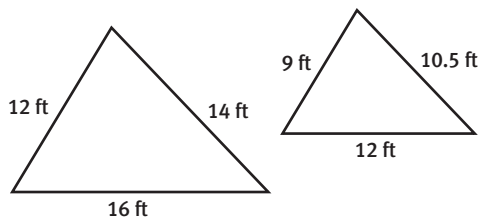
The factor by which Alice's height is changed, or dilated, is known as a **scale factor**.

The **scale factor of dilation**, typically represented by the variable k , determines the size of the image of a dilated figure.

If $0 < k < 1$, then the image will be smaller than the original figure. In this case, the dilation is called a **reduction**.

If $k > 1$, then the image will be larger than the original figure, and dilation is called an **enlargement**.

1. Consider the similar triangles shown.



- a. By what scale factor is the smaller triangle enlarged? Explain why the factor given must result in an enlargement.
- b. By what scale factor is the larger triangle reduced? Explain why the factor given must result in a reduction.
- c. What is the relationship between the two scale factors?

2. Suppose a point with coordinates (x, y) is a vertex of a geometric figure and that figure is dilated by a scale factor of k with the **center of dilation** at the origin.
 - a. Create an ordered pair to represent the coordinates of the corresponding point on the image.
 - b. Predict the size of the image as it compares to the preimage if k is 10.
 - c. Predict the size of the image as it compares to the preimage if k is 0.5.

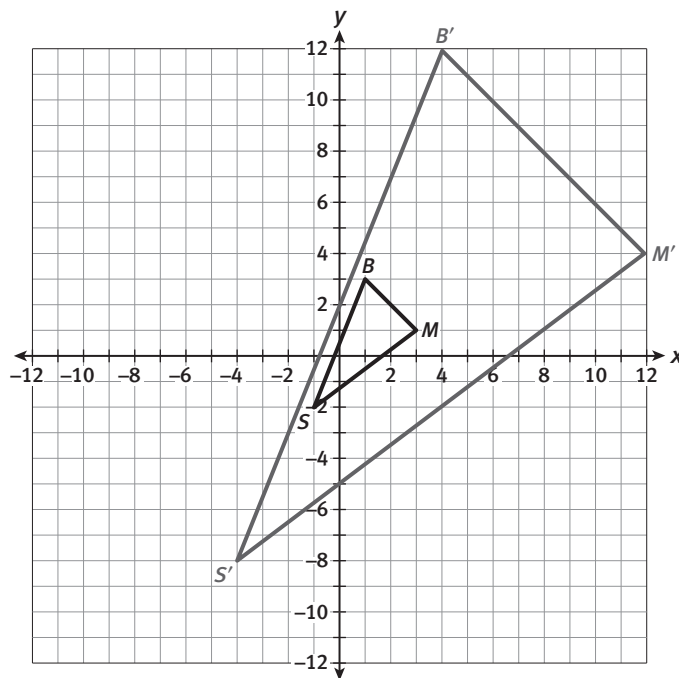
My Notes

MATH TERMS

The **center of dilation** is a fixed point in the plane about which all points are expanded or reduced. It is the only point under a dilation that does not move. The center of dilation determines the location of the image.

Example A

Triangle $S'B'M'$ is a dilation of $\triangle SBM$ with a scale factor of 4. Using the coordinates of the vertices of $\triangle SBM$, determine the coordinates of the vertices of $\triangle S'B'M'$. Then plot $\triangle S'B'M'$ on the coordinate plane.



- Step 1:** Determine if the dilation is a reduction or enlargement.
Since the scale factor is 4 and $4 > 1$, the dilation is an enlargement.
- Step 2:** Multiply the coordinates of the vertices of $\triangle SBM$ by the scale factor.
 $\triangle SBM$: $S(-1, -2), B(1, 3), M(3, 1)$
 Multiply each coordinate by 4.
 $\triangle S'B'M'$: $S'(-4, -8), B'(4, 12), M'(12, 4)$
- Step 3:** Plot the coordinates of the vertices of $\triangle S'B'M'$ on the coordinate plane.

My Notes

Try These A

- a. Suppose the scale factor of dilation of $\triangle SBM$ in Example A is $\frac{1}{2}$. Determine if the resulting image, $\triangle S'B'M'$, will be a reduction or an enlargement of $\triangle SBM$. Then, determine the coordinates of $\triangle S'B'M'$.

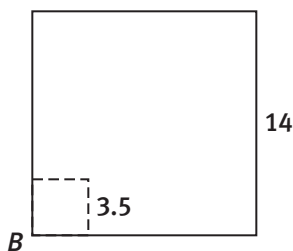
- b. Figure $A'B'C'D'$ is a dilation of figure $ABCD$ with a scale factor of 5. Given the coordinates of the vertices of $A(0, 0)$, $B(0, 2)$, $C(-2, -2)$, $D(-2, 0)$, determine the coordinates of the vertices of figure $A'B'C'D'$.

Check Your Understanding

3. Compare the ratio of the side lengths of figure $A'B'C'D'$ and figure $ABCD$ to the scale factor in Try These part b. Make a conjecture about the ratio of side lengths of dilated figures and the scale factor of dilation.
4. Triangle $P'Q'R'$ is a dilation image of $\triangle PQR$. The scale factor for the dilation is 0.12. Is the dilation an enlargement or a reduction? Explain.
5. **Make use of structure.** A geometric figure contains the point $(0, 0)$ and is dilated by a factor of m with the center at the origin. What changes will occur to the point $(0, 0)$?

The scale factor of dilation describes the size change from the original figure to the image. The scale factor can be determined by comparing the ratio of corresponding side lengths.

6. The solid line figure shown is a dilation of the figure formed by the dashed lines. Describe a method for determining the scale factor used to dilate the figure.



Lesson 21-2

Effects of Scale Factor

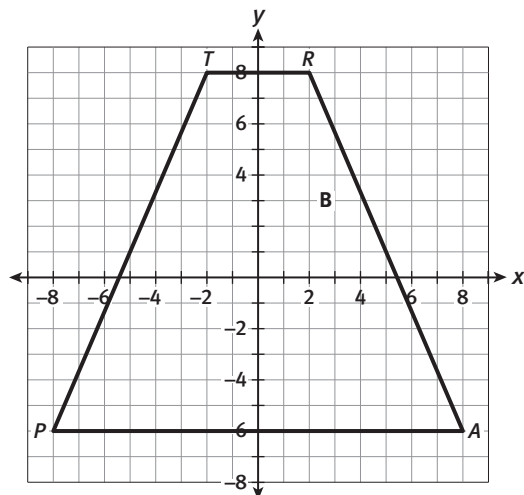
ACTIVITY 21

continued

7. **Critique the reasoning of others.** Josie found the scale factor in Item 6 to be $\frac{1}{4}$. Explain why Josie got the wrong scale factor.

There exists a relationship between the area of dilated figures and the perimeter of dilated figures.

8. Make a prediction about the effect of the scale of dilation on the area and perimeter of two figures.
9. Trapezoid $TRAP$, shown on the coordinate plane, has vertices $(-2, 8)$, $(2, 8)$, $(8, -6)$, $(-8, -6)$. Suppose trapezoid $TRAP$ is dilated by a scale factor of $\frac{1}{4}$.



- a. Plot and label the vertices of the image $T'R'A'P'$.
- b. Determine the area of trapezoids $TRAP$ and $T'R'A'P'$.
- c. What is the ratio of the area of $TRAP$ to the area of $T'R'A'P'$?
- d. **Reason quantitatively.** Make a conjecture about the relationship between scale factor of dilation and the area of dilated figures.

My Notes

MATH TIP

The area of a trapezoid can be found using the formula $\text{Area} = \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the bases.

My Notes

Check Your Understanding

10. Suppose a polygon is dilated by a scale factor of k . Write an expression for the ratio of the perimeters. Then, write an expression to represent the ratio of the areas.
11. A triangle is dilated by a scale factor of $\frac{2}{5}$.
 - a. What is the ratio of the perimeters?
 - b. What is the ratio of the areas?
12. **Construct viable arguments.** Suppose that a dilation is executed with a scale factor of 1. How would the preimage relate to the image? Using an example, justify your answer.

LESSON 21-2 PRACTICE

13. A rectangle has a perimeter of 24 ft. Following a dilation, the new perimeter of the rectangle is 36 ft.
 - a. Determine the scale factor of dilation.
 - b. What is the ratio of the areas?
14. A triangle has an area of 40 cm^2 . Following a dilation, the new area of the triangle is 360 cm^2 . What is the scale factor of dilation?
15. The vertices of trapezoid $ABCD$ are $A(-1, -1)$, $B(-1, 1)$, $C(2, 2)$, and $D(2, -1)$.
 - a. Draw the trapezoid and its dilation image for a dilation with center $(0, 0)$ and scale factor 3.
 - b. Determine the ratio of the perimeter.
 - c. Determine the ratio of the areas.
16. **Make sense of problems.** Eye doctors dilate patients' pupils to get a better view inside the eye. If a patient's pupil had a 3.6-mm diameter before dilation and 8.4-mm diameter after dilation, determine the scale factor used to dilate the pupil. Explain why this created an enlargement.

MATH TIP

The area of a circle can be found using the formula $\text{Area} = \pi r^2$, where r is the radius of the circle.

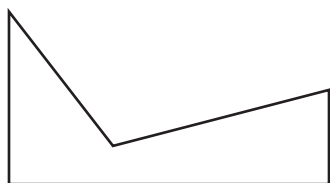
ACTIVITY 21 PRACTICE

Write your answers on notebook paper.

Show your work.

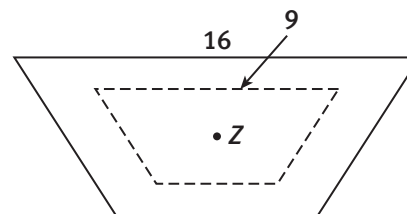
Lesson 21-1

1. Use appropriate tools strategically. Sketch the dilation of the image of the figure below using a scale factor of $\frac{2}{3}$.



2. Does the size of a preimage increase or decrease when
 - a. dilated by a factor greater than 1?
 - b. dilated by a factor between 0 and 1?
3. The ratio of the area of $\triangle X'Y'Z'$ to the area of $\triangle XYZ$ is $\frac{2}{9}$. Explain how you can use this information to determine if the image is greater or smaller in area than the preimage.

4. The solid line figure is a dilation of the dashed line figure. Tell whether the dilation is an enlargement or a reduction. Then find the scale factor of the dilation.

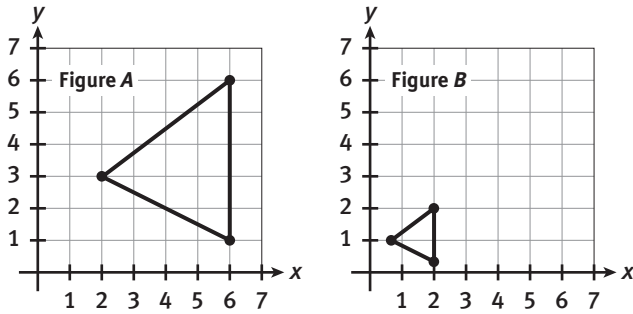


5. Explain how dilations are different from other types of transformations you have studied.
6. If the radius of a circle is 24 ft, how many circles can be the dilations of this circle? Why?

Lesson 21-2

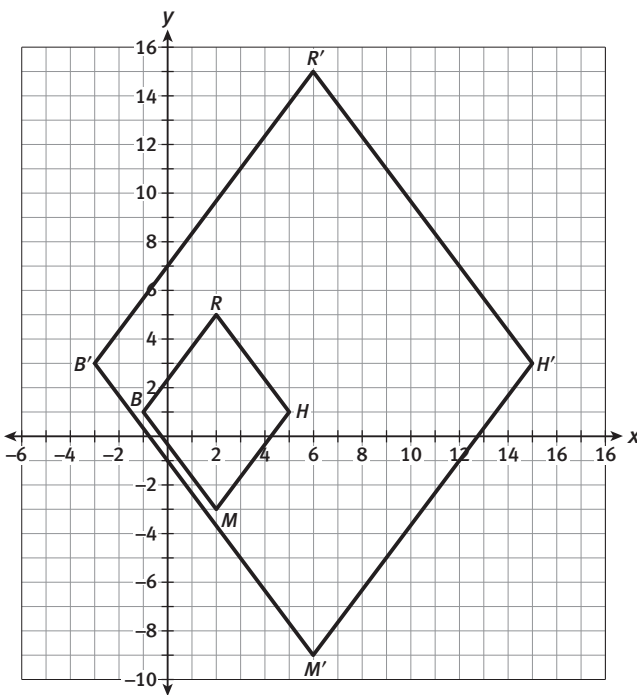
7. A dilation has a center $(0, 0)$ and scale factor 1.5. What is the image of the point $(-3, 2)$?
8. A triangle has vertices $(-1, 1)$, $(6, -2)$, and $(3, 5)$. If the triangle is dilated with a scale factor of 3, which of the following are the vertices of the image?
 - A. $(-3, 3)$, $(18, -6)$, $(9, 15)$
 - B. $(3, 3)$, $(18, 6)$, $(9, 15)$
 - C. $(-3, 3)$, $(18, 6)$, $(9, 15)$
 - D. $(3, 3)$, $(18, -6)$, $(9, 15)$

9. Figure B is the result of a dilation of Figure A.



What is the scale factor of dilation?

- A. 3 B. 2
C. $\frac{1}{3}$ D. $\frac{1}{2}$
10. Rhombus $RHMB$ has vertices $(2, 5)$, $(5, 1)$, $(2, -3)$, and $(-1, 1)$. This figure has been dilated to rhombus $R'H'M'B'$, as shown on the coordinate plane.



The area of rhombus $RHMB$ is 24 square units. Which of the following is the area of rhombus $R'H'M'B'$?

- A. 216 square units
B. 72 square units
C. 8 square units
D. 2.7 square units

11. The diagonals of rhombus $ABCD$ are 6 ft and 8 ft. Rhombus $ABCD$ is dilated to rhombus $RSTU$ with the scale factor 8. What is the perimeter of rhombus $RSTU$?

MATHEMATICAL PRACTICES

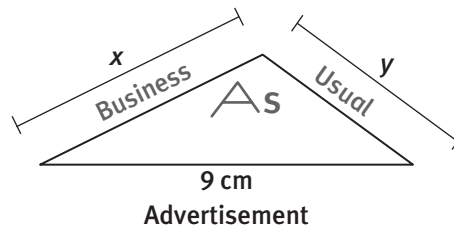
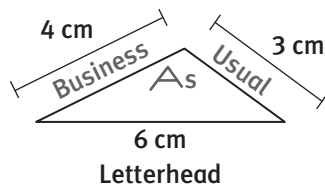
Reason Abstractly and Quantitatively

12. The endpoints of \overline{AB} are $A(78, 52)$ and $B(26, -52)$. \overline{AB} is dilated to \overline{GH} with endpoints at $G(30, 20)$ and $H(10, -20)$. Then, \overline{GH} is dilated to \overline{PQ} with endpoints at $P(42, 28)$ and $Q(14, -28)$. If \overline{AB} is dilated directly to \overline{PQ} , what will be the scale factor?

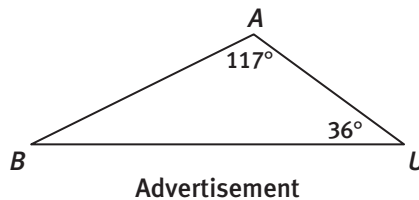
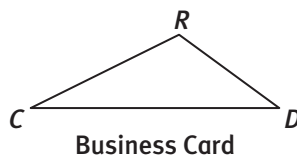
Liz is a commercial artist working for Business as Usual. The company specializes in small-business public relations. Liz creates appealing logos for client companies. In fact, she helped create the logo for her company. Business As Usual will use its logo in different sizes, with each design including a triangle similar to the one shown.



- The advertisement and stationery letterhead-size logos are shown below with the measurements of some of the side lengths. Determine the missing measures of the sides.



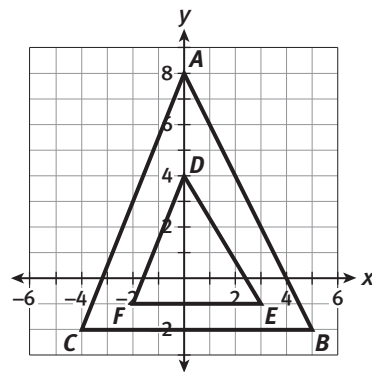
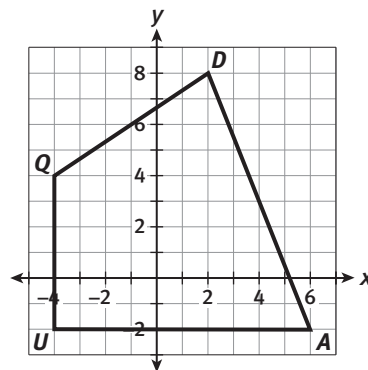
- To create the triangles in the design, Liz wants to determine the measure of each angle in the designs. The advertisement logo is shown below including the measures of two of its angles. The business card logo will be similar to the advertisement so that $\triangle BAU \sim \triangle CRD$. Determine the measure of each angle.
 - $m\angle C =$ _____
 - $m\angle R =$ _____
 - $m\angle D =$ _____



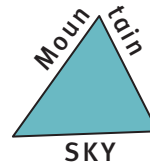
Liz tries to incorporate triangles and quadrilaterals into many of the logos she designs for her clients. She begins her layout by laying it out on a coordinate plane.

3. Quadrilateral $QUAD$ is shown.
 - a. Quadrilateral $Q'U'A'D'$ is a dilation of $QUAD$ with scale factor $\frac{1}{2}$. List the coordinates of $Q'U'A'D'$ and sketch the graph on a coordinate plane.
 - b. Determine the ratio of the perimeter of $Q'U'A'D'$ to the perimeter of $QUAD$.
 - c. Determine the ratio of the area of $Q'U'A'D'$ to the area of $QUAD$.

4. The coordinates of $\triangle ABC$ are $A(0, 8)$, $B(5, -2)$, and $C(-4, -2)$, and the coordinates of $\triangle DEF$ are $D(0, 4)$, $E(3, -1)$, and $F(-2, -1)$. Determine whether or not $\triangle ABC$ is similar to $\triangle DEF$. Defend your answer.



5. You have been chosen to work with Liz on a logo for a new client, Mountain Sky, a company that provides camping equipment and guides. Using either the logo design shown or a design of your own, recreate the design in sizes appropriate for a business card, business stationery letterhead, and an advertisement. Use properties of similar triangles to explain to Liz how you know the designs are dilations of the original. Include scale factors for each design.



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
Mathematics Knowledge and Thinking (Items 1, 2a-c, 3a-c, 4, 5)	<ul style="list-style-type: none"> Accurately finding side lengths and angle measures in similar triangles. Accurately using dilations and scale factors. 	<ul style="list-style-type: none"> Finding side lengths and angle measures in similar triangles. Using dilations and scale factors with few errors. 	<ul style="list-style-type: none"> Difficulty finding side lengths and angle measures in similar triangles. Difficulty using dilations and scale factors. 	<ul style="list-style-type: none"> Little or no understanding of finding side lengths in similar triangles. Little or no understanding of dilations.
Problem Solving (Items 3b-c, 4, 5)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer. 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but is correct. 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers. 	<ul style="list-style-type: none"> No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 3a, 5)	<ul style="list-style-type: none"> Modeling dilations accurately and clearly. 	<ul style="list-style-type: none"> Drawing similar figures correctly. 	<ul style="list-style-type: none"> Difficulty drawing similar figures accurately. 	<ul style="list-style-type: none"> Incorrectly transforming pre-images and drawing the images.
Reasoning and Communication (Items 4, 5)	<ul style="list-style-type: none"> Using precise language to justify that two triangles are similar. 	<ul style="list-style-type: none"> Explaining why two triangles are similar. 	<ul style="list-style-type: none"> A confusing explanation of triangle similarity. 	<ul style="list-style-type: none"> An inaccurate explanation of triangle similarity.

My Notes

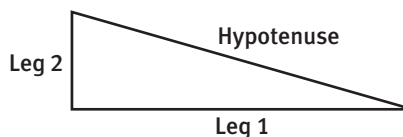
MATH TIP

It does not matter which leg is labeled Leg 1 and which is labeled Leg 2.

MATH TERMS

The **hypotenuse** is the longest side of a right triangle. It is the side that is opposite the right angle.

The **legs** of a right triangle are the two sides that create the right angle.



2. The **hypotenuse** of a right triangle is the side that is opposite the right angle. It is always the longest side of the triangle. The **legs** of a right triangle are the sides that form the right angle. Both Figures 1 and 2 have been formed using four congruent right triangles like the one above.

a. Use grid paper to cut out four congruent right triangles with Leg 1 equal to seven units and Leg 2 equal to two units. Recreate Figures 1 and 2 on another piece of graph paper by tracing your four congruent triangles and adding line segments to complete L and M. Then complete Case 1 in Table A at the bottom of this page.

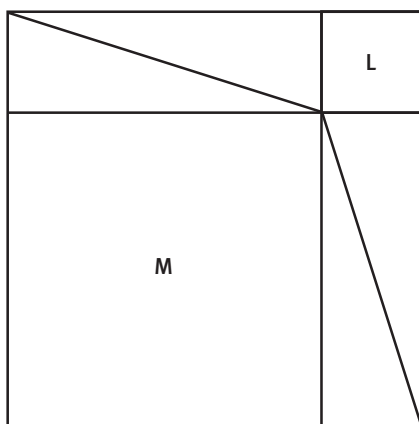


Figure 1

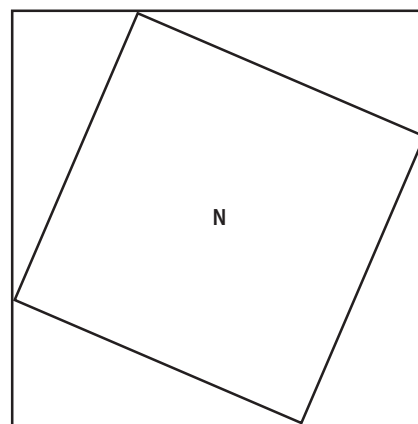


Figure 2

Table A

Case	Length Leg 1	Length Leg 2	Width Figure 1	Length Figure 1	Area Figure 1	Width Figure 2	Length Figure 2	Area Figure 2
1	7	2						
2	6	3						
3	4	3						
4								

- b. Complete Cases 2 and 3 in Table A by cutting out triangles to recreate Figures 1 and 2 using the lengths given in the table.
- c. Complete Case 4 in Table A by choosing your own leg lengths for a right triangle.
- d. What do you notice about Figure 1 and Figure 2 in each case?

Lesson 22-1

Pythagorean Theorem: Squares of Lengths

ACTIVITY 22

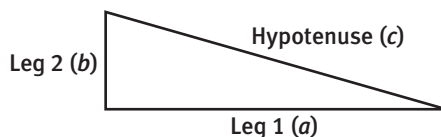
continued

3. Now use the figures you drew for Cases 1 through 4 to complete the first seven columns (Case through Area of Shape M) in Table B. For Case 5, use the variables a and b as the lengths of Leg 1 and Leg 2.

Table B

Case	Length Leg 1	Length Leg 2	Dimensions Shape L	Area Shape L	Dimensions Shape M	Area Shape M	Area Shape N
1	7	2					
2	6	3					
3	4	3					
4							
5	a	b					

4. Describe the relationship between the areas of shapes L, M, and N and complete the Area of Shape N column of Table B.
5. Describe the lengths of the sides of shapes L, M, and N in terms of the sides of the right triangles.
6. Find the area of shapes L, M, and N in terms of the lengths of the sides of the right triangles.
7. **Make use of structure.** Use a for the length of Leg 1, b for the length of Leg 2, and c for the length of the hypotenuse to write an equation that relates the areas of shapes L, M, and N.



The relationship that you have just explored is called the **Pythagorean Theorem**.

My Notes

MATH TERMS

The **Pythagorean Theorem** states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

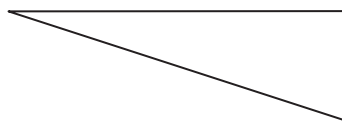
CONNECT TO HISTORY

Although the Pythagorean Theorem is named for Pythagoras, a Greek mathematician who lived about 500 B.C.E., the ancient Babylonians, Chinese, and Egyptians understood and used this relationship even earlier.

My Notes

Check Your Understanding

8. Label this triangle using a for leg 1, b for leg 2, and c for the hypotenuse:



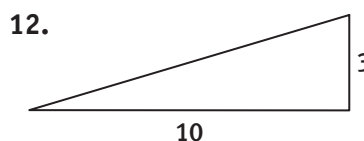
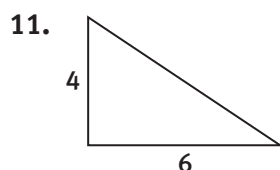
Use the figure to answer Items 9–10.



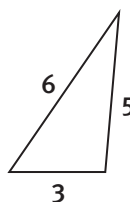
9. If the right triangle used to make the figure has leg lengths of 6 units and 8 units, what is the area of the inner square, S ?
10. Write an equation in the form $a^2 + b^2 = c^2$ for the figure.

LESSON 22-1 PRACTICE

Find c^2 for the following right triangles.



14. What does the Pythagorean Theorem state? Explain in your own words.
15. **Construct viable arguments.** Riley drew a triangle with the following dimensions:



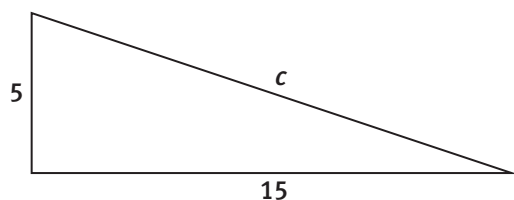
Is this triangle a right triangle? Explain your reasoning.

Learning Targets:

- Investigate the Pythagorean Theorem.
- Find missing side lengths of right triangles using the Pythagorean Theorem.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Visualization, Look for a Pattern, Critique Reasoning, Sharing and Responding

The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. This relationship can be used to determine the missing length of a side of a right triangle when you are given two lengths.

Example A

Find the length of the hypotenuse, c .

Step 1: Substitute the given lengths into the equation: $a^2 + b^2 = c^2$.

$$5^2 + 15^2 = c^2$$

Step 2: Square the lengths and add.

$$25 + 225 = c^2$$

$$250 = c^2$$

Step 3: Find the square root to solve for c . Since 250 is not a perfect square, round to the nearest tenth when finding the square root.

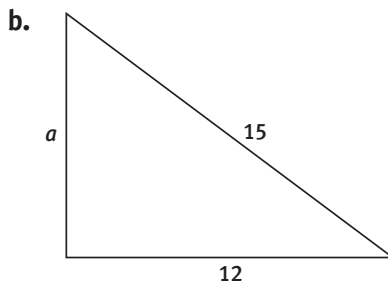
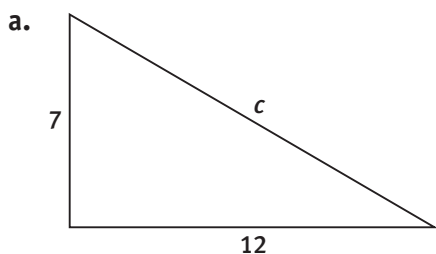
$$\sqrt{250} = c$$

$$15.8 = c$$

Solution: The length of the hypotenuse, c , is 15.8.

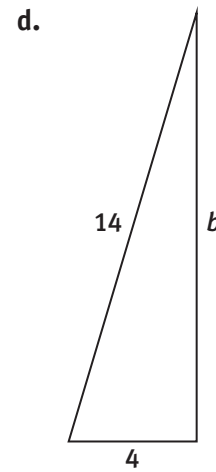
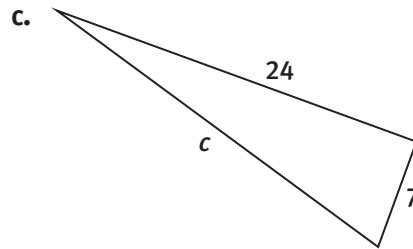
Try These A

Use the Pythagorean Theorem to find the unknown length to the nearest tenth.



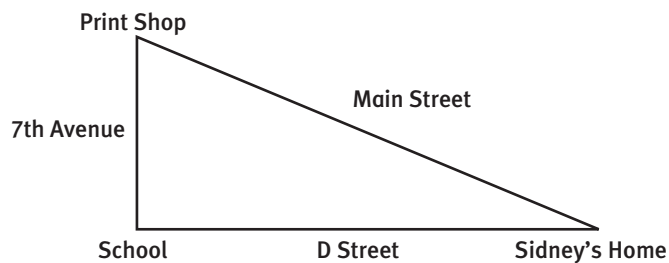
My Notes

My Notes



- e. Leg 1 = 6
Leg 2 = 10

1. Now that you know the relationship of the lengths of the three sides of any right triangle, you can figure out whether Sidney will make it to the print shop before it closes using the Pythagorean Theorem. Recall that Sidney leaves his house at 3:45 P.M. to try to make it to the print shop before 4:00 P.M. He starts biking down Main Street to the print shop. As he is pedaling, he wonders how far it is to the print shop. His house is 12 blocks away from the school and the print shop is five blocks away from the school. He can travel, at the most, one block per minute on his bike.



- a. How many blocks is it from the school to the print shop?
- b. How many blocks is it from the school to Sidney's home?
- c. How many block lengths down Main Street will Sidney have to bike to get to the print shop?

Lesson 22-2

Pythagorean Theorem: Missing Lengths

ACTIVITY 22

continued

d. **Model with mathematics.** Can Sidney make it to the print shop on time? Explain your reasoning.

2. When you used the Pythagorean Theorem to find the distance from Sidney's house to the print shop, the formula gave you the square of the distance. What did you have to do to get the actual distance?

My Notes

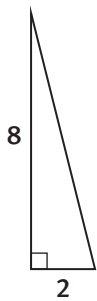
CONNECT TO AP

The Pythagorean Theorem is fundamental to the development of many more advanced mathematical topics such as the distance formula, complex numbers, and arc length of a curve.

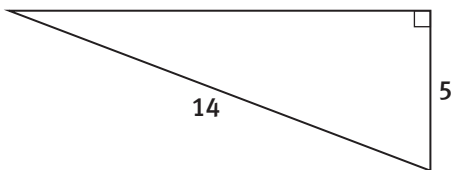
Check Your Understanding

Use the Pythagorean Theorem to find the unknown length to the nearest tenth.

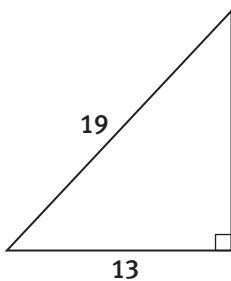
3.



4.



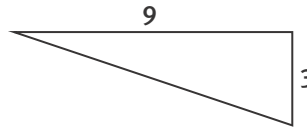
5.



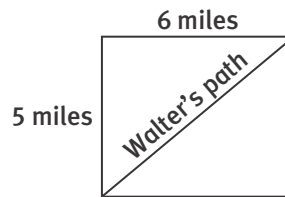
My Notes

LESSON 22-2 PRACTICE

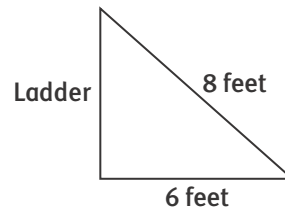
- Explain in your own words how the Pythagorean Theorem can be used to find a missing length of a right triangle.
- Find the length of the hypotenuse in this right triangle:



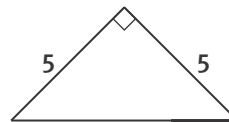
- Walter is riding his bike across a park as shown. How far does he travel?



- A playground slide measures 8 feet long. The slide ends 6 feet from the ladder. What is the length of the ladder?



- Critique the reasoning of others.** Shanti says she can use the Pythagorean Theorem to find the missing length on this isosceles triangle. Do you agree with her reasoning? Explain.

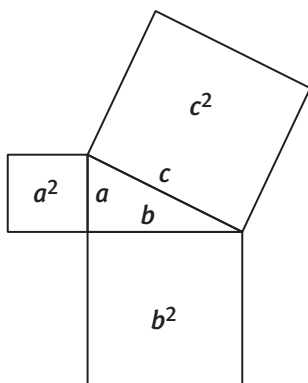


ACTIVITY 22 PRACTICE

Write your answers on notebook paper.
Show your work.

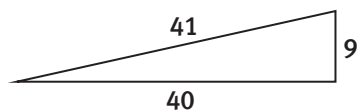
Lesson 22-1

- This diagram shows the squares of the lengths of the sides of a right triangle. Copy the table and refer to the diagram to complete.



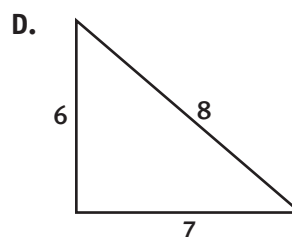
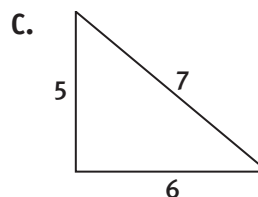
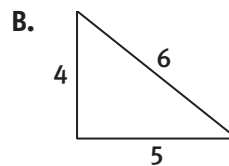
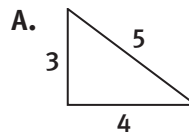
Case	a	b	c	a^2	b^2	c^2
1	3	4	5			
2				81	144	225
3				36	64	100
4	8	15	17			

- Find c^2 given a triangle whose legs measure 5 units and 8 units.
- Write the Pythagorean Theorem equation for this right triangle.



- If you know the lengths of the sides of a triangle, how might you use the Pythagorean Theorem to tell if the triangle is or is not a right triangle?

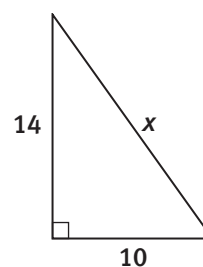
- Which of the following is a right triangle?



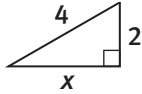
- Roman says the Pythagorean Theorem applies to all triangles. Do you agree with his statement? Explain your reasoning.

Lesson 22-2

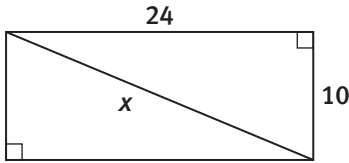
- Find x in the triangle below.



8. Find x in the triangle below.



9. Find x in the triangle below.



10. A painter uses a ladder to reach a second-story window on the house she is painting. The bottom of the window is 20 feet above the ground. The foot of the ladder is 15 feet from the house. How long is the ladder?
11. Which length is the greatest?
- the diagonal of a square with 4-in. sides
 - the hypotenuse of a right triangle with legs of length 3 in. and 4 in.
 - the diagonal of a rectangle with sides of 5 in. and 12 in.
 - the perimeter of a square with side lengths of 1 in.
12. A hiker leaves her camp in the morning. How far is she from camp after walking 9 miles west and then 10 miles north?
- 19 miles
 - 4.4 miles
 - 181 miles
 - 13.5 miles
13. A brick walkway forms the diagonal of a square playground. The walkway is 20 m long. To the nearest tenth of a meter, how long is one side of the playground?
14. The screen size of a television is measured along the diagonal of the screen from one corner to another. If a television has a length of 28 inches and a diagonal that measures 32 inches, what is the height of the television set to the nearest tenth?
15. Tim's cousin lives 8 blocks due south of his house. His grandmother lives 6 blocks due east of him. What is the distance in blocks from Tim's cousin's house to Tim's grandmother's house?
16. A rectangular garden is 6 meters wide and 12 meters long. Sean wants to build a walkway that goes along the diagonal of the garden. How long will the walkway be?

MATHEMATICAL PRACTICES

Attend to Precision

17. Use grid paper to draw a right triangle. Count the units for the legs. Calculate the length of the hypotenuse using the Pythagorean Theorem.

Diamond in the Rough

Lesson 23-1 The Pythagorean Theorem in Two and Three Dimensions

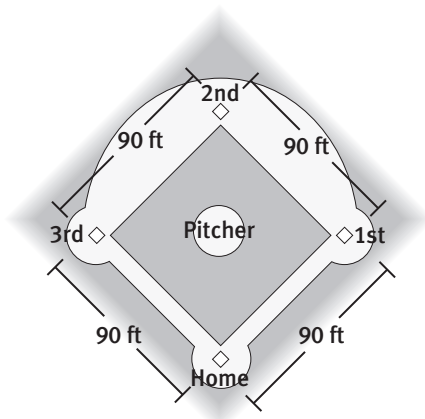
Learning Targets:

- Apply the Pythagorean Theorem to solve problems in two dimensions.
- Apply the Pythagorean Theorem to solve problems in three dimensions.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Close Reading, Paraphrasing, Identify a Subtask, Think-Pair-Share

Cameron is a catcher trying out for the school baseball team. He has played baseball in the community and is able to easily throw the ball from home plate to second base to throw out a runner trying to steal second base. However, the school baseball diamond is a regulation-size field and larger than the field he is accustomed to.

The distance between each consecutive base on a regulation baseball diamond is 90 feet and the baselines are perpendicular. The imaginary line from home plate to second base divides the baseball diamond into two right triangles. There is a relationship between the lengths of the three sides of any right triangle that might be helpful for determining if Cameron can throw across a regulation baseball diamond.



1. Sketch a diagram of a regulation baseball diamond showing the baselines and the imaginary line from home plate to second base. Identify and label the hypotenuse and legs of any right triangles. What are the lengths of the legs of the triangles?

My Notes

DISCUSSION GROUP TIPS

In discussion groups, read the text carefully to clarify the meaning of math terms and other vocabulary.

My Notes

MATH TIP

The Pythagorean Theorem states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs of the triangle.

MATH TIP

If you take the square root of a number that is not a perfect square, the result is a decimal number that does not terminate or repeat and is therefore an irrational number.

2. Write an equation that can be used to find the distance from home plate to second base.
3. **Use appropriate tools strategically.** Can the distance from home plate to second base be found without a calculator? Why or why not?
4. Is this value from Item 3 a rational or irrational number? Using a calculator, give the approximate length of the distance from home plate to second base.
5. If Cameron can throw the baseball 130 feet, will he be able to consistently throw out a runner trying to steal second base? Explain your reasoning.
6. On a regulation softball diamond, the distance between consecutive bases is 60 feet and the baselines are perpendicular.
 - a. Sketch and label a scale drawing of a softball diamond.
 - b. Use your sketch to approximate the distance from home plate to second base on a softball field. Show all your work.

Lesson 23-1

The Pythagorean Theorem in Two and Three Dimensions

ACTIVITY 23

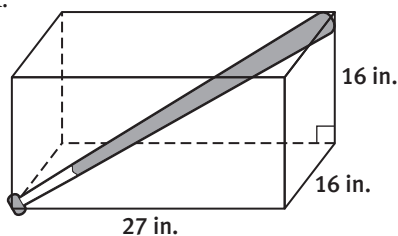
continued

Check Your Understanding

- A rectangular garden is 6 meters wide and 12 meters long. Sean wants to build a walkway that goes through the diagonal of the garden. How long will the walkway be? Round to the nearest hundredth.
- A rectangular computer screen has a diagonal length of 21 inches. The screen is 11 inches wide. To the nearest tenth of an inch, what is the length of the screen?

During summer vacation, Cameron's parents take him to see his favorite baseball team play. On their last day of vacation, he discovers that he will not be able to carry the autographed bat that he won home on the plane. His dad suggests that he speak to the concierge at the hotel about options for shipping the bat home.

The concierge has only one box that he thinks might be long enough. After measuring the dimensions of the box to be 16 in. \times 16 in. \times 27 in., the concierge apologizes for not having a box long enough for the 34 inch bat. Cameron thinks he might still be able to use the box. His idea is to put the bat in the box at an angle as shown in the diagram below. He wonders if the bat will fit in the box.



- The diagonal of the box is the hypotenuse of a right triangle. Outline this triangle in the diagram above.
- What are the lengths of the legs of this right triangle? Show any work needed to find these lengths.
- Find the length of the diagonal of the box. Show any necessary calculations.
- Will Cameron be able to use the box to ship his bat? Justify your response.

My Notes

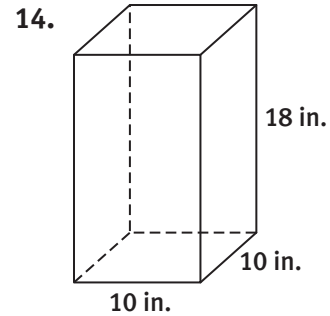
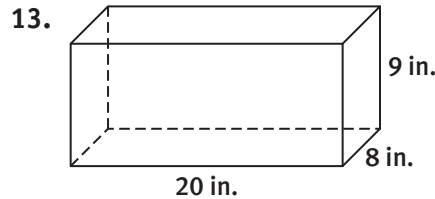
CONNECT TO TRAVEL

In a hotel, a concierge is a person who helps guests with various tasks ranging from restaurant reservations to travel plans.

My Notes

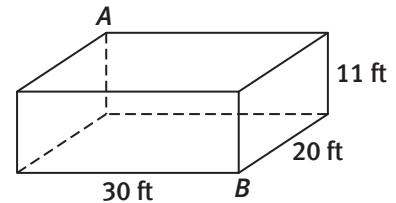
Check Your Understanding

Cameron brought some collapsible fishing rods on his vacation. Find the length of the longest fishing rod that he can fit in each of the boxes shown below. Round to the nearest tenth.

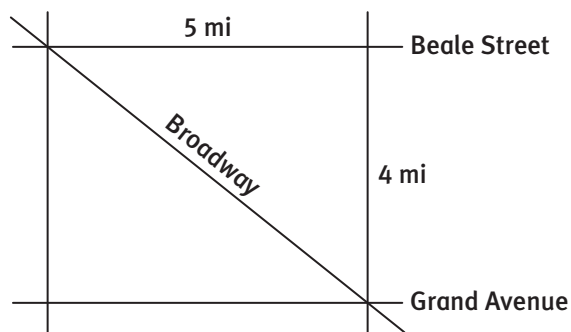


LESSON 23-1 PRACTICE

15. A rectangular photograph has a diagonal length of 18 centimeters. The photograph is 10 centimeters wide. What is the length of the photograph to the nearest hundredth of a centimeter?
16. A square window is 2 meters long on each side. To protect the window during a storm, Marisol plans to put a strip of duct tape along each diagonal of the window. To the nearest tenth of a meter, what is the total length of duct tape Marisol will need?
17. The figure shows the dimensions of a classroom. What is the distance that a moth travels if it flies in a straight line from point A to point B? Round to the nearest tenth.



18. **Make sense of problems.** A city employee is organizing a race down Broadway, from Beale Street to Grand Avenue. There will be a water station at the beginning and end of the race. There will also be water stations along the route, with no more than one mile between stations. What is the minimum number of water stations for this race?



My Notes

Cameron's coach wants to know approximately how far the players will run and slide as they go from base C back to base A .

1. Plot and label the bases on the coordinate plane on the previous page.
2. What is the distance between bases A and B ? What is the distance between the bases B and C ?
3. Can you use the same method that you used in Item 2 to find the distance between bases C and A ? Why or why not?
4. **Make use of structure.** Calculate the shortest distance between bases C and A . Explain and justify your method.

Check Your Understanding

Use the My Notes column on this page to plot the points to find the length of the hypotenuse in each right triangle. Round to the nearest tenth, if necessary.

5. $D(-3, 0)$, $E(0, 0)$, $F(0, 4)$
6. $G(-4, 2)$, $H(3, 2)$, $J(3, -2)$

Lesson 23-2

The Pythagorean Theorem and the Coordinate Plane

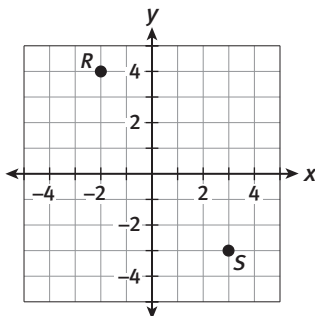
ACTIVITY 23

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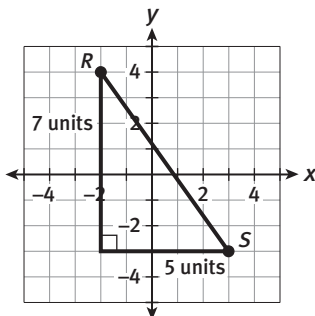
Example A

Find the distance between $R(-2, 4)$ and $S(3, -3)$.

Step 1: Plot the points.



Step 2: Draw a right triangle. Find the length of the legs.



Step 3: Use the Pythagorean Theorem to find the length of the hypotenuse.

$$RS^2 = 7^2 + 5^2$$

$$RS^2 = 74$$

$$RS \approx 8.6 \text{ units}$$

Solution: The distance between the points is approximately 8.6 units.

Try These A

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

- $(-1, 3)$ and $(2, 1)$
- $(6, 2)$ and $(-4, -2)$

My Notes

WRITING MATH

Write \overline{RS} to represent the line segment with endpoints R and S . Write RS to represent the length of the line segment or the distance between R and S .

My Notes

Check Your Understanding

- Carlos looked at the figure in Example A and said that there is a different way to draw a right triangle that has \overline{RS} as its hypotenuse. Draw a figure to show what Carlos means. Does this right triangle give the same result? Explain.
- Use what you know about triangles to explain why the answer to Example A is reasonable.

LESSON 23-2 PRACTICE

Find the length of the hypotenuse in each right triangle. Round to the nearest tenth, if necessary.

- $P(1, 3)$, $Q(5, 3)$, $R(5, -1)$
- $L(-3, 2)$, $M(-3, -3)$, $N(4, -3)$

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

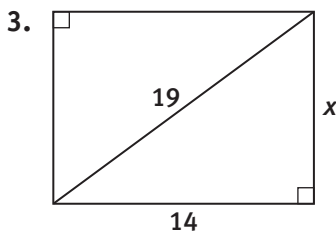
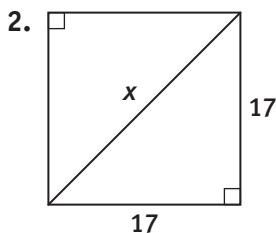
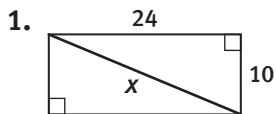
- $(-2, -2)$ and $(-1, 3)$
- $(1, -3)$ and $(6, 5)$
- During a drill, Cameron's coach has players sprint from $J(3, 2)$ to $K(-4, 2)$ to $L(-4, -3)$ and back to J . Each unit of the coordinate plane represents 10 feet. To the nearest foot, what is the total distance players sprint during this drill?
- Attend to precision.** On a map of Ayana's town, the library is located at $(-5, -3)$ and the middle school is located at $(0, 2)$. Each unit of the map represents one mile. Ayana wants to bike from the middle school to the library. She knows that it takes her about 5 minutes to bike one mile. Will she be able to make the trip in less than half an hour? Explain.

ACTIVITY 23 PRACTICE

Write your answers on notebook paper.
Show your work.

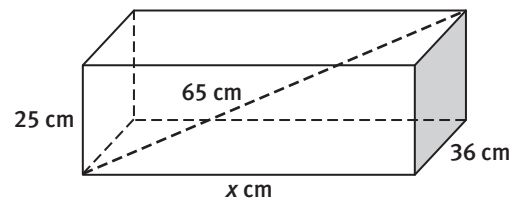
Lesson 23-1

Find x in each of the following figures. Round to the nearest tenth, if necessary.



4. Riyo wants to place a string of lights across the ceiling of her bedroom. The room is a rectangle that is 18 feet long and 15 feet wide. Her string of lights is 20 feet long. Will the string of lights be long enough to hang diagonally from one corner of the ceiling to the other? Explain.
5. Which of the following lengths is the greatest?
 - A. the diagonal of a square with 4-in. sides
 - B. the hypotenuse of a right triangle with legs of length 3 in. and 4 in.
 - C. the diagonal of a rectangle with sides of length 5 in. and 12 in.
 - D. the perimeter of a square with side lengths of 1 in.

6. Gary has a rectangular painting that is 21 inches wide and 36 inches long. He wants to place wire in the shape of an X on the back of the painting along its diagonals so that he can hang the painting on the wall. Which is the best estimate of the total amount of wire Gary will need?
 - A. 42 inches
 - B. 57 inches
 - C. 84 inches
 - D. 114 inches
7. What is the length of the longest fishing pole that will fit in a box with dimensions 18 in., 24 in., and 16 in.?
8. The box below has dimensions 25 cm, 36 cm, and x cm. The diagonal shown has a length of 65 cm. Find the value of x . Round to the nearest tenth, if necessary.



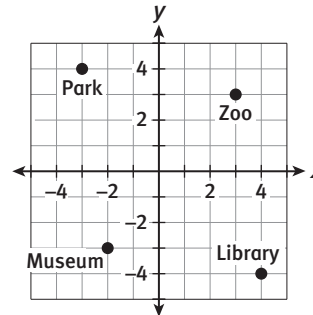
9. A brick walkway forms the diagonal of a square playground. The walkway is 20 m long. To the nearest tenth of a meter, how long is one side of the playground?
10. A television set's screen size is measured along the diagonal of the screen from one corner to another. If a television screen has a length of 28 inches and a diagonal that measures 32 inches, what is the height of the screen to the nearest tenth?
11. A rectangle has sides of length p and q . Which expression represents the length of the diagonal of the rectangle?
 - A. $2(p + q)$
 - B. $p^2 + q^2$
 - C. $\sqrt{p + q}$
 - D. $\sqrt{p^2 + q^2}$

ACTIVITY 23*continued***Applying the Pythagorean Theorem
Diamond in the Rough****Lesson 23-2**

For Items 12–17, find the distance between each pair of points. Round to the nearest tenth, if necessary.

12. $(0, 0)$ and $(3, 2)$
13. $(-3, -1)$ and $(0, 2)$
14. $(-1, 1)$ and $(3, -2)$
15. $(2, -1)$ and $(2, 5)$
16. $(6, -2)$ and $(-2, 4)$
17. $(-3, -5)$ and $(5, 5)$
18. Which is the best estimate of the distance between the points $A(4, -5)$ and $B(-2, 1)$?
 A. 7 units B. 7.5 units
 C. 8 units D. 8.5 units
19. Which point lies the farthest from the origin?
 A. $(-6, 0)$ B. $(-3, 8)$
 C. $(5, 1)$ D. $(-4, -3)$
20. How far from the origin is the point $(-2, -4)$? Round to the nearest tenth, if necessary.
21. For a baserunning drill, a coach places bases at $A(1, 1)$ and $B(4, 1)$, where each unit of the coordinate plane represents 10 feet. The coach wants to locate base C so that the distance from B to C is 40 feet and so that the three bases form a right triangle.
 - a. What is a possible location for base C ?
 - b. Is there more than one possibility for the location of base C ? Explain.
 - c. What is the distance from base A to base C ? Does this distance depend upon which of the possible locations for base C the coach chooses? Justify your response.

The coordinate plane shows a map of Elmville. Each unit of the coordinate plane represents one mile. Use the map for Items 22–24.



22. What is the distance from the zoo to the library? Round to the nearest tenth of a mile.
23. Assuming it is possible to walk between locations in a straight line, how much longer is it to walk from the museum to the zoo than to walk from the museum to the park?
24. Donnell plans to walk from the park to the library along a straight route. If he walks at 4 miles per hour, can he complete the walk in less than 2 hours? Explain.

MATHEMATICAL PRACTICES**Reason Abstractly and Quantitatively**

25. Consider the points $A(5, 0)$, $B(-3, 4)$, and $C(-4, 3)$.
 - a. Find the distance of each point from the origin.
 - b. Give the coordinates of four additional points that are this same distance from the origin.
 - c. Plot the given points and the points you named in part b.
 - d. Suppose you continued to plot points that are this same distance from the origin. What geometric figure would the points begin to form?

My Notes

4. Use paper clips to create triangles having the given side lengths. Use a corner of an index card to test the largest angle of each triangle for a right angle and predict whether or not the given triangles are right triangles. Draw and label a sketch of each triangle formed.

Triangle Side Lengths	Right Triangle?	Pictorial Representation
6, 8, 10		
5, 9, 10		
5, 12, 13		

Triangle Side Lengths	Right Triangle?	Pictorial Representation
4, 12, 14		
9, 15, 16		
8, 15, 17		

Lesson 24-1

The Converse of the Pythagorean Theorem

ACTIVITY 24

continued

5. Using $c^2 = a^2 + b^2$, where c is the longest side, support your predictions for each triangle in Item 4. Use the chart below to show your work.

Triangle Side Lengths	c^2	(?) = or \neq	$a^2 + b^2$	Prediction Correct?
6, 8, 10				
5, 9, 10				
5, 12, 13				
4, 12, 14				
9, 15, 16				
8, 15, 17				

My Notes

READING MATH

The symbol $=$ is read "is equal to" and the symbol \neq is read "is not equal to."

My Notes

- 6. Express regularity in repeated reasoning.** If the sides of a triangle satisfy the equation $c^2 = a^2 + b^2$, what can be said about the triangle? What must be true about c ?

The relationship that you have just explored is called the Converse of the Pythagorean Theorem. It states that if the sum of the squares of the two shorter sides of a triangle equal the square of the longest side, then the triangle is a right triangle.

Check Your Understanding

Tell whether each set of side lengths forms a right triangle. Justify your response.

7. 7, 24, 25

8. 6, 12, 13

LESSON 24-1 PRACTICE

Tell whether each set of side lengths forms a right triangle. Justify your response.

9. 8, 12, 16

10. 10, 24, 26

11. Isabella has sticks that are 10 cm, 11 cm, and 13 cm long. Can she place the sticks together to form a right triangle? Justify your answer.
12. The triangular sail of a toy sailboat is supposed to be a right triangle. The manufacturer says the sides of the sail have lengths of 4.5 inches, 6 inches, and 7 inches. Is the sail a right triangle? If not, how could you change one of the lengths to make it a right triangle?
13. **Model with mathematics.** Alan made a small four-sided table for his office. The opposite sides of the table are 27 inches long and 36 inches long. If the diagonal of the table measures 40 inches, does the table have right angles at the corners? Why or why not?

Learning Targets:

- Verify whether a set of whole numbers is a Pythagorean triple.
- Use a Pythagorean triple to generate a new Pythagorean triple.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Visualization, Discussion Group, Create Representations, Note Taking

A **Pythagorean triple** is a set of three whole numbers that satisfies the equation $c^2 = a^2 + b^2$.

- 1. Make use of structure.** Choose 3 Pythagorean triples from Lesson 24-1 and list them in the first column of the table below. Multiply each Pythagorean triple by 2. Is the new set of numbers a Pythagorean triple? Repeat by multiplying each original set of numbers by 3.

Pythagorean Triple	Multiply by 2	Pythagorean Triple?	Multiply by 3	Pythagorean Triple?

- 2.** What do you notice when you multiply each value in a Pythagorean triple by a whole-number constant? Make a conjecture based on your results in the table.

My Notes

My Notes

Check Your Understanding

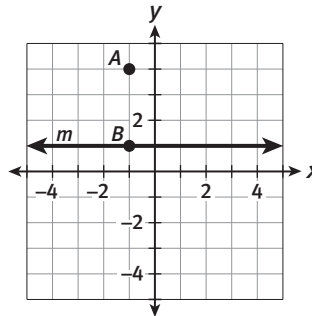
- How many Pythagorean triples can be created by multiplying the side lengths in a known triple by a constant? Explain your answer.
- Do the numbers 65, 156, and 169 form a Pythagorean triple? Why or why not?

LESSON 24-2 PRACTICE

- Below are sets of triangle side lengths. Sort the sets of lengths into two groups. Explain how you grouped the sets.

3, 4, 5	6, 8, 10	5, 12, 13	14, 48, 50
10, 24, 26	8, 15, 17	9, 12, 15	16, 30, 34
7, 24, 25	20, 48, 52	24, 45, 51	12, 16, 20

- What number forms a Pythagorean triple with 14 and 48?
- Point C is located on line m . What is the location of point C if the side lengths of $\triangle ABC$ form a Pythagorean triple? Is there more than one possibility? Explain.



- Lisa says that if you start with a Pythagorean triple and add the same whole number to each number in the set, then the new set of numbers will also be a Pythagorean triple. Explain why Lisa is correct or provide a counterexample to show that she is not correct.
- Critique the reasoning of others.** Devon knows that 5, 12, 13 is a Pythagorean triple. He states that he can form a new Pythagorean triple by multiplying each of these values by 1.5. Is Devon correct? Justify your answer.

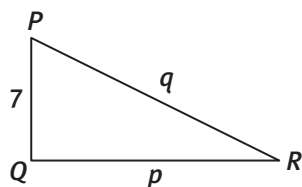
ACTIVITY 24 PRACTICE

Write your answers on notebook paper.

Show your work.

Lesson 24-1

- Is a triangle with sides measuring 9 feet, 12 feet, and 18 feet a right triangle? Justify your answer.
- Determine whether $\frac{4}{5}$, $\frac{3}{5}$, and 1 can be the sides of a right triangle. Justify your answer.
- The lengths of four straws are listed below. Which three of the straws can be placed together to form a right triangle? Why?
5 cm
6 cm
12 cm
13 cm
- The lengths of the three sides of a right triangle are three consecutive even integers. What are they?
- Which equation guarantees that $\triangle PQR$ is a right triangle?



- $q^2 + 49 = p$
- $q^2 - 7 = p^2$
- $q^2 - 49 = p^2$
- $q^2 + 7 = p$

Determine whether each statement is true or false. If the statement is false, explain why.

- If a triangle has sides of length 8 cm, 10 cm, and 12 cm, then the triangle does not contain a right angle.
- If you have sticks that are 15 in., 36 in., and 39 in. long, you can place the sticks together to form a triangle with three acute angles.
- A triangle that has sides of length 7.5 cm, 10 cm, and 12.5 cm must be a right triangle.
- The converse of the Pythagorean theorem says that in a right triangle the sum of the squares of the lengths of the legs equals the square of the hypotenuse.

Lesson 24-2

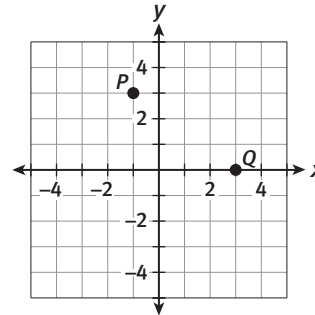
- Is 9, 40, 41 a Pythagorean triple? Explain your reasoning.
- The numbers 3, 4, 5 form a Pythagorean triple. Give four other Pythagorean triples that can be generated from this one.
- Keiko said that the numbers 3.6, 4.8, and 6 form a Pythagorean triple since $6^2 = 3.6^2 + 4.8^2$. Do you agree or disagree? Explain.
- Consider the following sets of whole numbers. Which sets form Pythagorean triples?
 - 6, 8, 10
 - 15, 36, 39
 - 10, 12, 14
 - 16, 30, 34
 - I only
 - II and III
 - III and IV
 - I, II, and IV

14. Which whole number should be included in the set $\{8, 15\}$ so that the three numbers form a Pythagorean triple?
A. 5 **B.** 12
C. 17 **D.** 19
15. Mario said the Pythagorean triple 7, 24, 25 is the only Pythagorean triple that includes the number 24. Do you agree or disagree? Justify your response.
16. Give an example of a Pythagorean triple that includes two prime numbers.
17. Explain the connection between Pythagorean triples and right triangles.

Determine whether each statement is always, sometimes, or never true.

18. A Pythagorean triple includes an odd number.
19. Two of the numbers in a Pythagorean triple are equal.
20. A Pythagorean triple includes a number greater than 3 and less than 4.
21. The greatest number in a Pythagorean triple can be the length of the hypotenuse of a right triangle while the other two numbers can be the lengths of the legs.

22. The lengths of the sides of $\triangle PQR$ form a Pythagorean triple. Which of the following could be the coordinates of point R ?



- A.** $(-3, -3)$ **B.** $(3, 3)$
C. $(3, -3)$ **D.** $(0, 3)$

MATHEMATICAL PRACTICES

Reason Abstractly and Quantitatively

23. Euclid's formula is a formula for generating Pythagorean triples. To use the formula, choose two whole numbers, m and n , with $m > n$. Then calculate the following values.

$$a = m^2 - n^2$$

$$b = 2mn$$

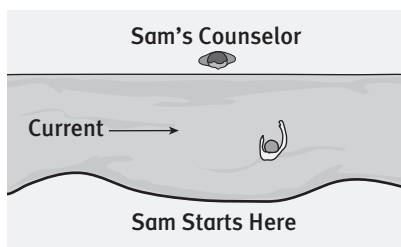
$$c = m^2 + n^2$$

- a.** Choose values for m and n . Then generate the numbers a , b , and c according to the formula. Is the resulting set of numbers a Pythagorean triple?
- b.** Does the formula work when $m = n$? Why or why not?

Sam is spending part of his summer vacation at Camp Euclid with some of his friends. On the first day of camp, they must pass an open-water swimming test to be allowed to use the canoes, kayaks, and personal watercraft. Sam and his friends must be able to swim across the river that they will be boating on.

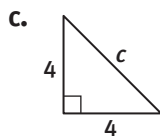
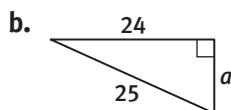
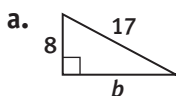
The river is 30 meters wide. On the day of the test, Sam begins on one bank and tries to swim directly across the river to the point on the opposite bank where his counselor is waiting. Because the river has a slight current, Sam ends up 35 meters downstream from his counselor.

- Copy and label the diagram for the problem situation.
- How far did Sam actually swim? Justify your answer.
- Sam's friend Alex started at the same spot but swam 50 meters. How far downstream was Alex from their counselor when he arrived at the opposite bank? Justify your answer.



In a lake fed by the river, a triangular area marked with buoys is roped off for swimming during free time at camp. The distances between each pair of buoys are 40 meters, 50 meters, and 60 meters.

- Draw and label a diagram for the problem situation.
- Is the swimming area a right triangle? Justify your answer.
- Find the missing side length in each of the following triangles. Show all your work.



- Determine which of the following sets of triangle side lengths form right triangles. Justify each response.
 - 9, 40, 41
 - 20, 21, 31
 - $\frac{6}{7}, \frac{8}{7}, \frac{10}{7}$
- After the swimming test, Alex makes his way back to camp. On a coordinate plane, Alex is at the point $(-4, 3)$ and camp is at the point $(3, -1)$. What is the shortest distance Alex will have to travel to get back to camp? Assume each unit of the coordinate plane represents one kilometer.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
The solution demonstrates these characteristics:				
Mathematics Knowledge and Thinking (Items 2, 3, 5, 6a-c, 7a-c, 8)	<ul style="list-style-type: none"> Using the Pythagorean Theorem to accurately find missing triangle side lengths and distance in the coordinate plane. Using the converse of the Pythagorean Theorem to correctly determine if a triangle is a right triangle. 	<ul style="list-style-type: none"> Using the Pythagorean Theorem to find missing triangle side lengths and distance in the coordinate plane with few errors. Using the converse of the Pythagorean Theorem to decide if a triangle is a right triangle. 	<ul style="list-style-type: none"> Difficulty in finding missing triangle side lengths and distance in the coordinate plane. Difficulty determining if a triangle is a right triangle. 	<ul style="list-style-type: none"> Little or no understanding of using the Pythagorean Theorem. Little or no understanding of using the converse of the Pythagorean Theorem.
Problem Solving (Items 2, 3, 5, 6, 7, 8)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer. 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but is correct. 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers. 	<ul style="list-style-type: none"> No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1, 4)	<ul style="list-style-type: none"> Precisely modeling a problem situation with an accurate diagram. 	<ul style="list-style-type: none"> Drawing a reasonably accurate diagram to model a problem situation. 	<ul style="list-style-type: none"> Difficulty drawing a diagram to model a problem situation. 	<ul style="list-style-type: none"> Drawing an incorrect diagram to model a problem situation.
Reasoning and Communication (Items 2, 3, 5, 7)	<ul style="list-style-type: none"> Correctly using the Pythagorean Theorem to justify answers to problems. 	<ul style="list-style-type: none"> Explaining an answer using the Pythagorean Theorem. 	<ul style="list-style-type: none"> Difficulty using the Pythagorean Theorem to justify answers. 	<ul style="list-style-type: none"> Little or no understanding of the Pythagorean Theorem.

Surface Area

Greenhouse Gardens

Lesson 25-1 Lateral and Surface Areas of Prisms

Learning Targets:

- Find the lateral and surface areas of rectangular prisms.
- Find the lateral and surface areas of triangular prisms.

SUGGESTED LEARNING STRATEGIES: Create Representations, Visualization, Think-Pair-Share

A greenhouse is a building used to grow plants. These buildings can vary widely in size and shape. By using a greenhouse, a gardener is able to grow a wider range of plants. The greenhouse shelters plants from weather and insects that can cause damage.

Marie and Ashton are planning to help build a greenhouse for their middle school. The local gardening club is donating funds and materials to get the greenhouse built.

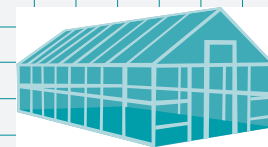
When a diagram like the one above accompanies a verbal description, use the visual along with the scenario to activate prior knowledge. For example, identify geometric shapes you see in the greenhouse and review formulas for finding perimeter and area of those figures. Review with your group any background information that will be useful in applying these concepts as you solve the item below.

1. Marie looks at the first design for the greenhouse. The design is a rectangular prism with a length of 12 feet, a width of 10 feet, and a height of 9 feet. Sketch a model of the greenhouse.

Marie and Ashton are asked to determine the cost of the glass that will be used to build the greenhouse. Glass will cover all of the walls of the greenhouse and the roof.

As you read Example A, clarify and make notes about any terms or descriptions you do not understand. Be sure to mark the text and label diagrams.

My Notes



My Notes

MATH TERMS

The **surface area** of a solid is the sum of the areas of all faces including the bases.

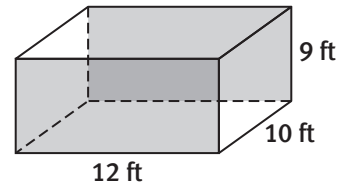
The **lateral area** of a solid is the sum of the areas of all faces excluding the bases. In a rectangular prism, you can assume the bases are the top and bottom faces, unless otherwise specified.

Example A

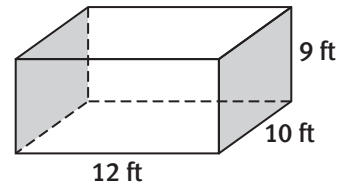
What is the surface area of the greenhouse that will be covered with glass? How would the lateral area differ from the surface area? Show all of your work.

Step 1: Identify the relevant faces. The **surface area** of a prism includes all faces. In this case, there is no glass on the bottom face, so find the area of the other five faces.

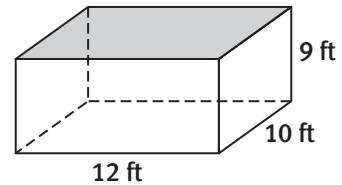
Step 2: Find the area of the front and back faces.
 $2(12 \times 9) = 216 \text{ ft}^2$



Step 3: Find the area of the left and right faces.
 $2(10 \times 9) = 180 \text{ ft}^2$



Step 4: Find the area of the top.
 $1(12 \times 10) = 120 \text{ ft}^2$

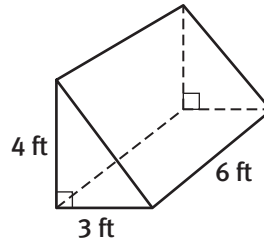


Step 5: Add the areas.
 $216 + 180 + 120 = 516 \text{ ft}^2$

Solution: 516 ft^2 will be covered with glass. The **lateral area** does not include the top or bottom faces, so the lateral area is $216 + 180 = 396 \text{ ft}^2$.

My Notes

3. In the right triangular prism below, mark the bases with a B . What two-dimensional shapes make up the lateral area of the figure?



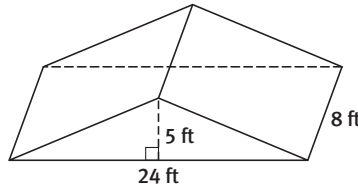
4. Ashton is building containers to hold plant food in the greenhouse. The design for these containers is shown above. Ashton will use plywood to cover the lateral area of the right triangular prism. What is the lateral area of the container? Show your work.

5. **Attend to precision.** Marie is working with Ashton on building the containers. She suggests that Ashton cover the bases with plywood as well. Ashton realizes he needs to find the total surface area of the right triangular prism to determine how much plywood he needs. Calculate the surface area of the container.

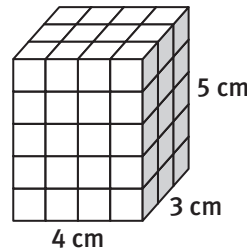
My Notes

LESSON 25-1 PRACTICE

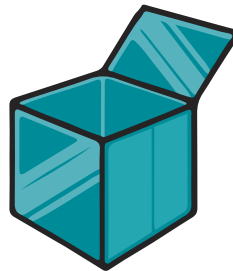
10. A cube is a rectangular prism with square faces. Suppose a cube has edges 9 cm long. What is its lateral area? What is its surface area?
11. Find the lateral area and surface area of the triangular prism.



12. The prism shown here is made from centimeter cubes. Find the lateral and surface area of the prism.



13. A gift box is a cube that measures 8 inches by 8 inches on all sides. What is the lateral and surface area of the gift box without the lid?



14. **Construct viable arguments.** Which cereal box requires more cardboard to manufacture? Justify your answer.

A.



depth = 3 in.
width = 9 in.
height = 12 in.

B.



depth = 2 in.
width = 7 in.
height = 14 in.

15. Use 8 cubes to create as many prisms as you can. Find and describe the prism with the least surface area and the greatest surface area.

My Notes

Example A

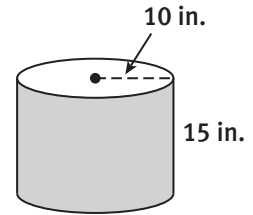
Calculate the surface area of the plant container from Item 2.

Step 1: Calculate the lateral area.

The lateral area is the area of the rectangle that covers the curved surface of the cylinder.

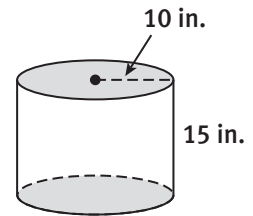
$$LA = (\text{height of cylinder}) \times (\text{circumference of base})$$

$$= (15)(2\pi \cdot 10) = 300\pi$$



Step 2: Calculate the area of the bases.

Each base is a circle with area πr^2 .
 The area of each base is $\pi(10)^2 = 100\pi$.
 So the total area of the two bases is $2 \cdot 100\pi = 200\pi$.



Step 3: Add the lateral area and the area of the bases.

$$300\pi + 200\pi = 500\pi \approx 1570.8 \text{ in.}^2$$

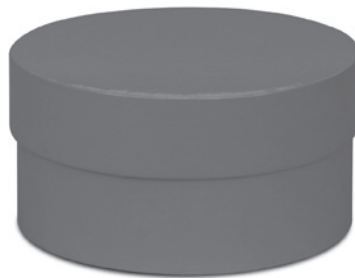
Solution: The surface area is $500\pi \text{ in.}^2$ or approximately $1,570.8 \text{ in.}^2$.

Try These A

Find the lateral area and surface area of the objects below. Give your answers in terms of π and rounded to the nearest tenth.

- a. a cylindrical hat box with diameter 30 cm and height 20 cm

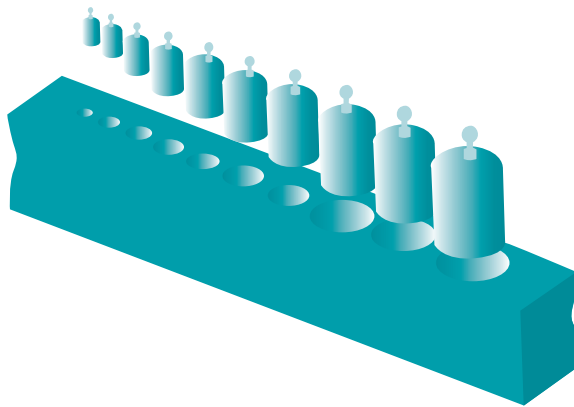
- b. a six-pack of juice cans where each can has a radius of 2 inches and a height of 7 inches



My Notes

LESSON 25-2 PRACTICE

This cylinder activity is used to promote fine-motor skills and spatial learning in young children. Use the figure for Items 8–11 and leave your answers in terms of π .



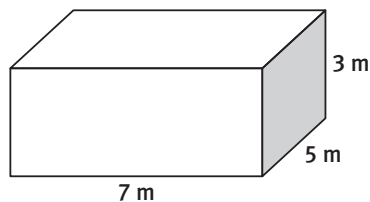
8. Calculate the lateral area of the largest peg to the far right of the diagram if its height is 2.5 inches and radius is 2 inches.
9. What is the surface area of the largest peg?
10. What is the lateral area of the smallest peg to the far left of the diagram if its height is $\frac{1}{2}$ inch and radius is $\frac{1}{4}$ inch?
11. What is the surface area of the smallest peg?
12. **Model with mathematics.** A water bottle has a label that is 3.5 inches high. If the bottle has a radius of 2 inches, how much paper would be needed to put labels on one dozen water bottles? Round to the nearest tenth.

ACTIVITY 25 PRACTICE

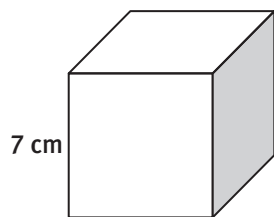
Write your answers on notebook paper.
Show your work.

Lesson 25-1

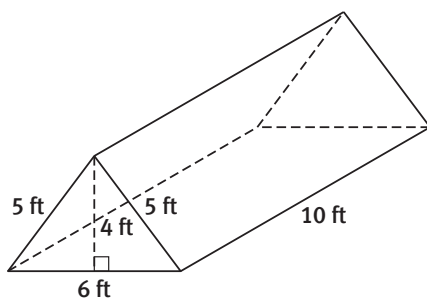
1. Find the surface area of the rectangular prism shown below.



2. Find the surface area of the cube shown below.

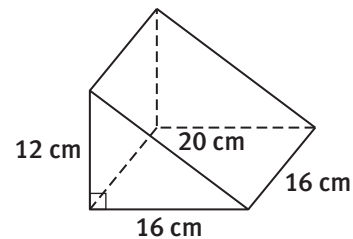


3. The dimensions of a nylon tent are shown in the figure.

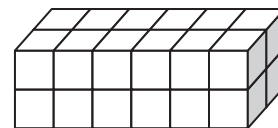


- How much nylon is needed to make the sides and floor of the tent?
 - How much nylon is needed to make the triangular flaps at the front and back of the tent?
 - What is the surface area of the triangular prism? How is this related to your responses to parts a and b?
4. A gift box is 6 inches long, 3 inches wide, and 3 inches tall.
- How much paper is needed to wrap the box? Assume the box is wrapped with the minimum amount of paper and no overlap.
 - How much wrapping paper should you buy to wrap the box if you assume you will need 15% extra for waste and overlap?

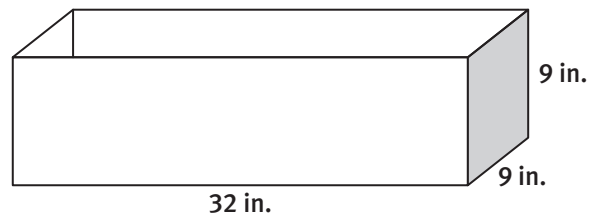
5. Find the lateral area and surface area of the triangular prism.



The figure below is made from one-inch cubes. Use the figure for Items 6 and 7.



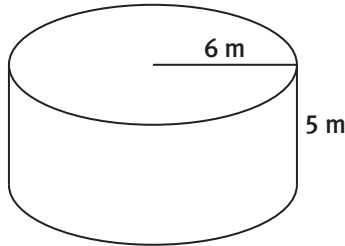
- What are the dimensions of the figure?
- What is the surface area of the figure?
- A cube has a surface area of 96 m^2 . What is the length of each edge of the cube?
 - 2 m
 - 4 m
 - 6 m
 - 8 m
- A window box for flowers is a rectangular prism with an open top, as shown. Tyrell wants to coat the inside and outside of the box with a special varnish that will protect it from the effects of water, cold, and harsh weather. The varnish comes in cans that can cover 500 square inches. How many cans of the varnish should Tyrell buy? Explain your answer.



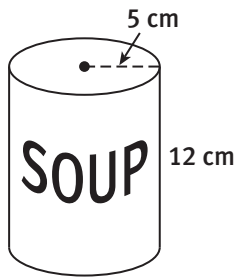
10. Which is the best estimate of the lateral area of a cube with edges that are 2.1 inches long?
- 9 in.^2
 - 16 in.^2
 - 25 in.^2
 - 36 in.^2

Lesson 25-2

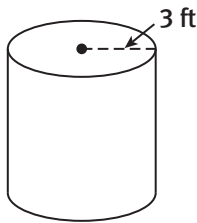
11. Find the lateral area and surface area of the cylinder. Leave your answers in terms of π .



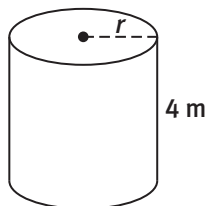
12. What is the area of the label on the soup can shown below? Round to the nearest tenth.



13. An orange juice can has a diameter of 4 inches and a height of 7 inches. The curved surface of the can is painted orange. How much paint is needed?
 A. $14\pi \text{ in.}^2$ B. $16\pi \text{ in.}^2$
 C. $28\pi \text{ in.}^2$ D. $36\pi \text{ in.}^2$
14. The height of the cylinder shown below is twice the radius. What is the lateral area of the cylinder? Round to the nearest tenth.

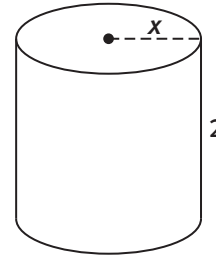


15. The lateral area of the cylinder shown below is $12\pi \text{ m}^2$. What is the radius of the cylinder?

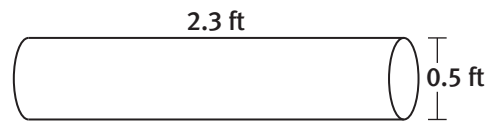


- A. 1.5 m B. 3 m
 C. 6 m D. 8 m

16. Which expression best represents the surface area of the cylinder shown below?



- A. $2\pi x + 2\pi x^2$ B. $2\pi x + 4\pi x^2$
 C. $4\pi x + 4\pi x^2$ D. $4\pi x + 2\pi x^2$
17. Mei is designing a cylindrical container for her ceramics class. The container will be open on top. She is considering a container with a radius of 5 cm and a height of 8 cm.
- Find the lateral area of the container. Leave your answer in terms of π .
 - What is the total area of the container? Leave your answer in terms of π .
 - Mei's friend Victor states that if she doubles the radius of the container she will double the total area of the container. Do you agree or disagree? Justify your response.
18. A pipe is made from a thin sheet of copper. The dimensions of the pipe are shown below. What is the amount of copper needed to make the pipe? Round to the nearest tenth.



MATHEMATICAL PRACTICES
Look For and Express Regularity in Repeated Reasoning

19. Consider a cylinder with a height of 1 cm.
- Find the lateral area of the cylinder if the radius is 2 cm, 4 cm, 8 cm, and 16 cm. Leave your answers in terms of π .
 - Use your results to make a conjecture about what happens to the lateral area of a cylinder when the radius is doubled.

My Notes

MATH TIP

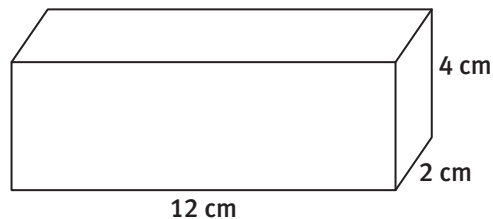
The formula for the volume V of a prism is $V = Bh$, where B is the area of the base and h is the height. For a rectangular prism with length ℓ , width w , and height h , the formula may be written as $V = \ell wh$.

MATH TIP

The formula for the volume V of a pyramid with base area B and height h is $V = \frac{1}{3}Bh$.

Example A

Find the volume of the rectangular prism shown below.



Step 1: Write the volume formula, $V = \ell wh$. Identify the values of the variables.

$$\ell = 12 \text{ cm}, w = 2 \text{ cm}, \text{ and } h = 4 \text{ cm}$$

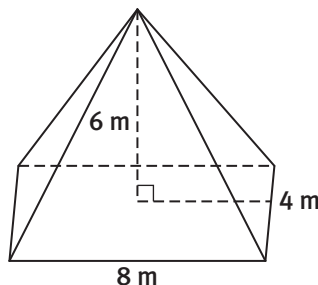
Step 2: Substitute the dimensions into the formula.

$$\begin{aligned} V &= \ell wh \\ &= 12 \cdot 2 \cdot 4 = 96 \end{aligned}$$

Solution: $V = 96 \text{ cm}^3$

Example B

Find the volume of the rectangular pyramid shown below.



Step 1: Write the volume formula, $V = \frac{1}{3}Bh$. Identify the known values of the variables.

$$h = 6 \text{ m}$$

Step 2: Calculate the area of the base.

$$B = 8 \times 4 = 32 \text{ m}^2$$

Step 3: Substitute the dimensions into the formula.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(32)(6) = 64 \end{aligned}$$

Solution: $V = 64 \text{ m}^3$

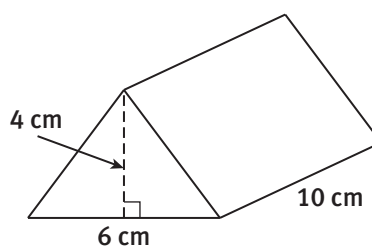
My Notes

Check Your Understanding

- Draw a square pyramid with a height of 8 centimeters and base side lengths of 6 centimeters. Find the volume.
- Draw a cube with side lengths of 4 inches. Find the volume.

LESSON 26-1 PRACTICE

- Julian measures the edges of a box in millimeters. What units should he use for the surface area of the box? What units should he use for the volume of the box?
- Find the volume of a triangular prism with a base area of 14 square centimeters and a height of 5 centimeters.
- A triangular pyramid has a volume of 20 in.^3 . The base area of the pyramid is 6 in.^2 . What is the height of the pyramid?
- Find the volume of the solid shown below.



- Reason quantitatively.** A toy manufacturer makes alphabet blocks in the shape of cubes with a side length of 1 inch.
 - The manufacturer plans to pack the blocks in a box that is a rectangular prism. The box is 7 inches long, 4 inches wide, and 3 inches tall. What is the volume of the box?
 - Suppose the manufacturer packs the blocks efficiently, so that as many blocks fit in the box as possible. How many blocks can fit? Describe how they would be packed.
 - Describe the connection between your answers to parts a and b.

Learning Targets:

- Apply the formula for the volume of a cone.
- Apply the formula for the volume of a cylinder.
- Apply the formula for the volume of a sphere.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Group Presentation, Quickwrite, Visualization

Shayla's friend Shelly continues to help Shayla prepare for the contest by showing her how to calculate the amount of sand needed to build various solids.

Example A

Find the volume of the cylinder.

Step 1: Write the volume formula, $V = Bh$.
 Identify the known values of the variables.

$$h = 11 \text{ in.}$$

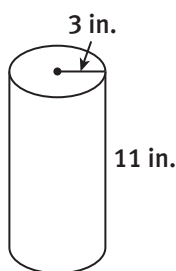
Step 2: Calculate the area of the base.

$$B = \pi r^2 = \pi(3)^2 = 9\pi \text{ in.}^2$$

Step 3: Substitute the dimensions into the formula.

$$\begin{aligned} V &= Bh \\ &= 9\pi(11) \\ &= 99\pi \end{aligned}$$

Solution: $V = 99\pi \text{ in.}^3 \approx 311.02 \text{ in.}^3$



Example B

Find the volume of the cone.

Step 1: Write the volume formula, $V = \frac{1}{3}Bh$.
 Identify the known values of the variables.

$$h = 7 \text{ mm}$$

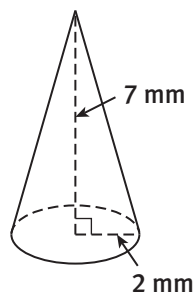
Step 2: Calculate the area of the base.

$$B = \pi r^2 = \pi(2)^2 = 4\pi \text{ mm}^2$$

Step 3: Substitute the dimensions into the formula.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(4\pi)(7) \\ &= \frac{28\pi}{3} \end{aligned}$$

Solution: $V = \frac{28\pi}{3} \text{ mm}^3 \approx 29.32 \text{ mm}^3$



My Notes

MATH TIP

The formula for the volume V of a cylinder is $V = Bh$, where B is the area of the base and h is the height. Since $B = \pi r^2$, the formula may be written as $V = \pi r^2 h$.

MATH TIP

The formula for the volume V of a cone is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. Since $B = \pi r^2$, the formula may be written as $V = \frac{1}{3}\pi r^2 h$.

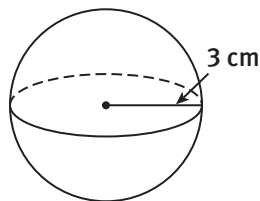
Compare the volume formulas for a cylinder and a cone. How are they the same? How are they different?

MATH TIP

The formula for the volume V of a sphere is $V = \frac{4}{3}\pi r^3$

Example C

Find the volume of the sphere shown below.



Step 1: Write the volume formula, $V = \frac{4}{3}\pi r^3$.

Identify the values of the variables.

$$r = 3 \text{ cm}$$

Step 2: Substitute the value of r into the formula.

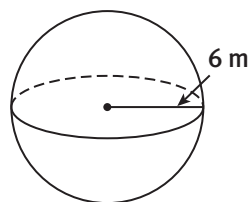
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(3)^3 \\ &= 36\pi \end{aligned}$$

Solution: $V = 36\pi \text{ cm}^3 \approx 113.10 \text{ cm}^3$

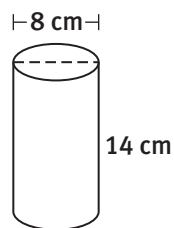
Try These A–B–C

Find the volume of each solid. Leave your answers in terms of π and round to the nearest tenth.

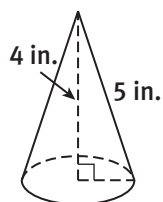
a.



b.



c.



Lesson 26-2

Volumes of Cylinders, Cones, and Spheres

ACTIVITY 26

continued

1. Shayla begins work on the cylindrical towers on either side of the front wall.
 - a. Draw and label a sketch of one of the cylinders used to create the two towers if the diameter of each cylinder is 10 inches and the height of each cylinder is 28 inches.
 - b. Use the volume formula for a cylinder to find the number of cubic inches of sand needed to build the two congruent cylindrical towers.
2. Shelly tells Shayla that she uses the formula $V = Bh$ to find the volume of both cylinders and prisms. Why does this work?
3. Now Shayla works on the turrets.
 - a. Draw and label a sketch of one of the cones used to create the turrets if the diameter of the base of each cone is 10 inches and the height of each cone is 16 inches.

- b. **Attend to precision.** Use the volume formula for a cone to find how many cubic inches of sand are needed to build the two congruent conical turrets. How many cubic feet are needed?

My Notes

MATH TIP

$$1 \text{ ft}^3 = 1,728 \text{ in.}^3$$

My Notes

4. Use the volume formula for a sphere to find how many cubic inches of sand are needed to build the three congruent decorative hemispheres on top of the wall if the radius of each hemisphere is 2 inches.

5. Finally, Shayla considers the posts in front of the drawbridge.
 - a. Draw and label a sketch of one of the posts in front of the drawbridge if the diameter of the base of the cylinder is 12 inches and the height of the cylinder is 15 inches. The sphere on top of each post has the same radius as the cylinder.

 - b. Use the volume formulas for a sphere and cylinder to find how many cubic inches of sand are needed to build the two posts in front of the drawbridge. How many cubic feet are needed?

Check Your Understanding

6. Draw a cone with a height of 12 cm and a radius of 5 cm. Find the volume.
7. Draw a sphere with a radius of 10 ft. Find the volume.
8.
 - a. Draw a cylinder with a height of 9 in. and a diameter of 4 in. Find the volume.
 - b. Draw a cone with a height of 9 in. and a diameter of 4 in. Find the volume.
 - c. How many times the volume of the cone is the volume of the cylinder?
 - d. State a rule for relating the volumes of a cylinder and cone that have the same height and diameter.
 - e. Would your rule also apply to the volumes of a cylinder and cone that have the same height and radius?

My Notes

Learning Targets:

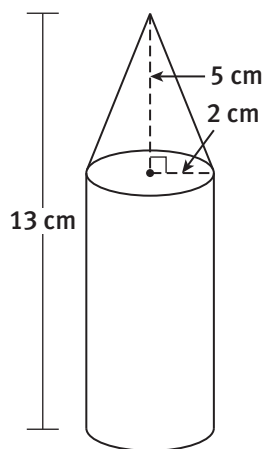
- Decompose composite solids into simpler three-dimensional figures.
- Find the volume of composite solids.

SUGGESTED LEARNING STRATEGIES: Visualization, Identify a Subtask, Think-Pair-Share, Group Presentation

A **composite solid** is a solid that consists of two or more simpler solids, such as prisms, pyramids, cylinders, cones, or hemispheres.

Example A

Find the volume of the composite solid shown below.



Step 1: Identify the solids that make up the composite figure. The composite figure consists of a cone and a cylinder.

Step 2: Find the volume of the cone.

$$\begin{aligned} V &= \frac{1}{3} Bh \\ &= \frac{1}{3} \pi (2)^2 (5) \\ &= \frac{20\pi}{3} \approx 20.9 \text{ cm}^3 \end{aligned}$$

Step 3: Find the volume of the cylinder.

Note that the height of the cylinder is $13 - 5 = 8$ cm.

$$\begin{aligned} V &= Bh \\ &= \pi (2)^2 (8) \\ &= 32\pi \approx 100.5 \text{ cm}^3 \end{aligned}$$

Step 4: Add the volume of the cone and the cylinder.

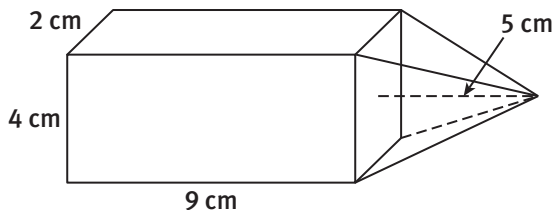
$$V = 20.9 + 100.5 = 121.4 \text{ cm}^3$$

Solution: $V \approx 121.4 \text{ cm}^3$

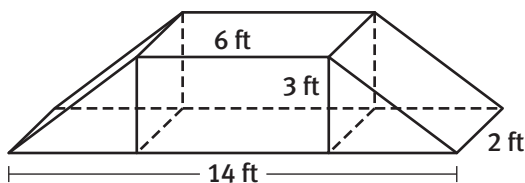
My Notes

LESSON 26-3 PRACTICE

6. Describe a composite solid you have seen in the real world. Explain how the composite figure is made up of simpler solids.
7. Find the volume of the composite solid shown below.



8. Find the volume of a composite figure comprised of two cones that are joined at the congruent circular bases, where one cone has a base radius of 8 inches and a height of 14 inches and the other cone has a height of 10 inches. Round to the nearest tenth.
9. A portable barrier that is used at construction sites is composed of three prisms, as shown below.



- a. What is the volume of the barrier?
 - b. The barrier is made of hollow, lightweight plastic for easy transportation. Once the barrier is placed at the construction site, it is filled with water. Given that water weighs 62.4 pounds per cubic foot, what is the weight of the barrier when it is filled?
10. **Construct viable arguments.** If Shayla builds a sand castle with dimensions twice as large as the dimensions of Shelly's winning castle, will Shayla need twice as much sand? Provide an argument to justify your response.

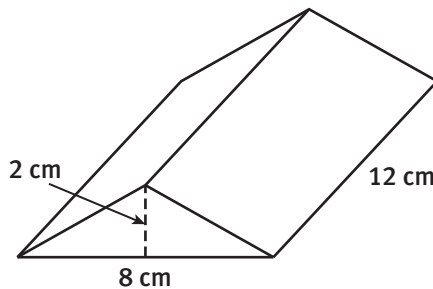
ACTIVITY 26 PRACTICE

Write your answers on notebook paper.

Show your work.

Lesson 26-1

1. Find the volume of a rectangular prism with a length of 5 inches, width of 8 inches, and height of 6 inches.
2. Find the volume of a cube with side lengths of 7.1 millimeters.
3. Find the volume of a square pyramid with a base edge length of 12 centimeters and a height of 20 centimeters.
4. Find the volume of the solid shown below.



5. A rectangular prism has a volume of 80 cubic feet. The length of the prism is 8 feet and the height of the prism is 4 feet. What is the width of the prism?
6. Jayden has a planter box in the shape of a cube. Each edge is 1.5 feet long. He fills the box with sand that weighs 100 pounds per cubic foot. Which of the following is the best estimate of the weight of the sand in the box once it is filled?

A. 150 pounds	B. 230 pounds
C. 300 pounds	D. 330 pounds
7. A cube has edges of length 6 inches. Casey calculates the surface area and the volume of the cube and states that the surface area equals the volume. Do you agree or disagree? Explain.

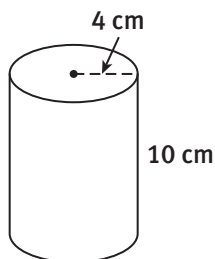
8. A square pyramid has a volume of 60 cubic meters. The height of the pyramid is 5 meters.
 - a. What is the area of the base of the pyramid?
 - b. What is the length of each edge of the square base?
9. A square pyramid has edges of length p and a height of p as well. Which expression represents the volume of the pyramid?

A. $\frac{1}{3}p^3$	B. $\frac{1}{3}p^2$
C. $\frac{1}{3}p^2 + p$	D. $\frac{1}{3}p + p^2$

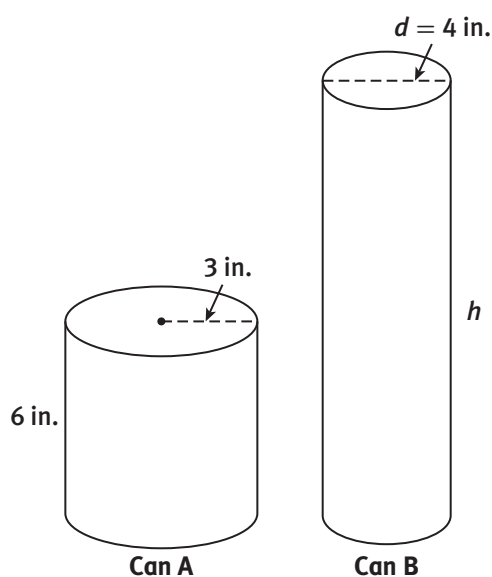
Lesson 26-2

10. Find the volume of a cone with a radius of 3 inches and a height of 12 inches. Round to the nearest tenth.
11. Find the volume of a sphere with a radius of 9 centimeters. Round to the nearest tenth.
12. Find the volume of a cone having a base circumference of 36π meters and height of 12 meters. Leave your answer in terms of π .
13. What is the formula for the volume of a cone with radius r and a height of $2r$?
14. A regulation NBA basketball has a diameter of 9.4 inches. What is the volume of one of these basketballs? Round to the nearest tenth.
15. A cylinder has a volume of 18π cubic inches. The radius of the cylinder is 3 inches. What is the height of the cylinder?

16. Which is the best estimate of the amount of soup that can fit in a soup can with the dimensions shown below?



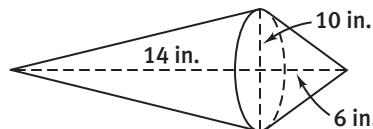
- A. 60 cm^3 B. 125 cm^3
 C. 170 cm^3 D. 500 cm^3
17. You buy two cylindrical cans of juice, as shown in the figure below. Each can holds the same amount of juice. What is the height of can B?



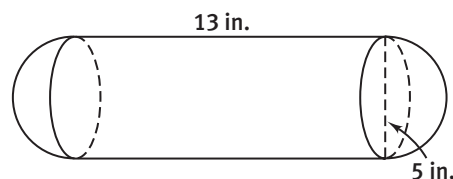
18. Which of these solids has the greatest volume?
 A. a cylinder with radius 3 cm and height 3 cm
 B. a cone with radius 3 cm and height 3 cm
 C. a sphere with radius 3 cm
 D. a cube with edges 3 cm long
19. A cylindrical glass has a radius of 3 cm and height of 14 cm. Elena pours water into the glass to a height of 8 cm.
 a. What is the volume of the water in the glass?
 b. What is the volume of the empty space in the glass?

Lesson 26-3

20. Find the volume of the composite solid below. Round to the nearest tenth.



21. Find the volume of the composite solid below. Round to the nearest tenth.



22. Create a sketch of a composite solid with a total volume greater than 500 cm^3 . Give the volume of the figure.
23. A composite solid consists of a cube with edges of length 6 cm and a square pyramid with base edges of length 6 cm and a height of 6 cm. Which is the best estimate of the volume of the solid?
 A. 100 cm^3 B. 200 cm^3
 C. 300 cm^3 D. 400 cm^3

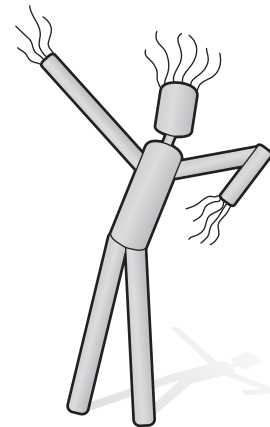
MATHEMATICAL PRACTICES

Use Appropriate Tools Strategically

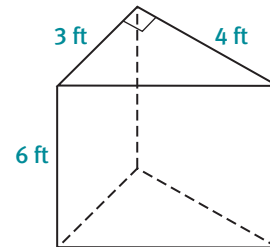
24. Can rounding make a difference in your results when you calculate a volume? Consider a sphere with a radius of 3.9 inches.
 a. Calculate the volume of the sphere by first finding r^3 . Then round to the nearest tenth. Calculate the volume using this value of r^3 and 3.14 for π .
 b. Now calculate the volume without rounding the value of r^3 and by using the π key on your calculator.
 c. How do the results compare? Which value do you think is more accurate? Why?

A group of students who will be attending the new Plato Middle School want to find a way to welcome the entire student body on the first day of school. After some investigation, the students decide an air dancer is a good idea and begin brainstorming ideas. The design they finally agree on has two cylindrical legs, a rectangular prism for a body, two right triangular prisms for arms, a cylindrical neck, a spherical head, and a cone-shaped hat.

Note: The drawing at the right does not necessarily represent the design that the students chose. To complete the items below, make your own drawing, showing the correct shape for each body part.



1. Sketch the air dancer design that the students chose.
2. The students must consider fans to keep a certain amount of air moving in the dancer. In order to determine the amount of air needed to inflate the air dancer, the students must calculate the volume.
 - a. Find the volume of the cylindrical legs if each one is 10 feet tall and 2 feet in diameter.
 - b. Find the volume of the rectangular prism to be used for the body. The dimensions are 6 feet long, 4 feet wide, and 8 feet high.
 - c. Find the volume of the right triangular prisms used for arms. A diagram for one of the arms is shown to the right.
 - d. Find the volume of the cylindrical neck if it is 2 feet tall and has a diameter of 1.5 feet.
 - e. Find the volume of the spherical head if it has a radius of 3 feet.
 - f. Find the volume of the cone-shaped hat if it has a radius of 3 feet and a height of 4 feet.
 - g. What is the total volume of the air dancer?
3.
 - a. Find the lateral area of one of the cylindrical legs using the dimensions from Item 2a.
 - b. Find the lateral area of each of the right triangular prisms that are used for arms using the dimensions from Item 2c.
 - c. The air dancer will be placed on a box that is a rectangular prism. The dimensions of the box are 12 feet long by 6 feet wide by 4 feet high. What is the total surface area of the box?



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
Mathematics Knowledge and Thinking (Items 1, 2a-g, 3a-c)	<ul style="list-style-type: none"> Accurately and efficiently finding the surface area and volume of three-dimensional figures. 	<ul style="list-style-type: none"> Finding the surface area and volume of three-dimensional figures with few, if any, errors. 	<ul style="list-style-type: none"> Difficulty finding the surface area and volume of three-dimensional figures. 	<ul style="list-style-type: none"> No understanding of finding the surface area and volume of three-dimensional figures.
Problem Solving (Items 2a-g, 3a-c)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer. 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but results in a correct answer. 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers. 	<ul style="list-style-type: none"> No clear strategy when solving problems.
Mathematical Modeling / Representations (Item 1)	<ul style="list-style-type: none"> Precisely modeling a problem situation with an accurate diagram. 	<ul style="list-style-type: none"> Drawing a reasonably accurate diagram to model a problem situation. 	<ul style="list-style-type: none"> Difficulty drawing a diagram to model a problem situation. 	<ul style="list-style-type: none"> Drawing an incorrect diagram to model a problem situation.
Reasoning and Communication (Items 3a-c)	<ul style="list-style-type: none"> Correctly understanding the difference between total surface area and lateral surface area. 	<ul style="list-style-type: none"> Distinguishing between total surface area and lateral surface area. 	<ul style="list-style-type: none"> Confusion in distinguishing between total surface area and lateral surface area. 	<ul style="list-style-type: none"> No understanding of the difference between total surface area and lateral surface area.